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Cognitive Robotics

Assignment 6

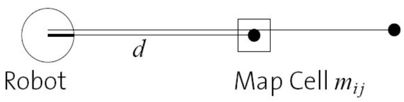
Due Tuesday, November 28th, before class.

6.1) Consider only one particular cell m_{ij} of a map at a given and fixed distance d away from the robot. Assume the robot does not move while mapping and it uses a range sensor to build the map. Suppose the inverse sensor model is given as

$$p(m = \text{occ} \mid z = d) = 0.8$$
$$p(m = \text{occ} \mid z > d) = 0.2$$

Accidentaly sum up to 1.0
($P(m=\text{occ}) + P(m=\text{free}) = 1.0$)

and the prior probability of the cell being occupied is $p(m_{ij}) = 0.3$.



(a) Formulate the update rule of the map cell with these concrete numbers using the log-odds ratio.

6 points

(b) What is the log-odds ratio after measuring 100 times if 70 measurements return the value d and 30 a value $> d$?
Compute also the resulting occupancy probability.

3 points

Counting approach

(c) What is the reflection probability of the cell?

3 points

(d) What are the benefits of the reflection map representation, and where are the problems?

3 points

6.2) A robot applies the so-called simple counting approach to build a grid map of a 1D environment consisting of the cells $c_0, \dots, c_3$.
While standing in cell c_0, the robot integrates four measurements $z_{t0}, \dots, z_{t3}$. After integrating these measurements, the resulting belief of the robot with regards to the occupancy of the four cells is $b_0 = 0.25$, $b_1 = \frac{1}{3}$, $b_2 = 0.5$, $b_3 = 1$. Given the three measurements $z_{t0} = 0$, $z_{t2} = 3$, $z_{t3} = 1$, compute the value of the measurement z_{t1}.

5 points

6.1) Consider only one particular cell m_{ij} of a map at a given and fixed distance d away from the robot. Assume the robot does not move while mapping and it uses a range sensor to build the map. Suppose the inverse sensor model is given as

$$p(m = \text{occ} \mid z = d) = 0.8$$
$$p(m = \text{occ} \mid z > d) = 0.2$$

and the prior probability of the cell being occupied is $p(m_{ij}) = 0.3$.

(a) Formulate the update rule of the map cell with these concrete numbers using the log-odds ratio.

6 points

→ Cell m_{ij} with distance d from the robot would be update as following:

$$\underbrace{l_{t,i} = l_{t-1,i}}_{\text{recursive term}} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - \underbrace{l_0}_{\text{prior}}$$

→ Before starting to build map, we set each cell to have $p(m = occ) = 0.5$. So in log-odds space it would be:

$$l_0 = \log\left[\frac{p(m = occ)}{p(m = free)}\right] = \log\left[\frac{p(m = occ)}{1 - p(m = occ)}\right] = \log \frac{0.5}{0.5} = 0$$

→ Recursive term in our case is for $\boxed{t-1 = 0}$ so we have:

$$l_{t-1}(m_{ij}) = l_0(m_{ij}) = 0$$

→ Since cell is at distance d, for inverse sensor model, we have:

$$ism(m_{ij}, x_t, z_t) = \log\left[\frac{p(m_{ij} = occ \mid z_t = d)}{p(m_{ij} = free \mid z_t = d)}\right]$$

$$= \log\left[\frac{p(m_{ij} = occ \mid z_t = d)}{1 - p(m_{ij} = occ \mid z_t = d)}\right]$$

$$= \log\left[\frac{0.8}{0.2}\right] \simeq 0.6$$

→ So we now have:

$$l_t(m_{ij}) = 0 + 0.6 - 0 = 0.6$$

→ From log-odds space we can go to probability space if needed

(b) What is the log-odds ratio after measuring 100 times if 70 measurements return the value d and 30 a value $> d$?
Compute also the resulting occupancy probability.

3 points

→ To solve this we would need to iteratively update our cell m_{ij}

for i in 0:100

$$\underbrace{l_{t,i}}_{} = \underbrace{l_{t-1,i}}_{\text{recursive term}} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - \underbrace{l_0}_{\text{prior}}$$

→ Final value for cell m_{ij} is calculated in .ipynb file and it is:

$$l_{100}(m_{ij}) = 55.44$$

→ We can compute probability of cell being occupied as well:

$$p_{100}(m_{ij} = occ) = \frac{1}{1 + \frac{1}{\exp l_{100}(m_{ij})}} \approx 1.0$$

→ This is expected result because we had much more measurements of distance d than $> d$ for cell m_{ij}

(c) What is the reflection probability of the cell?

3 points

- **hits(x,y)**: number of cases where a beam ended in cell <x,y>
- **misses(x,y)**: number of cases where a beam passed through the cell <x,y>

$$Bel(m^{[xy]}) = \frac{\text{hits}(x,y)}{\text{hits}(x,y)+\text{misses}(x,y)}$$

→ Reflection probability

$$P(m_{ij} = occ) = \frac{hits(i,j)}{hits(i,j) + misses(i,j)} = \frac{70}{70+30} = 0.7$$

(d) What are the benefits of the reflection map representation, and where are the problems?

6.2) A robot applies the so-called simple counting approach to build a grid map of a 1D environment consisting of the cells $c_0, \dots, c_3$. While standing in cell c_0, the robot integrates four measurements $z_{t0}, \dots, z_{t3}$. After integrating these measurements, the resulting belief of the robot with regards to the occupancy of the four cells is $b_0 = 0.25$, $b_1 = \frac{1}{3}$, $b_2 = 0.5$, $b_3 = 1$. Given the three measurements $z_{t0} = 0$, $z_{t2} = 3$, $z_{t3} = 1$, compute the value of the measurement z_{t1}.

5 points

$Z_{t0} = 0$ m $\qquad b_0 = 0.25$

$Z_{t1} = ?$ $\qquad b_1 = 0.33$ $\qquad \frac{a}{a+b} = \frac{1}{3}$

$Z_{t2} = 3$m $\qquad b_2 = 0.5$

$Z_{t3} = 1$ m $\qquad b_3 = 1.0$

$C_0 \qquad C_1 \qquad C_2 \qquad C_3$

$$b_i = \frac{hits(i)}{hits(i) + misses(i)}$$

For $Z_0 = 0$, C_0 has hit ; C_1, C_2, C_3 do not have miss (since ray did not pass through them)

For $Z_2 = 3$, C_3 has hit; C_0, C_1, C_2 do have miss

For $Z_3 = 1$, C_1 has hit ; C_0 does have miss
C_1, C_2 do not have miss

⊛ $Z_1 = 0$

$$b_0 = \frac{2}{2+2} = 0.5 \neq 0.25 \qquad \perp$$

⊛ $Z_1 = 1$

$$b_0 = \frac{1}{1+3} = 0.25 = 0.25 \qquad T$$

$$b_1 = \frac{2}{2+1} = \frac{2}{3} = 0.66 \neq 0.33 \qquad \perp$$

⊛ $Z_1 = 2$

$$\left.\begin{aligned} b_0 &= \frac{1}{1+3} = \frac{1}{4} = 0.25 = 0.25 \quad T \\ b_1 &= \frac{1}{1+2} = \frac{1}{3} = 0.33 = 0.33 \quad T \\ b_2 &= \frac{1}{1+1} = \frac{1}{2} = 0.5 = 0.5 \quad T \\ b_3 &= \frac{1}{1+0} = 1 = 1 \quad T \end{aligned}\right\} \quad T \quad \boxed{Z_1 = 2}$$

⊛ $Z_1 = 3$

$$b_0 = \frac{1}{1+3} = 0.25 = 0.25 \qquad T$$

$$b_1 = \frac{1}{1+2} = 0.33 = 0.33 \qquad T$$

$$b_2 = \frac{0}{0+2} = 0 \neq 0.5 \qquad \perp$$

$$b_3 = \frac{2}{2+0} = 1 = 1 \qquad T$$