ASE 381P Problem Set #8

Posting Date: December 5, 2019

You need not hand in anything. Instead, be prepared to answer any of these problems on an upcoming take-home exam. You may discuss your solutions with classmates up until the time that the exam becomes available, but do not swap work.

1. A sounding rocket deploys a payload that spins about its z-axis as it follows a ballistic trajectory through the upper atmosphere. The payload's z-axis dynamics can be modeled as

$$\begin{bmatrix} \dot{\theta}_z(t) \\ \dot{\omega}_z(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} \begin{bmatrix} \theta_z(t) \\ \omega_z(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u(t)$$

where $u(t)$ is a control torque, $a = c/h_z$ and $b = 1/h_z$, with c a drag coefficient and h_z the z-axis moment of inertia.

A malfunction occurs on the payload shortly after deployment. The latching mechanism on the booms that hold several ionospheric sensing instruments fails to lock in the deployed position. As a consequence the boom's position is uncertain. The boom design engineers are able to narrow the possibilities down to the following three modes:

1. The booms remain stowed, in which case $c = 0.1$ and $h_z = 1$.
2. The booms have locked at an intermediate position, in which case $c = 0.5$ and $h_z = 2$.
3. The booms have locked at a nearly-deployed position, in which case $c = 1$ and $h_z = 3$.

An onboard sensor provides noisy absolute angular measurements about the payload's z-axis. The measurement model is

$$z(t_k) = [1 \quad 0]\boldsymbol{x}(t_k) + w(t_k)$$

where $\boldsymbol{x}^T(t_k) = [\theta_z(t_k), \omega_z(t_k)]$ and $w(t_k)$ is zero-mean and white with $w(t_k) \sim N(0, R)$.

To salvage this multi-million dollar mission, good estimates of $\theta_z(t)$ and $\omega_z(t)$ will be required. Develop a multiple-model filter to do the estimation. Use the following steps as a guide:

(a) Assume the control input is zero-order hold (i.e., $u(t) = u(k)$ for $t_k \leq t < t_{k+1}$). Discretize the dynamics to arrive at the usual form

$$\boldsymbol{x}(k+1) = F(k)\boldsymbol{x}(k) + G(k)\boldsymbol{u}(k) + \boldsymbol{v}(k)$$

where $\boldsymbol{v}(k) \sim N(0, Q)$ is a zero-mean white process noise term that has been added to account for modeling errors.

(b) Develop a truth-model simulator that generates truth states and measurements according to the discrete-time dynamics and measurement models. Use an initial state `x1 = [0;0.1]` and covariance matrices `Q = 0.001*diag([0.1 1])` and `R = 0.1`. You may either write the simulator from scratch or fill in the missing code wherever the symbol `????` appears in the relevant parts of the file `mmExample_temp.m`.

(c) Develop a static multiple-model Kalman filter that provides MMSE state estimates by considering the three possible modes. Assume an initial state error covariance matrix of `P1 = 10*eye(nx)`. You may either write the simulator from scratch or fill in the missing code wherever the symbol `????` appears in the relevant parts of the file `mmExample_temp.m`.

(d) Experiment with different true modes and covariance values until you are familiar with the behavior of the multiple model KF.

Now suppose the boom isn't permanently locked at any one of the three possible positions. Instead, it settles at one of the possible positions but then comes dislodged and settles at another. Assume that the transitions between possible positions are immediate and infrequent ($\sim$ 20 seconds between transitions). Modify your static multiple-model Kalman filter to accommodate this mode-switching behavior. Use the ad-hoc technique discussed in Bar Shalom pg. 443 whereby the mode probabilities $\mu_j(k)$ are artificially lower bounded. Experiment with the choice of lower bound. Note that the simulator in `mmExample_temp.m` is capable of simulating such mode switching if the vectors `switchHist` and `switchIndex` are set up properly.

2. Do the sigma-point and particle filtering problems (problems I through IV) designed by Isaac Miller. These are found in the document SPF-PF_Problems.pdf.