# Exploration of 2D and 3D Environments using Voronoi Transform and Fast Marching Method

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**3 authors:**


Santiago Garrido
University Carlos III de Madrid



Luis E Moreno
University Carlos III de Madrid


Dolores Blanco
University Carlos III de Madrid

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# Exploration of 2D and 3D Environments using Voronoi Transform and Fast Marching Method

**S. Garrido · L. Moreno · D. Blanco**

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**Abstract** Robot navigation in unknown environments requires an efficient exploration method. Exploration involves not only to determine towards the robot must to move but also motion planning, and simultaneous localization and mapping processes. The final goal of the exploration task is to build a map of the environment that previously the robot didn't know. This work proposes the Voronoi Fast Marching method, that uses a Fast Marching technique on the Logarithm of the Extended Voronoi Transform of the environment's image provided by sensors, to determine a motion plan. The Logarithm of the Extended Voronoi Transform imitates the repulsive electric potential from walls and obstacles, and the Fast Marching Method propagates a wave over that potential map. The trajectory is calculated by the gradient method. The robot is directed towards the most unexplored and free zones of the environment so as to be able to explore all the workspace. Finally, to build the environment map while the robot is carrying out the exploration task, a SLAM (Simultaneous Localization and Modelling)algorithm is implemented, the Evolutive Localization Filter (ELF) based on a differential evolution technique.

The combination of these methods provide a new autonomous exploration strategy to construct consistent maps of 2D and 3D indoor environments.

Keywords:
Mobile robots, exploration, SLAM, motion planning, robot mapping.

## 1 INTRODUCTION

Autonomous exploration and map building are fundamental capabilities that a mobile robot needs to carry out tasks in unknown environments in an efficient way. There is a variety of potential applications for autonomous mobile robots in such diverse areas as forestry, space, nuclear reactors, environmental disasters, industry, and offices. A mobile robot is a useful addition to these domains only when it is capable of functioning robustly under a wide variety of environmental conditions [37], operating without human intervention for long periods

S. Garrido and L. Moreno and D. Blanco
Robotics Lab. Carlos III University
Av. de la Universidad, 30 - 28911 Leganés (Madrid) - Spain
E-mail: sgarrido@ing.uc3m.es

of time [1], and providing some guarantee of task performance [24]. The environments in which mobile robots must ***function*** are dynamic, unpredictable and not completely specifiable by a map beforehand. In order to autonomously acquire and create maps, robots have to explore their environment and to perform Simultaneous Localization and Mapping (SLAM) at the same time.

When the robot moves through a completely unknown environment the exploration task requires the guidance of an agent -a robot - by discovering and negotiating the environment so as to reach a goal location. The goal must to be dynamically defined to drive the robot towards the most unexplored areas. For this purpose, this paper presents sensor-based exploration and path planning methods based on the Logarithm of the Extended Voronoi Transform and the Fast Marching Method.

In each step of the exploration process the sensors provide a binary image of the visible environment having distinguished the detected obstacles of the free space. The Extended Voronoi Transform of that binary image gives a grey scale image darker the more near to the obstacles. The Logarithm of the Extended Voronoi Transform imitates the repulsive electric potential from walls and obstacles. This potential impels the robot to follow a trajectory far from obstacles. The Fast Marching Method is applied to calculate the trajectory in the image generated by the Logarithm of the Extended Voronoi Transform. Then, the path obtained verifies the smoothness and safety considerations required for mobile robot path planning.

The Fast Marching Method has been applied to Path Planning [37], and their trajectories are of minimal distance, but they are not very safe because the path is too close to obstacles and what is more important, the path is not smooth enough. The advantages of the proposed planning method, combination of Voronoi and Fast Marching, are its easy implementation, its speed and above all, the quality of the trajectories. The method works in 2D and 3D, and can be used on a local scale operating with sensor information. This makes it especially appropriate for exploration tasks.

The construction of the map of an unknown environment requires to perform Simultaneous Localization and Mapping (SLAM). This enables navigation of an autonomous mobile robot in a structured environment without relying on maps given a priori. For this purpose our approach uses the Evolutive Localization Filter (ELF) [25]. This SLAM approach is based on the stochastic search for solutions in the state space to the global localization problem by means of a differential evolution algorithm.

## 2 Previous and related works

### 2.1 Representations of the world

Roughly speaking there are two main forms for representing the spatial relations in an environment: metric maps and topological maps. Metric maps are characterized by a representation where the position of the obstacles are indicated by coordinates in a global frame of reference. Some of them represent the environment with grids of points, defining regions that can be occupied or not by obstacles or goals [24] [1]. Topological maps represent the environment with graphs that connect landmarks or places with special features [21] [14]. In our approach we choose the grid-based map to represent the environment. The clear advantage is that with grids we already have a discrete environment representation and ready to be used in conjunction with the Extended Voronoi Transform function and Fast Marching Method for path planning. The pioneer method for environment representation in a grid-based model was the certainty grid method developed at Carnegie Mellon University by

Moravec [24]. He represents the environment as a 3D or 2D array of cells. Each cell stores the probability of the related region being occupied. The uncertainty related to the position of objects is described in the grid as a spatial distribution of these probabilities within the occupancy grid. The larger the spatial uncertainty, the greater the number of cells occupied by the observed object. The update of these cells is performed during the navigation of the robot or through the exploration process by using an update rule function. Many researchers have proposed their own grid-based methods. The main difference among them is the function used to update the cell. Some of them are, for example: Fuzzy [26], Bayesian [11], Heuristic Probability [5], Gaussian [6], etc. In the Histogramic In-Motion Mapping (HIMM), each cell, has a certainty value, which is updated whenever it is being observed by the robots sensors. The update is performed by increasing the certainty value by 3 (in the case of detection of an object) or by decreasing it by 1 (when no object is detected), where the certainty value is an integer between 0 and 15.

## 2.2 Approaches to exploration

This section relates some interesting techniques used for exploratory mapping. They mix different localization methods, data structures, search strategies and map representations. Kuipers and Byun [15] proposed an approach to explore an environment and to represent it in a structure based on layers called Spatial Semantic Hierarchy (SSH) [14]. The algorithm defines distinctive places and paths, which are linked to form an environmental topological description. After this, a geometrical description is extracted. The traditional approaches focus on geometric description before the topological one. The distinctive places are defined by their properties and the distinctive paths are defined by the twofold robot control strategy: follow-the-mid-line or follow-the-left-wall. The algorithm uses a lookup table to keep information about the place visited and the direction taken. This allows a search in the environment for unvisited places. Lee [18] developed an approach based on Kuipers work [15] on a real robot. This approach is successfully tested in indoor office-like spaces. This environment is relatively static during the mapping process. Lee's approach assumes that walls are parallel or perpendicular to each other. Furthermore, the system operates in a very simple environment comprised of cardboard barriers. Mataric [21] proposed a map learning method based on a subsumption architecture. Her approach models the world as a graph, where the nodes correspond to landmarks and the edges indicate topological adjacencies. The landmarks are detected from the robot movement. The basic exploration process is wall-following combined with obstacle avoidance. Oriolo et al. [27] developed a grid-based environment mapping process that uses fuzzy logic to update the grid cells. The mapping process runs on-line [26], and the local maps are built from the data obtained by the sensors and integrated into the environment map as the robot travels along the path defined by the $A^*$ algorithm to the goal. The algorithm has two phases. The first one is the perception phase. The robot acquires data from the sensors and updates its environment map. The second phase is the planning phase. The planning module re-plans a new safe path to the goal from the new explored area. Thrun and Bucken [40] [41] developed an exploration system which integrates both evidence grids and topological maps. The integration of the two approaches has the advantage of disambiguating different positions through the grid-based representation and performing fast planning through the topological representation. The exploration process is performed through the identification and generation of the shortest paths between unoccupied regions and the robot. This approach works well in dynamic environments, although, the walls have to be flat and cannot form angles that differ more than 15° from the perpendicular. Feder et al. [7] proposed a probabilistic approach to treat the concurrent mapping and localization using a sonar. This approach is an example of a feature-based approach. It uses the extended Kalman filter to estimate the localization of the robot. The essence of this approach is to take actions that maximize the total knowledge about the system in the presence of measurement and navigational uncertainties. This approach was tested successfully in wheeled land robot and autonomous underwater vehicles (AUVs). Yamauchi [42] [43] developed the Frontier-Based Exploration to build maps based on grids. This method uses a concept of frontier, which consists of boundaries that separate the explored free space from the unexplored space. When a frontier is explored, the algorithm detects the nearest unexplored frontier and attempts to navigate towards it by planning an obstacle free path. The planner uses a depth-first search on the grid to reach that frontier. This process continues until all the frontiers are explored. Zelek [46] proposed a hybrid method that combines a local planner based on a harmonic function calculation in a restricted window with a global planning module that performs a search in a graph representation of the environment created from a CAD map. The harmonic function module is employed to generate the best path given the local conditions of the environment. The goal is projected by the global planner in the local windows to direct the robot. Recently, Prestes el al. [30] have investigated the performance of an algorithm for exploration based on partial updates of a harmonic potential in an occupancy grid. They consider that while the robot moves, it carries along an activation window whose size is of the order of the sensors range.

Prestes and coworkers [31] propose an architecture for an autonomous mobile agent that explores while mapping a two-dimensional environment. The map is a discretized model for the localization of obstacles, on top of which a harmonic potential field is computed. The potential field serves as a fundamental link between the modelled (discrete) space and the real (continuous) space where the agent operates.

### 2.3 Approaches to motion planning

The motion planning method proposed in this paper can be included in the sensor-based global planner paradigm. It is a potential method but it does not have the typical problems of these methods enumerated by Koren- Borenstein [13]: 1) Trap situations due to local minima (cyclic behavior). 2) No passage between closely spaced obstacles. 3) Oscillations in the presence of obstacles. 4) Oscillations in narrow passages. The proposed method is conceptually close to the navigation functions of Rimon-Koditscheck [35], because the potential field has only one local minimum located at the single goal point. This potential and the paths are smooth (the same as the repulsive potential function) and there are no degenerate critical points in the field. These properties are similar to the characteristics of the electromagnetic waves propagation in Geometrical Optics (for monochromatic waves with the approximation that length wave is much smaller than obstacles and without considering reflections nor diffractions).

The Fast Marching Method has been used previously in Path Planning by Sethian [39] [38], but using only an attractive potential. This method has some problems. The most important one that typically arises in mobile robotics is that optimal motion plans may bring robots too close to obstacles (including people), which is not safe. This problem has been dealt with by Latombe [16], and the resulting navigation function is called *NF2*. The Voronoi Method also tries to follow a maximum clearance map [10]. Melchior, Poty and Oustaloup [23,29], present a fractional potential to diminish the obstacle danger level and improve the smoothness of the trajectories, Philippsen [28] introduces an interpolated Navigation Function, but with trajectories too close to obstacles and without smooth properties and Petres [32], introduces efficient path-planning algorithms for Underwater Vehicles taking advantage of the underwaters currents.

LaValle [17], treats on the feedback motion planning concept. To move in the physical world the actions must be planned depending on the information gathered during execution.

Lindemann and Lavalle [19] [20] present a method in which the vector field globally solves the navigation problem and provides robustness to disturbances in sensing and control. In addition to being globally convergent, the vector field's integral curves (system trajectories) are guaranteed to avoid obstacles and are $\mathscr{C}^\infty$ smooth, except in the changes of cells. They construct a vector field with these properties by using existing geometric algorithms to partition the space into simple cells; they then define local vector fields for each cell, and smoothly interpolate between them to obtain a global vector field that can be used as a feedback control for the robot.

Yang and Lavalle [44] presented a randomized framework motion strategies, by defining a global navigation function over a collection of spherical balls in the configuration space. Their key idea is to fill the collision free subset of the configuration space with overlapping spherical balls, and define collision free potential functions on each ball. A similar idea has been developed for collision detection in [33] and [34].
The proposed method constructs a vectorial field as in the work by Lindemann, but the field is done in the global map instead of having local cells maps with the problem of having trajectories that are not $\mathscr{C}^\infty$ in the union between cells. The method has also similitudes with the Yang and Lavalle method. They proposed a series of balls with a Lyapunov potential associated to each of them. These potentials are connected in such a way that it is possible to find the trajectory using in each ball the gradient method. The method that we propose, has a unique global Lyapunov potential associated with the vectorial field that permits build the $\mathscr{C}^\infty$ trajectory in a single pass with the gradient method.

To achieve a smooth and safe path,it is necessary to have smooth attractive and repulsive potentials, connected in such a way that the resulting potential and the trajectories have no local minima and curvature continuity to facilitate path tracking design. The main improvement of the proposed method are these good properties of smoothness and safety of the trajectory. Moreover, the associated vector field allows the introduction of nonholonomic constraints.

It is important to note that in the proposed method the important ingredients are the attractive and the repulsive potentials, the way of connecting them describing the attractive potential using the wave equation (or in a simplified way, the eikonal equation). This equation can be solved in other ways: Mauch [22] uses a Marching with Correctness Criterion with a computational complexity that can reduced to $\mathscr{O}(N)$. Covello [3] presents a method that can be used on nodes that are located on highly distorted grids or on nodes that are randomly located.

## 3 Introduction to the Extended Voronoi Transform

The Distance Transform [36] is a useful tool in digital picture processing. It has found a wide range of uses in image analysis, pattern recognition, and robotics. In Computer Vision, it is known as Distance Transform, but this term is also used in Robotics to designate a different concept. For this reason, in Robotics, this concept is called Extended Voronoi Transform (EVT).

The Extended Voronoi Transform computes the Euclidean Distance of the binary image. For each pixel in the image, the EVT assigns a number which is the distance to the nearest nonzero pixel of the image. In a binary image, a pixel is referred to as background if its value is zero. For a given distance metric, the EVT of an image produces a distance map of the same size. For each pixel inside the objects in the binary image, the corresponding pixel in the distance map has a value equal to the minimum distance to the background.

Clearly, the Extended Voronoi Transform is closely related to the Voronoi diagram. The Voronoi diagram concept is involved in many EVT approaches either explicitly or implicitly.

For any topologically discrete set $S$ of points in Euclidean space and for almost any point $x$, there is one point of $S$ to which $x$ is closer than $x$ is to any other point of $S$. The word "almost" is occasioned by the fact that a point $x$ may be equally close to two or more points of $S$. If $S$ contains only two points, $a$, and $b$, then the set of all points equidistant from $a$ and $b$ is a hyperplane, i.e. an affine subspace of codimension 1. That hyperplane is the boundary between the set of all points closer to $a$ than to $b$, and the set of all points closer to $b$ than to $a$.

In general, the set of all points closer to a point $c$ of $S$ than to any other point of $S$ is the interior of a convex polytope (in some cases unbounded) called the **Dirichlet domain** or **Voronoi cell** for $c$. The set of such polytopes tesselates the whole space, and is the **Voronoi tessellation** corresponding to the set $S$. If the dimension of the space is only 2, then it is easy to draw pictures of Voronoi tessellations, and in that case they are sometimes called **Voronoi diagrams**.

The close relation between EVT and the Voronoi Diagram implies that it is possible to compute one of them from the other. The majority of the EVT methods use the Voronoi Diagram as an intermediate step.

The algorithm used in this work for two dimensions, is based on the second algorithm work carried out by Breu et al. [2]. In this work, the special properties of the Euclidean metric are exploited. They designed two linear-time algorithms based on Voronoi transforms where the second algorithm could have been improved if they had used the result of the previous row to reduce the set of possible candidates. It is an $O(m \times n)$ algorithm, where the image size is $m \times n$.

For more than two dimensions, the Extended Voronoi Transform uses a nearest-neighbor search on an optimized kd-tree, as described by Friedman [8].

The Voronoi approach to path planning has the advantage of providing the safest trajectories in terms of distance to obstacles but because its nature is purely geometric and it does not achieve enough smoothness.

## 4 Intuitive introduction of the Eikonal Equation and the Fast Marching Planning Method

Intuitively, Fast Marching Method gives the propagation of a front wave in an inhomogeneous media. Let us imagine that the curve or surface moves in its normal direction with a known speed $F$ (see fig. 1a). The objective would be to follow the movement of the interface while this one evolves. A large part of the challenge, in the problems modelled as fronts in evolution, consists in defining a suitable speed, which faithfully represents the physical system.

A way to characterize the position of a front in expansion is to compute the time of arrival $T$, in which the front reaches each point of the underlying mathematical space of the interface. It is evident that for one dimension (see fig. 1b) the equation for the arrival

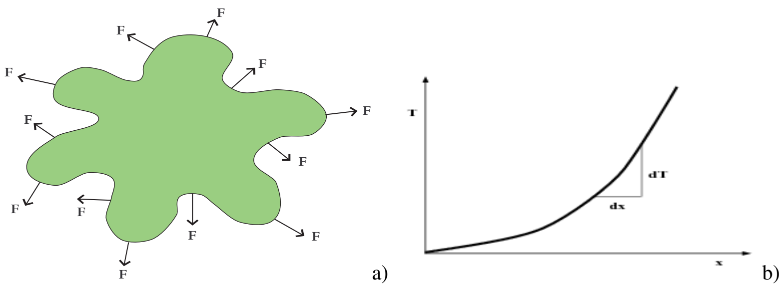



**Fig. 1** Wavefront propagating with velocity F (a), and Formulation of the arrival function $T(x)$, for an unidimensional wavefront.(b).

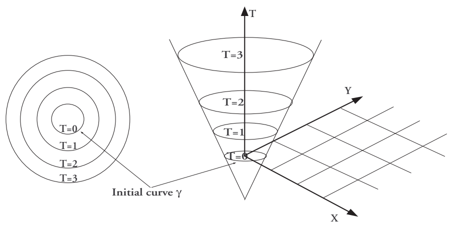



**Fig. 2** Movement of a circular wavefront, as a problem of boundary conditions

function $T$ can be obtained in an easy way, simply by considering the fact that the distance $x$ is the product of the speed $F$ by the time $T$: $x = F \cdot T$. The spatial derivative of the solution function becomes the gradient: $1 = F\frac{dT}{dx}$ and therefore we have that the magnitude of the gradient of the arrival function $T(x)$ is inversely proportional to the speed, $\frac{1}{F} = |\nabla T\,|$. For multiple dimensions, the same concept is valid because the gradient is orthogonal to the level sets of the arrival function $T(x)$. In this way, the movement of the front can be characterized as the solution of a boundary conditions problem. The speed $F$ depends only on the position, then the equation $\frac{1}{F} = |\nabla T|$ or the Eikonal equation:

$$|\nabla T|F = 1. \tag{1}$$

As a simple example we define a circular front $\gamma_t = \{(x,y)/T(x,y) = t\}$ for two dimensions that advance with unitary speed. The evolution of the value of the arrival function $T(\theta)$ can be seen as the time increases (i.e. $T = 0$, $T = 1$, $T = 2$, ...) and the arrival function comes to points of the plane in more external regions of the surface as can be seen in fig. 2. The boundary condition is that the value of the wave front is zero in the initial curve.

The direct use of the Fast Marching method does not guarantee a smooth and safe trajectory. Due to the way the front wave is propagated the shortest geometrical path is determined. This makes the trajectory unsafe because it touches corners, walls and obstacles, as is shown in fig. 7. This problem can be easily solved by enlarging the obstacles, but even in that case the trajectory tends to get close to the walls and it is not smooth and safe enough.

In Geometrical Optics, Fermat's *least time principle* for light propagation in a medium with space varying refractive index $\eta(\mathbf{x})$ is equivalent to the eikonal equation and can be written as $||\nabla \Phi(\mathbf{x})|| = \eta(\mathbf{x})$ where the eikonal $\Phi(\mathbf{x})$ is a scalar function whose isolevel

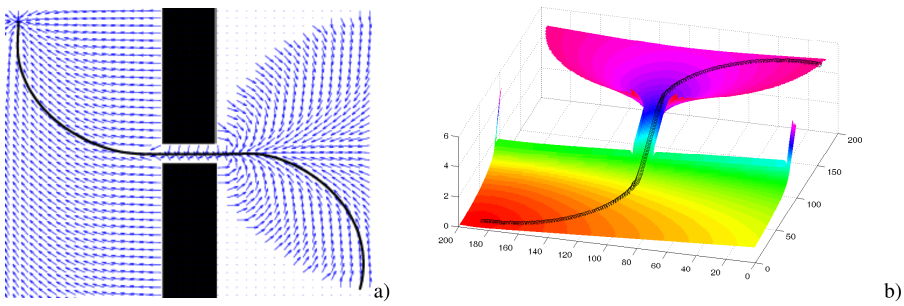



**Fig. 3** Union of the two potentials: the second one having the first one as refractive index. a) Associated vector field and a typical trajectory obtained with this method. b) The Funnel shaped potential applied to the first potential of a H-shaped environment and the path calculated with the gradient method.

contours are normal to the light rays. This equation is also known as Fundamental Equation of the Geometrical Optics.

The eikonal (from the Greek "eikon", which means "image") is the phase function in a situation for which the phase and amplitude are slowly varying functions of position. Constant values of the eikonal represent surfaces of constant phase, or wavefronts. The normals to these surfaces are rays (the paths of energy flux). Thus the eikonal equation provides a method for "ray tracing" in a medium of slowly varying index of refractive (or the equivalent for other kinds of waves).

The use of the Fast Marching method over a slowness (refraction or inverse of velocity) potential improves the quality of the calculated trajectory considerably. On one hand, the trajectories tend to go close to the Voronoi skeleton because of the optimal conditions of this area for robot motion [9]. On the other hand, the trajectories are also considerably smooth. For a small and easy H-shaped environment, the slowness (velocity inverse) potential in 3D is shown in fig. 3b and the funnel shaped potential given by the wave propagation of and the trajectory calculated by the gradient method is shown in fig. 3a.

For further details and summaries of level set and fast marching techniques for numerical purposes, see [38]. The Fast Marching Method is an $O(n)$ algorithm as has been demonstrated by Yatziv [45].

## 5 Intuitive introduction to the Voronoi Fast Marching Method (VFM)

Which properties and characteristics are desirable for a Motion Planner of a mobile robot? The first one is that the planner always drives the robot in a smooth and safe way to the goal point. In Nature there are phenomena with the same way of working: electromagnetic waves. If there is an antenna that emits an electromagnetic wave in the goal point, then the robot could drive itself to the destination following the waves to the source. The concept of the electromagnetic wave is especially interesting because the potential and its associated vector field have all the good properties desired for the trajectory, such as smoothness (it is $C^\infty$) and the absence of local minima.

This attractive potential still has some problems. The most important one that typically arises in mobile robotics is that optimal motion plans may bring robots too close to obstacles, which is not safe. This problem has been dealt with by Latombe [16], and the resulting

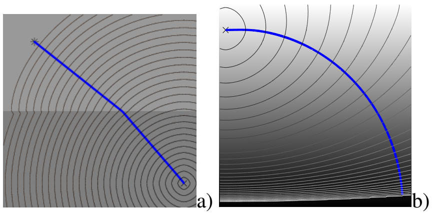



**Fig. 4** Propagation of a wave and the corresponding minimum time path when there are two media of different slowness (diffraction) index. (a), the same with an vertical gradient (b).

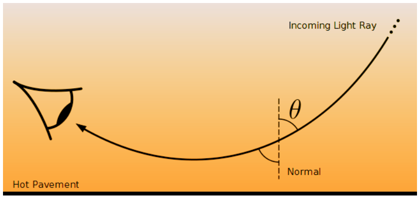



**Fig. 5** Light rays bending due to changing refraction index in air with higher temperature near road surface.

navigation function is called *NF2*. The Voronoi Method also tries to follow a maximum clearance map [9]. To generate a safe path, it is necessary to add a component that repels the robot away from obstacles. In addition, this repulsive potential and its associated vector field should have good properties such as those of the electrical field. If we consider that the robot has an electrical charge of the same sign as the obstacles, then the robot would be pushed away from obstacles. The properties of this electric field are very good because it is smooth and there are no singular points in the interest space ($C_{free}$).

The third part of the problem consists in how to mix the two fields together. This union between an attractive and a repulsive field has been the biggest problem for the potential fields in path planning since the works of Khatib [12]. In the VFM Method, this problem has been solved in the same way that Nature does so: the electromagnetic waves, such as light, have a propagation velocity that depends on the media. For example, flint glass has a refraction index of 1.6, while in the air it is approximately one. This refraction index of a medium is the quotient between the velocity of light in the vacuum and the velocity in the medium under consideration. That is the slowness index of the front wave propagation in a medium.

A light ray follows a straight line if the medium has a constant refraction index (the medium is homogeneous) but refracts when there is a transition of medium (sudden change of refraction index value) as shown in fig. 4a. In the case of a gradient change in refraction index in a given medium, the light ray follows a curved line as shown in fig. 4b. This phenomenon can be seen in nature in hot road mirages. In this phenomenon, the air closer to the road surface is warmer than the higher level layers. The warmer air has lower density and lower refraction index. For this reason, light rays coming from the sun are curved near the road surface and cause what is called the hot road mirage, as illustrated in fig. 5. This is the idea that inspires the way in which the attractive and the repulsive fields are merged in our work.

For this reason, in the *VFM* method, the repulsive potential is used as refraction index of the wave emitted from the goal point. In this way, a unique field is obtained and its associated vector field is attractive to the goal point and repulsive from the obstacles. This method inherits the properties of the electromagnetic field. Intuitively, the *VFM* Method gives the propagation of a front wave in an inhomogeneous media as shown in fig. 4b.

## 6 Details of the VFM algorithm

This method starts with the calculation of the Logarithm of the Extended Voronoi Transform of the 2D a priori map of the environment (or the Extended Voronoi Transform in case of 3D maps). Each white point of the initial image (which represents free cells in the map) is associated with a level of grey that is the logarithm of the 2D distance to nearest obstacles (or the Extended Voronoi Transform in 3D). As a result of this process, a kind of potential proportional to the distance to the nearest obstacles to each cell is obtained, see fig. 6. Zero potential indicates that a given cell is part of an obstacle and maxima potential cells corresponds to cells located in the Voronoi diagrams (which are the cells located equidistant to the obstacles).

This function introduces a potential similar to a repulsive electric potential (in 2D) as shown in figure 6, that can be expressed by

$$\phi = c_1 \log(r) + c_2. \tag{2}$$

where $c_1$ is a negative constant.

If $n > 2$ , ($n$ is the space dimension), the potential is

$$\phi = \frac{c_3}{r^{n-1}} + c_4. \tag{3}$$

where $r$ is the distance from the origin.

In a second step, the technique proposed here uses Fast Marching to calculate the shortest trajectory in the potential surface defined by logarithm of the Extended Voronoi Transform. The calculated trajectory is the geodesic one in the potential surface, i.e. with a viscous distance. This viscosity is done by the grey level. If the Fast Marching Method were used directly on the environment map, we would obtain the shortest geometrical trajectory, as shown in fig. 7, but the trajectory is not safe nor smooth.

The potential created has local minima as shown in fig. 6, but the trajectories are not stuck in these points because the Fast Marching Method gives the trajectories that correspond to the propagation of a wave front which is faster in lighter regions and slower in the darker ones.

The trajectories obtained by using the logarithm of the EVT tend to go by the Voronoi diagram but properly smoothed as shown figure 8.

This method to plan a trajectory can be used in 3D environments, the figure 9 shows an example of this case.

## 7 Properties

The proposed $VFM$ algorithm has the following key properties:

- *Fast response*. The planner needs to be fast enough to be used reactively in case unexpected obstacles make it necessary to plan a new trajectory. To obtain this fast response, a fast planning algorithm and fast and simple treatment of the sensor information is necessary. This requires a low complexity order algorithm for a real time response to

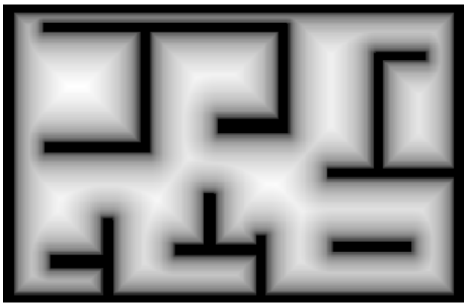


**Fig. 6** Potential of the Logarithm of the inverse of Extended Voronoi Transform.

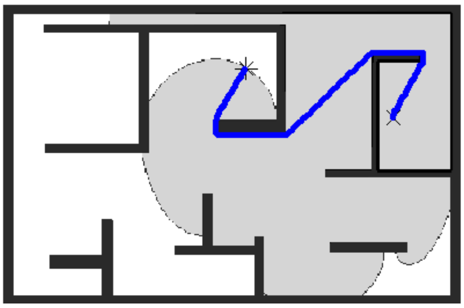


**Fig. 7** Trajectory calculated with Fast Marching without the Logarithm Extended Voronoi Transform.

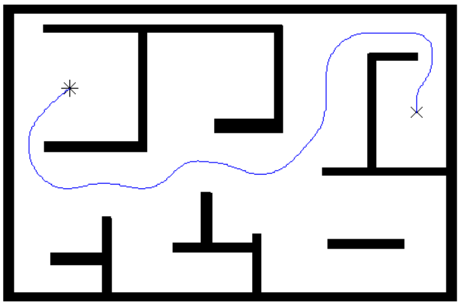


**Fig. 8** Trajectory calculated with Fast Marching with the Logarithm Extended Voronoi Transform.

unexpected situations. As shown in table I, the proposed algorithm has a fast response time to allow its implementation on real time, even in environments with moving obstacles using a normal PC computer.

*Smooth trajectories*. The planner must be able to provide a smooth motion plan which can be executed by the robot motion controller. In other words, the plan does not need to be refined, avoiding the need for a local refinement of the trajectory. The solution of the eikonal equation used in the proposed method is given by the solution of the wave equation:

$$\phi = \phi_0 e^{ik_0(\eta x - c_0 t)}$$

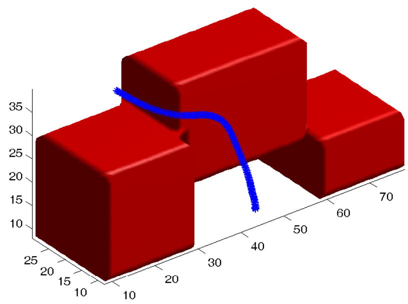


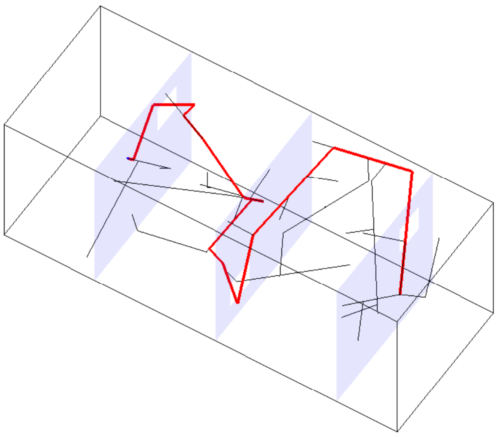

**Fig. 9** Trajectory calculated with Fast Marching with the Logarithm EVT in 3D.

As this solution is an exponential, if the potential $\eta(x)$ is $\mathscr{C}^{\infty}$ then the potential $\phi$ is also $\mathscr{C}^{\infty}$ and therefore the trajectories calculated by the gradient method over this potential would be of the same class.
This smoothness property can be observed in fig. 8, where trajectory is clearly good, safe and smooth. One advantage of the method is that it not only generates the optimum path, but also the velocity of the robot at each point of the path. The velocity reaches its highest values in the light areas and minimum values in the greyer zones. The $VFM$ Method simultaneously provides the path and maximum allowable velocity for a mobile robot between the current location and the goal.
Figure 10 shows performance of the classical RRT planning method in a 3D environment and the proposed VFM method.

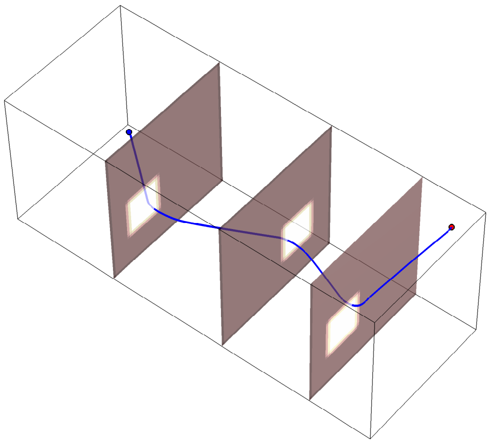
- *Reliable trajectories*. The proposed planner provides a safe (reasonably far from a priori and detected obstacles) and reliable trajectory (free from local traps). This avoids the coordination problem between the local collision avoidance controllers and the global planners, when local traps or blocked trajectories exist in the environment. This is due to the refraction index, which causes higher velocities far from obstacles.
- *Completeness.* As the method consists of the propagation of a wave, if there is a path from the the initial position to the objective, the method is capable of finding it.

## 8 Implementation of the explorer

In order to solve the problem of the exploration of an unknown environment, our algorithm can work in two different ways. First, the exploration process can be directed giving to the algorithm one or several successive goal points in the environment which the robot must drive to during the exploration process. Second, that is the second form to work of our algorithm, the exploration can be carried out without having any previously fixed objective point. In such case, the algorithm must automatically determine towards where the robot must drive in order to complete the exploration process. In each movement of the robot, information about the environment is used to build a binary image distinguishing occupied space represented by value 0 (obstacles and walls) from free space, with value 1. The Extended Voronoi Transform of the known map at that moment, gives a grey scale that is darker



a)



b)

**Fig. 10** Trajectory calculated with a) RRT and b) Fast Marching Method.

near the obstacles and walls and lighter far from them. The Voronoi Fast Marching Method gives the trajectory from the pose of the robot to the goal point using the known information.

*A. Case I*

In this first case, the robot has a final goal: the exploration process the robot performs in the algorithm described in the flowchart of figure 11. In this way, the robot has a general direction of movement towards the goal.

In this first case, the SLAM algorithm described in [25] is used to avoid localization errors being translated into the map built during the exploration process.

*B. Case II*

In the second way of working of the algorithm, the goal location is unknown and robot behavior is truly exploratory. We propose an approach based on the incremental calculation

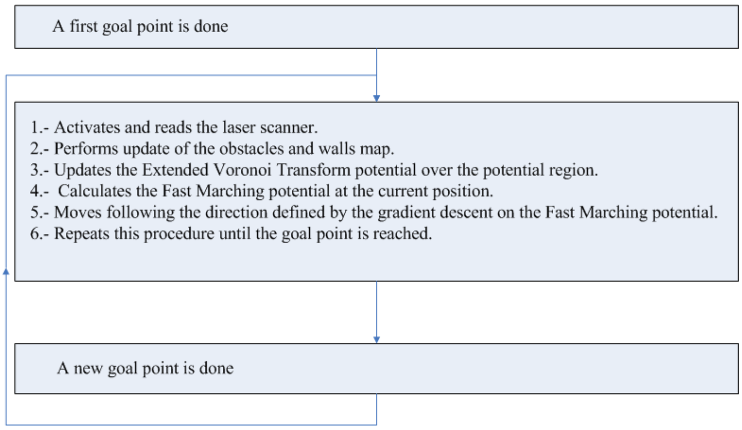



**Fig. 11** Flowchart of case 1.

of a map for path planning. We define an neighborhood window, that travels with the robot, with roughly the size of its laser sensor range. This window indicates the new grid cells that are recruited for update, i.e., if a cell was at a given time in the neighborhood window, it becomes part of the explored space by participating in the EVT and Fast Marching Method calculation for all times. The set of activated cells that compose the explored space is called the neighborhood region. Cells that were never inside the neighborhood window indicate unexplored regions. Their potential values are set to zero and define the knowledge frontier of the state space, the real space in our case. The detection of the nearest unexplored frontier comes naturally from the Extended Voronoi Transform calculation. It can also be understood from the physical analogy with electrical potentials that obstacles repel while frontiers attract.

Consider that the robot starts from a given position in an initially unknown environment. In this second method, there is no direction of the place where the robot must go. The first step consists of calculating a first matrix $W$ that gives us the EVT of the obstacles found up until the moment. A matrix with zeros in the obstacles and value 1 in the free zones is considered. The EVT is applied and a matrix $W$ of grays with values between 0 (obstacles) and 1 is obtained. The second matrix $VT$ is built darkening the zones that the robot already has visited for which it assigns value 1 to the points of the trajectory. Then, it calculates the EVT of the obtained image. Finally, matrix $WV$ is the sum of the matrices $VT$ and $W$, with weights 0.5 and 1 respectively.

$$WV = 0.5 * VT + W$$

In this way, it is possible to darken the zones already visited for the robot and impel it to go to the unexplored zones. The whitest point of matrix $WV$ is calculated as $max(WV)$, that is, the most unexplored region that is in a free space. This is the point chosen as the new goal point. Applying Fast Marching method on $WV$, the trajectory towards that goal is calculated. The robot moves following this trajectory. In the following steps, the trajectory to follow is computed, calculating at every moment first $W$ and $VT$, and therefore $WV$, but without changing the objective point. Once the robot has been arrived at the objective, (that is to say, that path calculated is very small), a new objective is selected as $max(WV)$.

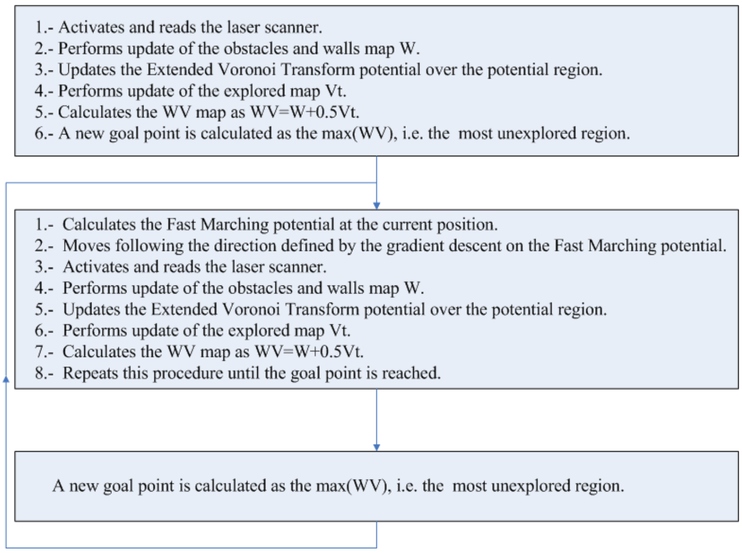



**Fig. 12** Flowchart of algorithm 2.

Therefore, the robot moves maximizing knowledge gain. In this case or in any other situation where there is no gradient to guide the robot, it simply follows the forward direction. The exploration process the robot performs in the second method described is summarized in the flowchart of figure 12.

The algorithms laid out in fig. 11 (Flowchart of Case 1) can be inefficient in very large environments. To increase speed it is possible to pick a goal point, put a neighborhood window the size of the sensor range, then run into the goal point, then look at the maximal initial boundary, recast and terminate when one reaches the boundary of the computed region. Similar improvements can be made to Algorithm 2.

## 9 Mapping. Differential Evolution approach to the SLAM

The robot needs to build a consistent map from data collected during the exploration. Localization and map building are key components in robot navigation and are required to successfully execute the path generated by the VFM planner in the exploration method proposed in this paper. Both problems are closely linked, and learning maps are required to solve both problems simultaneously; this is the SLAM (simultaneous localization and mapping) problem. Uncertainty in sensor measures and uncertainty in robot pose estimates make the use of a SLAM method necessary to create a consistent map of the explored environment.

The SLAM algorithm used in this work is described in [25]. It is based on the stochastic search of solutions in the state space to the localization problem by means of a differential evolution algorithm. A non linear evolutive filter, called Evolutive Localization Filter (ELF), searches stochastically along the state space for the best robot pose estimate. The proposed SLAM algorithm operates in two steps: in the first step the ELF filter is used at a local level to re-localize the robot based on the robot odometry, the laser scan at a given position and

**Fig. 13** The robot Manfred has been used to test the algorithms.

a local map where only a low number of the last scans have been integrated. In a second step, the aligned laser measures, together with the corrected robot poses, are used to detect when the robot is revisiting a previously crossed area. Once a cycle is detected, the Evolutive Localization Filter is used again to reestimate the robot position and orientation in order to integrate the sensor measures in the global map of the environment.

This approach uses a differential evolution method to perturb the possible pose estimates contained in a given set until the optimum is obtained. By properly choosing the cost function, a maximum a posteriori estimate is obtained. This method is applied at a local level to re-localize the robot and at a global level to solve the data association problem. The method proposed integrates sensor information in the map only when cycles are detected and the residual errors are eliminated, avoiding a high number of modifications in the map or the existence of multiple maps, thus decreasing the computational cost compared to other solutions.

## 10 Results

The proposed method, has been tested using the manipulator robot Manfred (see fig. 13). It has a coordinated control of all degree of freedom in the system (the mobile base has 2 DOF and the manipulator has 6 DOF) to achieve smooth movement. This mobile manipulator use a sensorial system based in vision and 3D laser telemetry to perceive and model 3D environments. The mobile manipulator will include all the capabilities needed to navigate, localize and avoid obstacles safely through the environment.

To illustrate the planning method possibilities, a trajectory in a typical office's indoor environment has been used for planning as shown in figure 14 . The dimensions of the environment are 116x14 meters (the cell resolution is 12 cm), that is the image has 966x120 pixels. For this environment the first step (Log of inverse Extended Voronoi Transform) takes 0.06 seconds in a Pentium 4 at 2.2 Ghz, and the second step (Fast Marching) takes 0.20 seconds for a long trajectory.

The motion planning method used provides smooth trajectories that can be used at low control levels without any additional smooth interpolation process. To illustrate the performance of the motion planner a known simulated environment has been used representing a office area (Robotics Lab) of the Carlos III University. The results are shown in figures 15

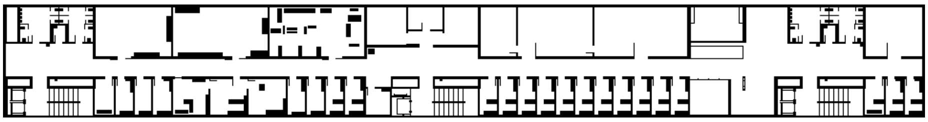


**Fig. 14** Environment map of the Robotics Lab.

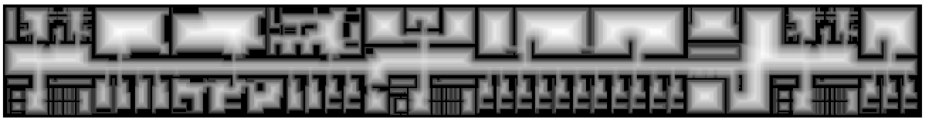


**Fig. 15** Log of the Extended Voronoi Transform applied of the environment map of the Robotics Lab.

to 17. Figure 15 shows the Log of the inverse Extended Voronoi Transform of the environment map (the Robotics Lab). Some of the steps of the exploration process are presented in the figure 16, the red line represents the crossed path and the blue one represents the calculated trajectory from the present position to the destination point, in this case a final goal is provided to the algorithm. In each moment the illuminated area represents the front wave propagation from the present position of the robot to the destination point. The EVT computation is made over the sensory map that the robot has in its memory. Finally in figure 17 the path obtained after applying the Fast Marching method to the previous potential image is shown.

A second test has been carried out on the same environment. In this case, the SLAM algorithm has been implemented to construct the environment map that has been considered unknown. Figures 18, 19 and 20 show the results.

For the case of exploration that this paper contemplates, the results of two different tests are presented to illustrate both cases described for the application of the proposed method. Tests have been carried out in a simulated room environment. In this case the size of image is 628*x*412 pixels.

Figures 21 and 22 represent the first case for implementing the exploration method. A final goal is provided for the robot, which is located with respect to a global reference system; the starting point of the robot movement is also known with respect to that reference system. The algorithm allows calculating the trajectory towards that final goal with the updated information of the surroundings that the sensors obtain in each step of the movement. When the robot reaches the defined goal, a new destination in an unexplored zone is defined.

The results of one of the tests done for the second case of exploration described are shown in figures 23, 24, and 25. Any final goal is defined. The algorithm leads the robot towards the zones that are free of obstacles and unexplored simultaneously. The final map built is shown in the fig. 25.

The proposed method is highly efficient from a computational point of view because the method operates directly over a 2D image map (without extracting adjacency maps), and due to the fact that Fast Marching complexity is $O(m \times n)$ and the Extended Voronoi Transform is also of complexity $O(m \times n)$, where $m \times n$ is the number of cells in the environment map. In table 1, illustrative results of the cost average in time appear (measured in ms), and each step of the algorithm for different trajectory lengths to calculate (the computational cost is depending on the number of points of the image).

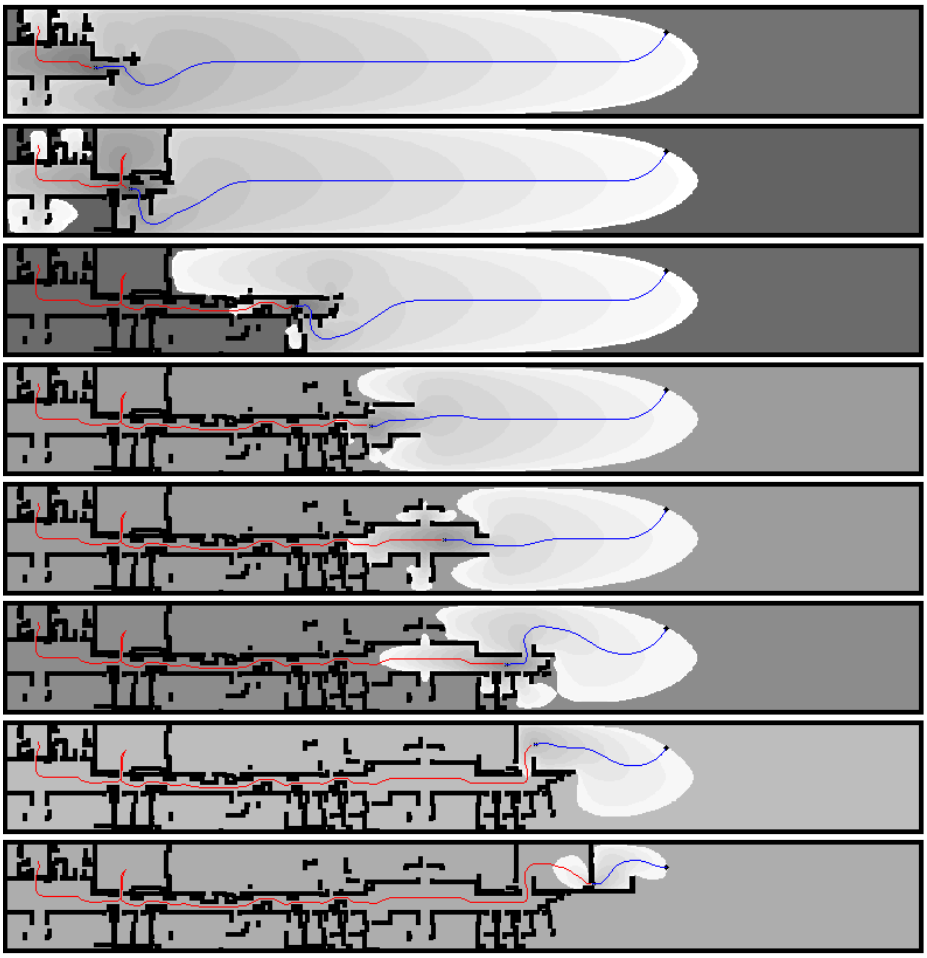


**Fig. 16** Consecutive steps of the process (the red line represents the crossed path and the blue one represents the calculated trajectory from the present position to the destination point).

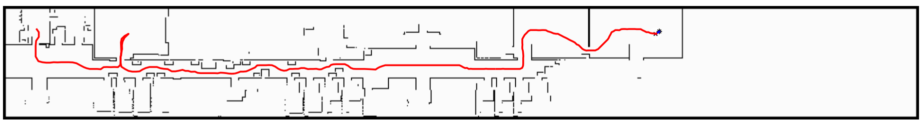


**Fig. 17** Trajectory calculated to avoid obstacles in a cluttered environment with Fast Marching and the Logarithm Extended Voronoi Transform (Global map).

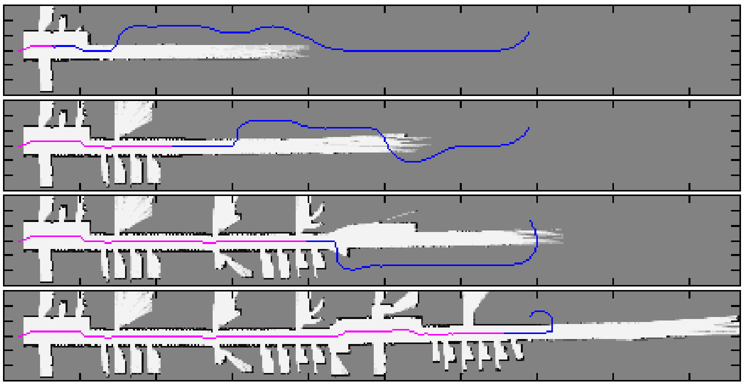


**Fig. 18** Consecutive steps of the process using the case I of the exploration algorithm. The red line represents the crossed path and the blue one represents the calculated trajectory from the present position to the destination point.

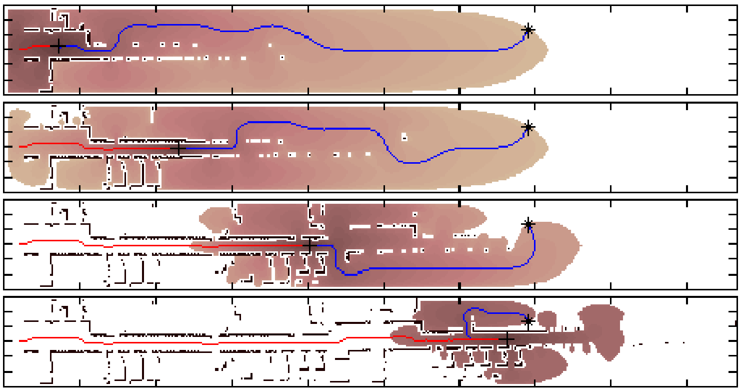


**Fig. 19** Consecutive steps of the process: the illuminated area represents the front wave propagation.

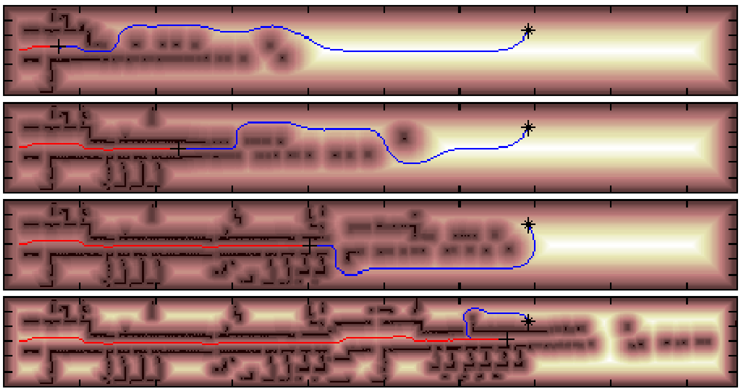


**Fig. 20** Map built in each step using the SLAM algorithm.

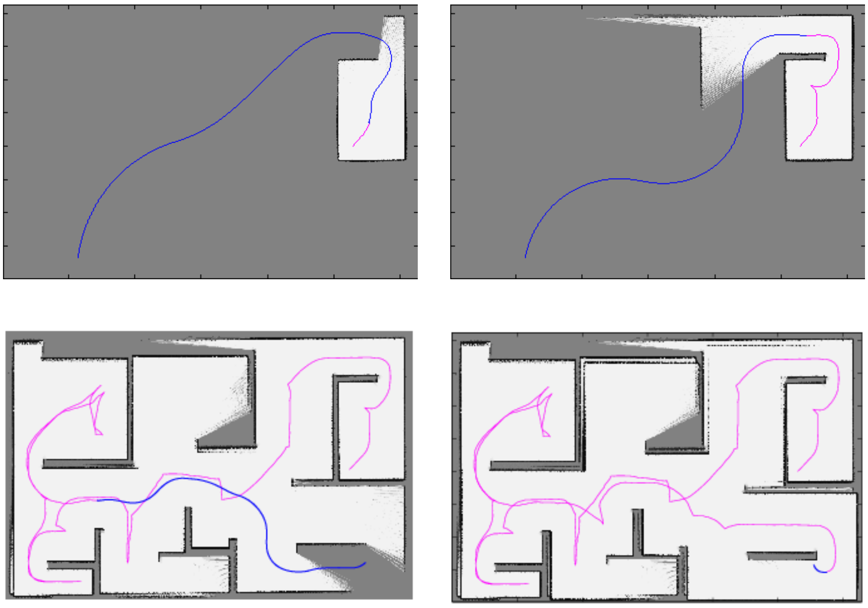


Fig. 21 Simulation results with method 1, with final objective. Trajectory calculated.

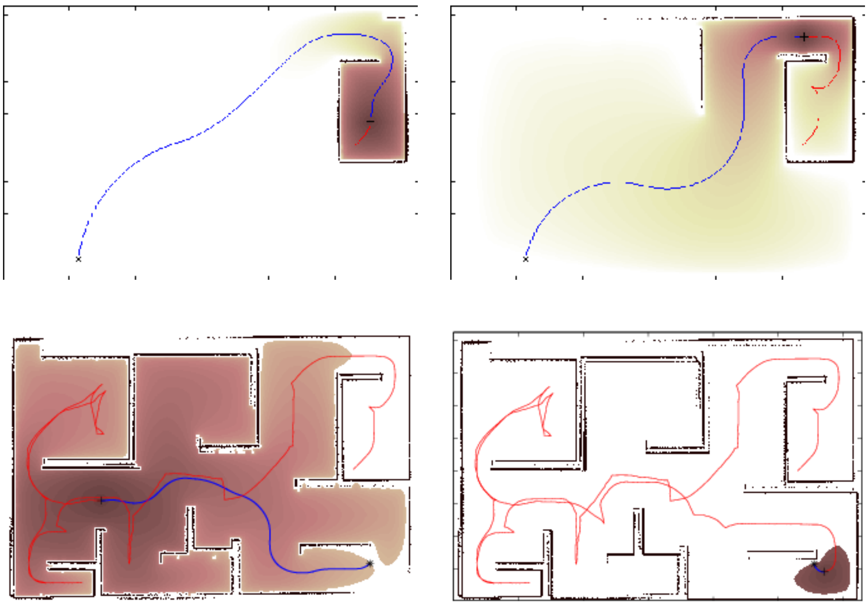


Fig. 22 Simulation results with method 1. Front wave expansion. The red line represents the crossed path and the blue one represents the calculated trajectory from the present position to the destination point.

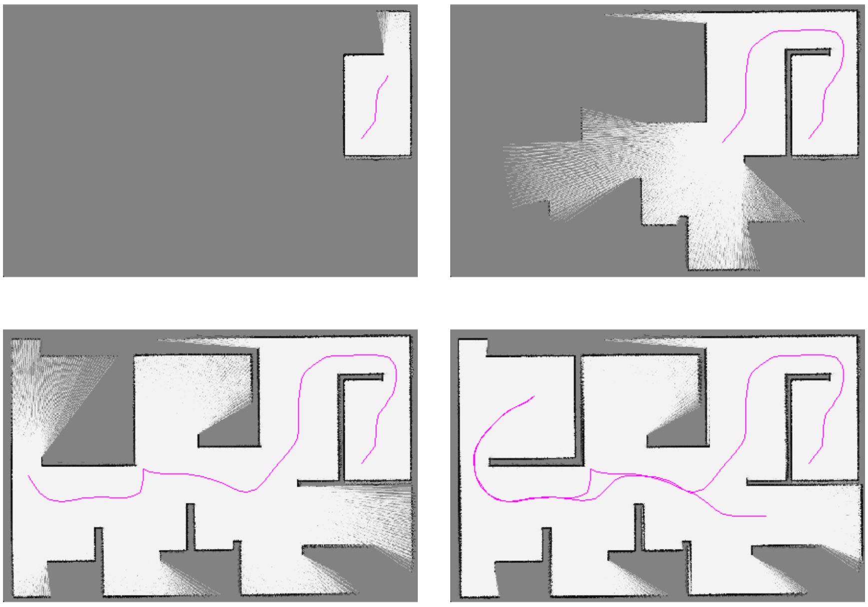


**Fig. 23** Simulation results with method 2, without final objective. Trajectory calculated.

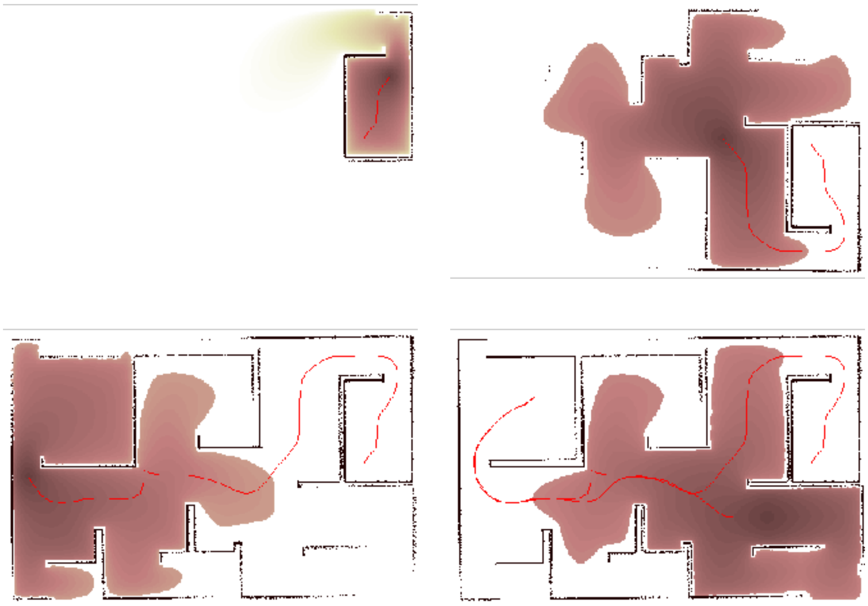


**Fig. 24** Simulation results with method 2. Front wave expansion.

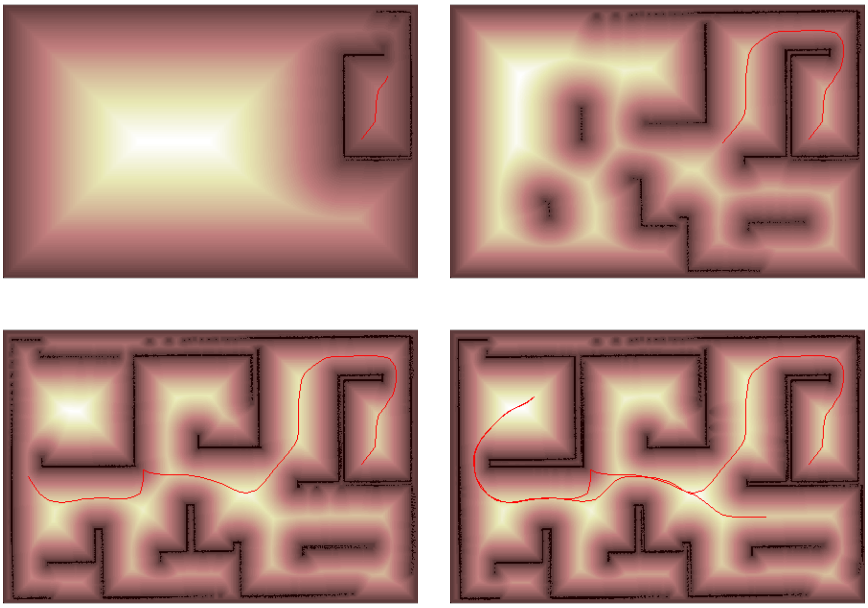


**Fig. 25** Simulation results with method 2. Map built.

**Table 1** Computational cost (ms) for the room environment of fig. 16 (966x120 pixels)

| Alg. Step/Trajectory length | Long | Medium | Short |
|---|---|---|---|
| Obst. Enlarging | 0.008 | 0.008 | 0.008 |
| Ext. Voronoi Transf. | 0.039 | 0.039 | 0.039 |
| FM Exploration | 0.172 | 0.078 | 0.031 |
| Path Extraction | 0.125 | 0.065 | 0.035 |
| Total time | 0.344 | 0.190 | 0.113 |

## 11 Conclusion

The results obtained show that the Logarithm of Extended Voronoi Transform can be used to improve the results obtained with Fast Marching method applied to environment exploration, providing smooth and safe trajectories along the exploratory process.

The algorithm complexity is $O(m \times n)$, where $m \times n$ is the number of cells in the environment map, which let us use the algorithm on line. Furthermore, the algorithm can be used directly with raw sensor data to implement a sensor based local path planning exploratory module.

This paper presents a new autonomous exploration strategy. The full exploratory method consist of the VFM motion planner applied to plan the trajectory towards the goal, a new exploratory strategy that drives the robot to the most unexplored region, and the SLAM algorithm [25] to build a consistent map of the environment. The proposed autonomous exploration method is a combination of the three tools which is able to completely construct consistent maps of unknown indoor environments in an autonomous way. The method can be used in 2D or 3D environments.

## References


1. Y. K. J. Borenstein, “Histogramic in-motion mapping for mobile robot obstacle avoidance,” *IEEE Journal of Robotics*, vol. 7, no. 4, 1991.
2. H. Breu, J. Gil, D. Kirkpatrick, and M. Werman, “Linear time euclidean distance transform algorithms,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 17, no. 5, pp. 529–533, 1995.
3. P. Covello and G. Rodrigue, “A generalized front marching algorithm for the solution of the eikonal equation,” *J. Comput. Appl. Math.*, vol. 156, no. 2, pp. 371–388, 2003.
4. J. L. Davis, *Wave propagation in solids and fluids*. Springer, 1988.
5. A. Elfes, “Sonar-based real world mapping and navigation,” *IEEE Journal of Robotics and Automation*, vol. 3, no. 3, pp. 249–265, 1987.
6. A. Elfes, “Using occupancy grids for mobile robot perception and navigation,” *Computer Magazine*, pp. 46–57, 1989.
7. C. S. H. Feder, J. Leonard, “Adaptive mobile robot navigation and mapping,” *International Journal of Robotics Research*, vol. 7, no. 18, pp. 650–668, 1999.
8. J. H. Friedman, J. L. Bentley, and R. A. Finkel, “An algorithm for finding best matches in logarithmic expected time,” *ACM Transactions on Mathematics Software*, vol. 3, no. 3, pp. 209–226, September 1997.
9. S. Garrido, L.Moreno, and D.Blanco., “Voronoi diagram and Fast Marching applied to path planning.” in *2006 IEEE International conference on Robotics and Automation. ICRA 2006.*, pp. 3049–3054.
10. S. Garrido, L. Moreno, M. Abderrahim, and F. Martin, “Path planning for mobile robot navigation using voronoi diagram and fast marching.” in *Proc of IROS’06. Beijing. China.*, 2006, pp. 2376–2381.
11. L. K. A. Howard, “Generating sonar maps in highly specular environments,” in *Proceedings of the Fourth International Conference on Control, Automation, Robotics and Vision*, 1996.
12. M. Khatib and R. Chatila, “An extended potential field approach for mobile robot sensor-based motions,” in *Proceedings of the IEEE Int. Conf. on Intelligent Autonomus Systems*, 1995.
13. Y. Koren and J. Borenstein, “Potential field methods and their inherent limitations for mobile robot navigation” in *Proceedings of the IEEE Int. Conf. on Robotics and Automation*, pp. 1398–1404, 2004.
14. T. L. B. Kuipers, “Navigation and mapping in large-scale space,” *AI Magazine*, vol. 9, pp. 25–43, 1988.
15. Y. B. B. Kuipers, “A robot exploration and mapping strategy based on a semantic hierarchy of spatial representations,” *Robotics and Autonomous Systems*, vol. 8, pp. 47–63, 1991.
16. J.-C. Latombe, *Robot motion planning*. Dordrecht, Netherlands: Kluwer Academic Publishers, 1991.
17. LaValle, S. M. 2006. *Planning Algorithms*, Cambridge University Press, Cambridge, U.K.
18. W. Lee, “Spatial semantic hierarchy for a physical robot,” Ph.D. dissertation, Department of Computer Sciences, The University of Texas, 1996.
19. S. R. Lindemann and S. M. LaValle,“ Simple and efficient algorithms for computing smooth, collision-free feedback laws”. International Journal of Robotics Research, 2007.
20. S. R. Lindemann and S. M. LaValle, “Smooth feedback for car-like vehicles in polygonal environments”. In Proceedings IEEE International Conference on Robotics and Automation, 2007.
21. M. Mataric, “Integration of representation into goal-driven behavior-based robots,” *IEEE Transaction on Robotics and Automation*, vol. 3, pp. 304–312, 1992.
22. S. Mauch, “Efficient algorithms for solving static hamilton-jacobi equations,” Ph.D. dissertation, California Inst. of Technology, 2003.
23. P. Melchior, B. Orsoni, O. Laviale, A. Poty, and A. Oustaloup, “Consideration of obstacle danger level in path planning using A* and Fast Marching optimisation: comparative study,” *Journal of Signal Processing*, vol. 83, no. 11, pp. 2387–2396, 2003.
24. A. E. H. Moravec, “High resolution maps from wide angle sonar,” in *Proceedings of the IEEE International Conference on Robotics and Automation*, 1985.
25. L. Moreno, S. Garrido and F. Martin. “E-SLAM solution to the grid-based Localization and Mapping problem”. *2007 IEEE International Symposium on Intelligent Signal Processing (WISP’2007)*. Alcala Henares. Spain. pp.897-903, 2007.
26. M. V. G. Oriolo, G. Ulivi, “On-line map-building and navigation for autonomous mobile robots,” in *Proceedings of the IEEE International Conference on Robotics and Automation*, 1995, pp. 2900–2906.
27. M. V. G. Oriolo, G. Ulivi, “Fuzzy maps: A new tool for mobile robot perception and planning,” *Journal of Robotic Systems*, vol. 3, no. 14, pp. 179–197, 1997.
28. R. S. R. Philippsen, “An interpolated dynamic navigation function,” in *2005 IEEE Int. Conf. on Robotics and Automation*, 2005.
29. A. O. A. Poty, P. Melchior, “Dynamic path planning by fractional potential,” in *Second IEEE International Conference on Computational Cybernetics*, 2004.
30. E. P. Silva., P. Engel, M. Trevisan, and M. Idiart, “Exploration method using harmonic functions,” *Journal of Robotics and Autonomous Systems*, vol. 40, no. 1, pp. 25–42, 2002.

31. E. P. Silva, P. Engel, M. Trevisan, and M. Idiart, “Autonomous learning architecture for environmental mapping,” *Journal of Intelligent and Robotic Systems*, vol. 39, pp. 243–263, 2004.
32. C. Petres, Y. Pailhas, P. Patron, Y. Petillot, J. Evans, D. Lane, “Path planning for autonomous underwater vehicles,” *IEEE Trans. on Robotics*, vol. 23, no. 2, pp. 331–341, 2007.
33. S. Quinlan and O. Khatib. “Elastic bands: Connecting path planning and control. ” in *IEEE Int. Conf Robot and Autom.*, pp 802-807, 1993.
34. S. Quinlan and O. Khatib, “Efficient distance computation between nonconvex objects,” in *IEEE Int. Conf Robot and Autom.*, pp. 3324-3329, 1994.
35. E. Rimon and D.E. Koditschek, “Exact robot navigation using artificial potential functions,” *IEEE Transactions on Robotics and Automation*, vol. 8, no. 5, pp.501–518, 1992.
36. J. L. P. A. Rosenfeld, “Sequential operations in digital picture processing,” *J. of the ACM*, vol. 13, no. 4, pp. 471–494, 1966.
37. J. A. Sethian, “A Fast Marching level set method for monotonically advancing fronts,” *Proc. Nat. Acad Sci. U.S.A.*, vol. 93, no. 4, pp. 1591–1595, 1996.
38. J. A. Sethian, “Theory, algorithms, and aplications of level set methods for propagating interfaces,” *Acta numerica*, pp. 309–395, 1996, cambridge Univ. Press.
39. J. Sethian, *Level set methods*. Cambridge University Press, 1996.
40. A. B. S. B. Thrun, “Integrating grid-based and topological maps for mobile robot,” in *Proceedings of the 13th National Conference on Artificial Intelligence (AAAI-96)*, 1996, pp. 944–950.
41. A. B. S. Thrun, “Learning maps or indoor mobile robot navigation,” CMU-CS-96-121, Carnegie Mellon University, Pittsburgh, PA, Tech. Rep., 1996.
42. B. Yamauchi, “A frontier-based exploration for autonomous exploration,” in *IEEE International Symposium on Computational Intelligence in Robotics and Automation, Monterey, CA*, 1997, pp. 146–151.
43. W. A. K. G. B. Yamauchi, A. Schultz, “Integrating map learning, localization and planning in a mobile robot,” in *Proceedings of the IEEE International Symposium on Computational Intelligence in Robotics and Automation, Gaithersburg, MD*, 1998, pp. 331–336.
44. L. Yang and S. M. LaValle, “A framework for planning feedback motion strategies based on a random neighborhood graph”. In Proceedings IEEE International Conference on Robotics and Automation, pages 544–549, 2000.
45. L. Yatziv, A. Bartesaghi, and G. Sapiro, “A fast O(n) implementation of the Fast Marching algorithm,” *Journal of Computational Physics*, vol. 212, pp. 393–399, 2005.
46. J. Zelek, “A framework for mobile robot concurrent path planning and execution in incomplete and uncertain environments,” in *Proceedings of the AIPS-98 Workshop on Integrating Planning, Scheduling and Execution in Dynamic and Uncertain Environments, Pittsburgh, PA*, 1988.