6 Joint Distributions and Conditional Expectation

With data coming from several groups, we should consider both *within* group variation and *between* group variation.

Using *conditional expectation*, we can predict the value of one random variable, given the information we have about other random variables.

6.1 Joint, marginal, and conditional distributions

We introduce multivariate analogs of the CDF, PMF, and PDF.

Key concepts:

- **Distribution** of RV X provides complete information about the probability of X into any subset of real line.
- **Joint distribution** of two RVs X and Y and provides complete information about the probability of the vector (X, Y).
- **Marginal distribution** of X is the individual distribution of X ignoring the value of Y.
- **Conditional distribution** of X given $Y = y$ is the updated distribution of X after observing $Y = y$.

Discrete joint CDF, PMF:

The **joint CDF** of RVs X and Y is the function F_{XY} given by:

$$F_{XY}(x, y) = P(X \leq x, Y \leq y)$$

analogously for joint CDF of n RVs.

The **joint PMF** of discrete RVs X and Y is the function p_{XY}, given by:

$$p_{XY}(x, y) = P(X = x, Y = y)$$

analogously for joint PMF of n RVs.

We require valid joint PMF to be nonnegative and sum to 1:

$$\sum_x \sum_y P(X = x, Y = y) = 1.$$

Marginal PMF: For discrete RVs X and Y, the *marginal PMF* of X is:

$$P(X = x) = \sum_y P(X = x, Y = y)$$

The operation of summering over the possible values of Y in order to convert the joint PMF to marginal PMF is *marginalizing* out of Y.