Optical Imaging and Holograms Display by Using Linear System

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***Abstract*—In our study, we developed a model to describe how rays propagate in free space and through a simple lens. Moreover, we used our knowledge of ray tracing to discover the objects within a holographic image of multiple objects. We selected a suitable sensor width, number of pixels, distance to objects, and focal length. In the future, there is an opportunity to introduce an algorithm that can measure the contrast of images in order to automatically discover the distance to the objects of interest.**

I. Introduction

In this project, we focus on using ray tracing to reflect an optical imaging system. The introduction of Linear Algebra can describe a ray in the magnitude of the x-direction and y-direction and the angle of the x-direction and y-direction. Therefore, the Linear System is a suitable tool to model a state with these variables. We can use a matrix M to represent the transformation of a ray of light as it passes through free space or a lens as shown in Equation 1.

$$\begin{bmatrix} x_2 \\ \theta_{x2} \\ y_2 \\ \theta_{y2} \end{bmatrix} = M * \begin{bmatrix} x_1 \\ \theta_{x1} \\ y_1 \\ \theta_{y1} \end{bmatrix} \tag{1}$$

The propagation of rays may involve multiple transformation states. For instance, if rays are transformed by M_1, M_2 ... M_n. Then the final state of the rays can be calculated by Equation 2.

$$\begin{bmatrix} x_2 \\ \theta_{x2} \\ y_2 \\ \theta_{y2} \end{bmatrix} = M_n * M_{n-1} * .... * M_1 \begin{bmatrix} x_1 \\ \theta_{x1} \\ y_1 \\ \theta_{y1} \end{bmatrix} \tag{2}$$

Therefore, we can explore how each ray traces in 3-dimensional space. In an imaging system, the lens is important because it focuses an object at a certain point. In this way, holograms can be visualized by choosing an accurate focal length and distance between the object and the detector. In this study, a hologram is represented as a light field, which is a collection of many light rays. A hologram can be important in sensing, art, imaging, 3D displays, and data storage. Based on the optical image system, we can view images from holograms on traditional 2D displays. We aim to transfer holograms to sharp images in a computational imaging system.

II. Methods

In this project, we apply the Linear system and Ray tracing to model the ray propagation, imaging system, and hologram. At first, we explored how changing the number of pixels, sensor width, and sensor distance affect the hologram, without any lenses. Later, we designed an optimal focusing system to display images from holograms sharply. Later in this paper, we will create images separately from holograms with our focusing system.

A. *Part 1a - Ray propagation in Free Space*

We suppose that rays propagate in the free space for distance d_1. The Z-axis is the direction of rays travelling. We only consider the x-axis and y-axis in our ray tracing system. Then, we can write a system of equations as follows:

$$\begin{cases} x_2 = x_1 + d * tan(\theta_{x1}) \approx x_1 + d \\ \theta_{x2} = \theta_{x1} \\ y_2 = y_1 + d * tan(\theta_{y1}) \approx y_1 + d \\ \theta_{y2} = \theta_{y1} \end{cases} . \tag{3}$$

Now, we can write our transformation matrix in free space in linear form which is needed for the subsequent calculation.

$$M_{d1} = \begin{bmatrix} 1 & d_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Then, we combine Equation 1 and Equation 4. We have Equation 5 that describes the rays traveling in the free space for d_1 distance.

$$\begin{bmatrix} x_2 \\ \theta_{x2} \\ y_2 \\ \theta_{y2} \end{bmatrix} = M_{d1} * \begin{bmatrix} x_1 \\ \theta_{x1} \\ y_1 \\ \theta_{y1} \end{bmatrix} \tag{5}$$

We can visualize this in 2 dimensions, as shown in Figure 2.

B. *Part 1b - Model of Entire Image System*

The entire image system also contains a lens that can focus an object. This makes it so each ray from one point on an object is directed to one point on a sensor, as shown in Figure 3. Each lens has a focal length f. To obtain a focused image at the detector, the following relationship has to be fulfilled.

$$\frac{1}{f} = \frac{1}{d_1} + \frac{1}{d_2} \tag{6}$$

d_1 denotes the distance between the object and lens. d_2 represents the distance between the lens and focused points.

Moreover, the lens would change the angle of a ray which can be described as the following:

$$\theta_2 = \theta_1 - \frac{y}{f} \tag{7}$$

In this way, we can generate our transformation matrix when rays propagate through the lens in Equation 8.

$$M_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 1 \end{bmatrix} \tag{8}$$

Now, we combine the Equation 2, Equation 4, and Equation 8. We can model the entire image system.

$$\begin{bmatrix} x_2 \\ \theta_{x2} \\ y_2 \\ \theta_{y2} \end{bmatrix} = M_{d2} * M_f * M_{d1} \begin{bmatrix} x_1 \\ \theta_{x1} \\ y_1 \\ \theta_{y1} \end{bmatrix} \tag{9}$$

We will visualize the 2-dimensional system in our results section.

C. Part 2 - Analysis of Hologram

In this section, we will use Matlab to visualize the hologram and analyze the relationship between the images of the hologram and sensor width, the number of pixels, and the distance of propagation in free space.

First, we use the function **rays2img()** to render an image of the rays.

$$[img, x, y] = rays2img(xRays, yRays, width, pixels) \tag{10}$$

xRays denoted the start state in the x-axis and yRays is the initial state in the y-axis. Width denotes the sensor width, pixel denotes the number of pixels. This function simulates a square sensor with a specified width and number of pixels per side. It then counts the number of rays within each pixel and reports this as a brightness value. Much like a real camera sensor, it does not care about which direction the ray is traveling, just its location at the time it intersects with the sensor.

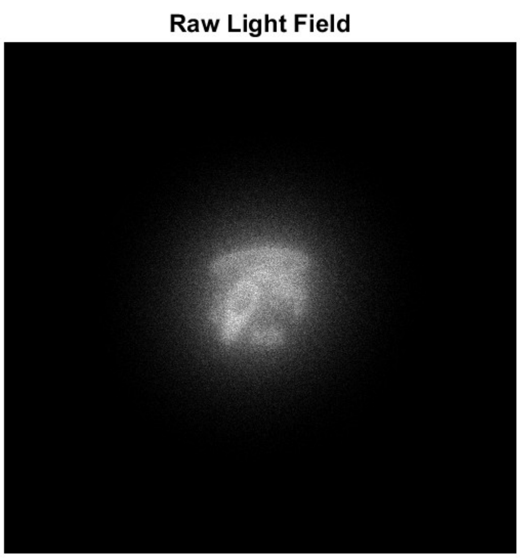

Fig. 1. Unprocessed light field, directly to camera sensor

As we can see in the figure, the unprocessed image is very blurry which requires us to experiment with the following variables and explore their effects on the image.

We tried different sensor widths, as shown in Figure 4. As the sensor size grows the central image becomes smaller and eventually, two other objects are visible. Later in this report, we will show how we distinguished between the three objects. Figure 5 shows the effect of changing the number of pixels. If there are not enough, the image is blocky and lacks resolution. With too many pixels the image becomes noisy. Finally, Figure 6 shows what happens if we propagate the image farther through free space. The image continues to blur, which indicates to us that this effect has already been applied. We will compensate for this in part 3.

However, these steps can't produce a clear image. Therefore, we need to add a lens into our focusing system by applying Equation 9. Because the lens reflects an inverse image, we will also need to invert the image before displaying it to get an image with the correct direction.

D. Part 3 - Conversion from Hologram to Separate Images

In part 3 we worked on figuring out the 3 subjects within the given lightfield. We followed the model outlined in Figure 3, of adding a lens to focus the light on the sensor, instead of directly exposing the sensor to the raw light field.

At the start, we did not know the distance the light had already traveled from the objects of the hologram to the presented light field, so the first step was to find d1. This was done in an iterative manner, by trying different values and searching for the distance that led to focused images. This process is shown in Figure 7.

As can be seen in Figure 4 at a sensor width of 0.1, there are three objects within the light field. With just a lens, they end up overlapping. An important aspect of a holographic image is that the image you see depends on the direction you are looking from. In order to separate the three images we divided the lightfield into thirds horizontally. This is equivalent in the real world to placing a card in front of part of the lens to block the light from the other portions of the hologram. This is how we separated the three objects.

This separation can also be achieved by dividing the incoming light into thirds based on each ray's x-angle. This is closer to how a hologram or 3D display would work in the real world but is not as simple to imagine in the camera model we are using.

III. Results and Discussion

A. Part 1 - Ray Tracing Models

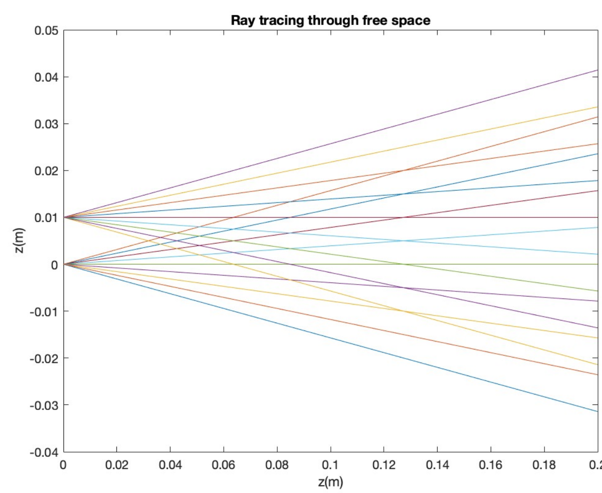

Fig. 2. Line Propagation in Free Space

Figure 2 shows ray propagation in the free space. Figure 3 visualizes how the lens changes the direction of rays.

B. Part 2 - Comparison

In this part, we explore how sensor width, the number of pixels, the distance, and the focus affect the hologram output. Now, I will analyze each variable based on our Matlab visualization.

Figure 4 shows how sensor width affects the image. When we have a larger sensor width, the detector can receive more rays. As we can see in the figure, there are three separate images when we change the sensor width to 0.1. However, the objects are too small to

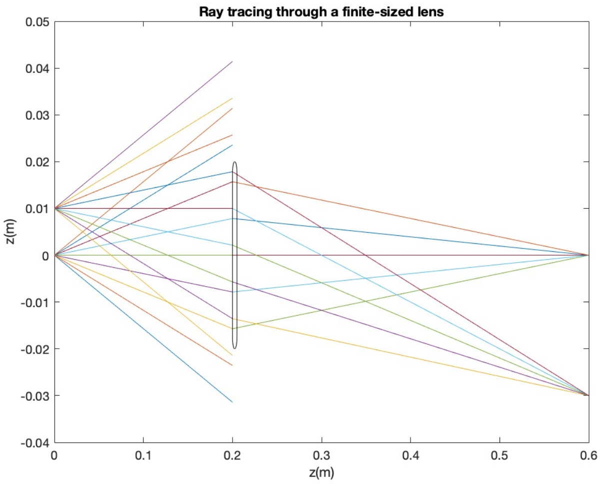

Fig. 3. The Model of Entire Image System

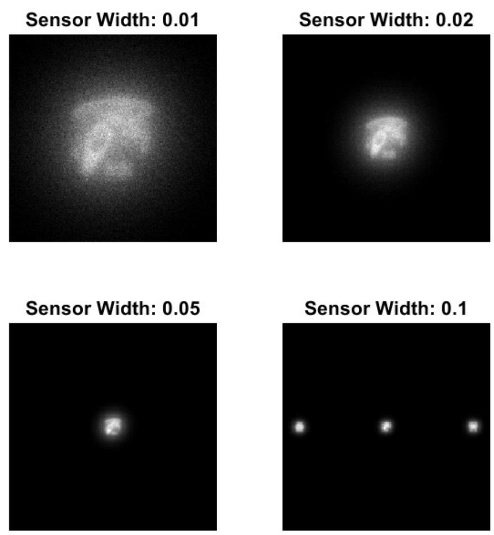

Fig. 4. The Relationship between Sensor Width and Hologram

be distinguished. Smaller sensor widths might see the details of an object. Changing the width can only change the detection area, which can't make a sharp image

In addition, we explore the relationship between the number of pixels with the hologram. If the pixel numbers are too small, the image is really blurry. However, if we have too many pixels, the boundary of an object can't be observed. The moderate number of pixels are essential for the quality of the hologram. Too many pixels lead to a noisy image as fewer light rays intersect with each pixel. Because the pixels ignore the incoming direction of each ray, changing them can not make a sharp image directly.

We can also adjust the distance of rays' propagation in free space. When the distance becomes larger, the rays become more and more dispersed. Therefore, a sharp image can't be created. As a result, we state that the lens is necessary to focus these rays on the detector.

Equations 4 and 5 show the magnitude of x and the magnitude of y are added a value $d_1 \times \theta_{x2}$. This makes the rays more and more dispersed. Therefore, we added the lens to our model to bring the rays

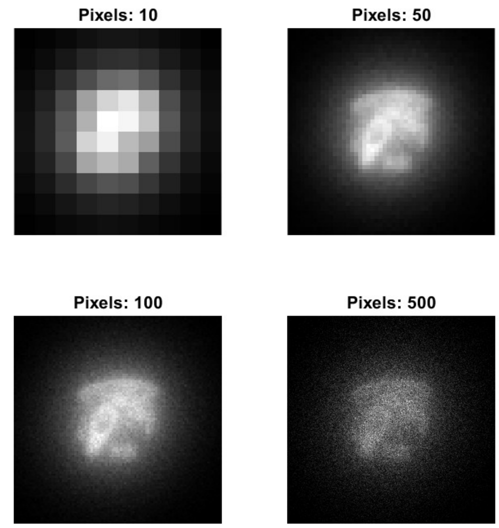

Fig. 5. The Relationship between Pixel Numbers and Hologram

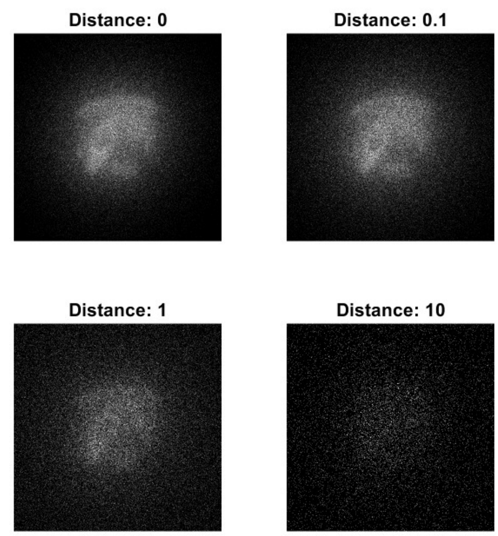

Fig. 6. The Relationship between Distance and Hologram

back together and focus them on the sensor. As equation 8 shows that the angle difference becomes less and less by $x_2 \times -\frac{1}{f}$. At the right position, everything can concentrate at one point. As we can see in Figure 3, the appropriate focal length can change the position of the reflected objects. Initially, the rays are separated.

As shown in Figure 7, when the focal distance is 0.4 we have a sharp image. We set the sensor width to 2cm, which is similar to a normal camera. d2 was set to 1.5 meters, to make the image large enough on a sensor. Real lenses usually have multiple lenses so they do not need as much distance to the sensor.

C. Part 3 - Separation of Images

As the method described, because we can't see the hologram on the screen, we separate each perspective of the hologram into three images (Figure 9, Figure 10, and Figure 11).

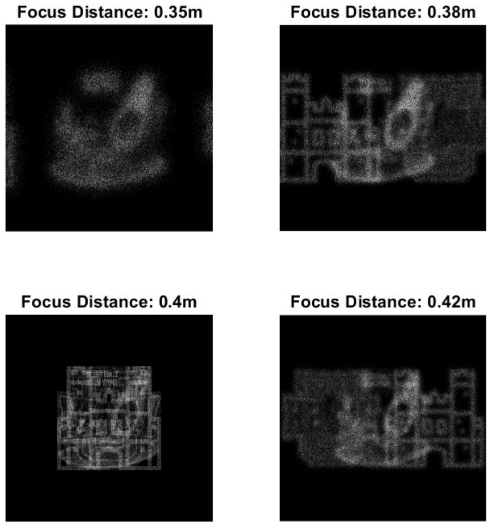

Fig. 7. The Relationship between Focal Length and Hologram

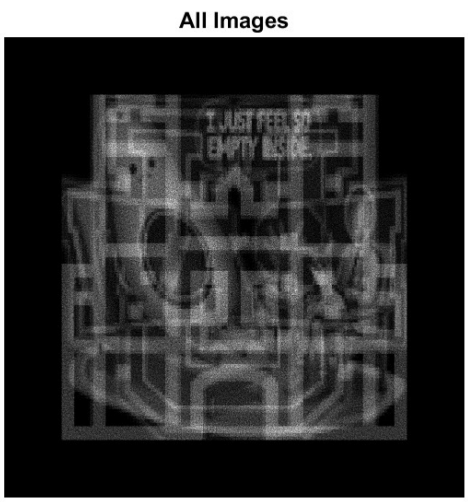

Fig. 8. Clear Image

Interestingly, Figure 9 is WashU athletics logo. Figure 10 is AI generated image by DALLE3. Figure 11 is from the Washu marketing icon library, Brookings.

D. Application

First of all, this project shows that we can model light ray propagation in different circumstances based on a linear system. In this way, we understand how the images are constructed on a monitor or the display, for instance during 3D rendering or video games. The theater and the cinema can also use this effect to create 3-dimensional visual effects. Moreover, architects can use ray tracing to evaluate the light collection ability of their designs and buildings.

Furthermore, we create the basic model to generate the hologram. The hologram can be significant in entertainment, medicine, and engineering. Specifically, holograms can create 3D game effects or displays that can make more diverse games and movies. People will be immersed in it. In addition, medicine is a complex system.

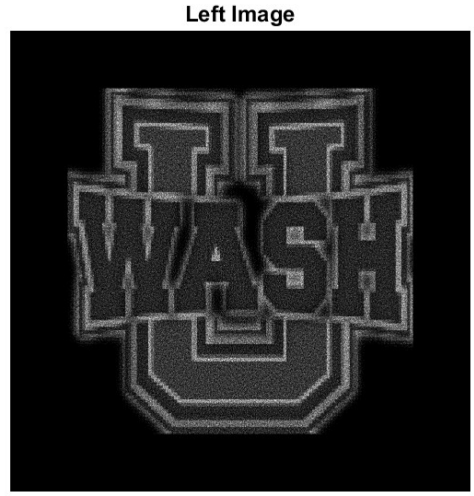

Fig. 9. The Left Image of the Hologram

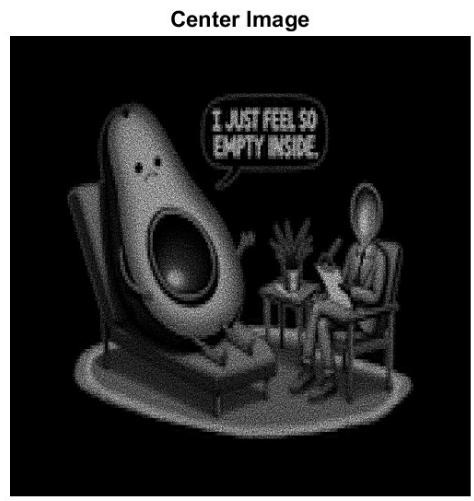

Fig. 10. The Center Image of the Hologram

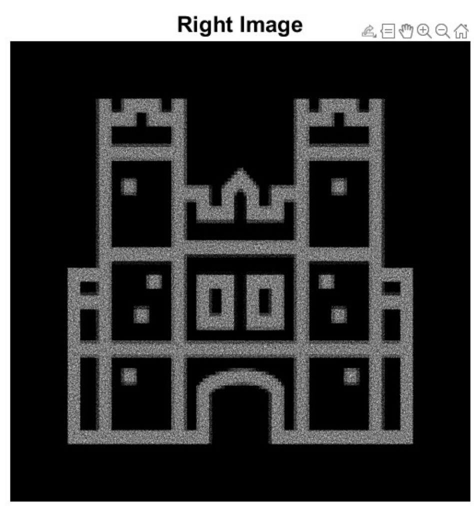

Fig. 11. The Right Image of the Hologram

People need to research human bodies and medical structures. 3D holograms can show different perspectives of the same object, which may enhance the performance of Biomedical Research or Medical innovations. Last but not least, engineers can visualize their designs from any angle and understand them better. 3D holograms may have a bright future in this field.

E. Limitations

In part 2, we needed hand tuning to discover the distance to the object. This is a timely and difficult process for a real camera. Failing to find this distance quickly can lead to blurry images or missed moments. Many cameras have algorithms to determine this directly, which we will discuss shortly.

IV. Conclusion

A. Summary

In our study, we developed a model to describe how rays propagate in the free space and through a lens. The paths are visualized by Matlab. Moreover, we used the ray tracing model to image a hologram. We experimented with different variables to observe their effects on the images. Eventually, we obtained the value for sensor width, the number of pixels, the distance, and the focal length from our observation. The hologram is composed of three images when it is focused and processed appropriately. Since we can't display the hologram, we separate each view of the hologram in computer images.

B. Future Projects

In this project, we mainly focus on modeling the ray tracing system and generating an image of the hologram. As we mentioned in the limitations, our constants are hand-tuned. A common simple technique for focusing a camera is to calculate the contrast of each image. In general, constrast is higher when an image is closer to being in focus. Other methods to focus the image involve Moreover, we can't see the effect of the hologram on the computer screen due to the 2D limitations of the monitor. Therefore, in the future, we can create the hardware that can display the hologram. In this way, the performance of the hologram can be better reflected.