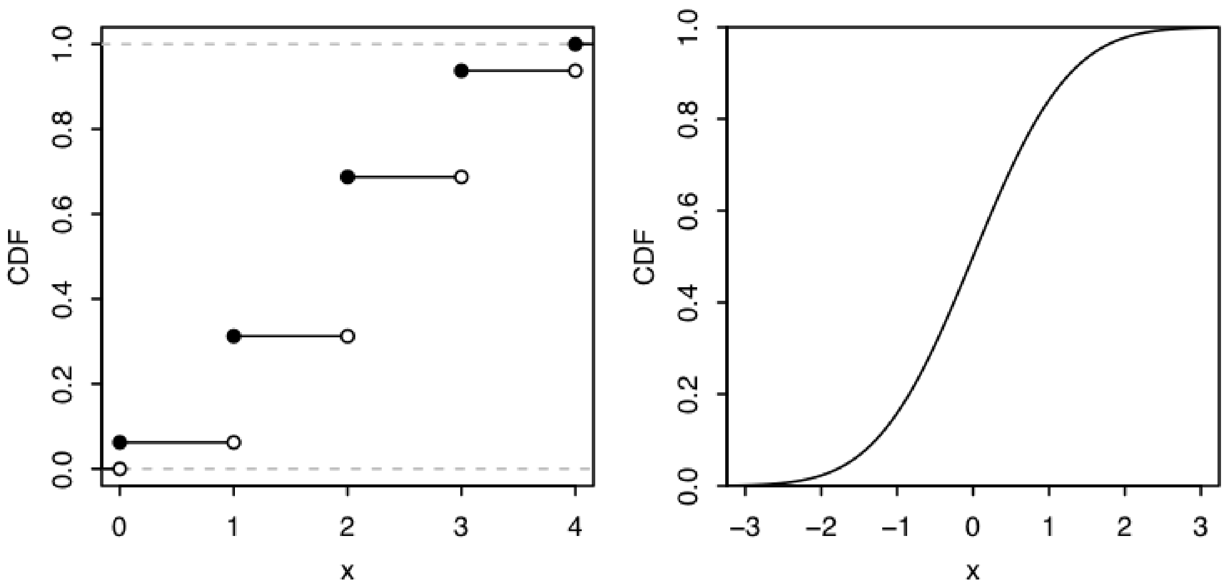
4 Continuous Random Variables

Together, discrete and continuous approaches form a powerful framework for modeling the world.

4.1 Probability density function

Continuous RVs!

An RV has a *continuous distribution* if its CDF is differentiable. Endpoints of CDF may be continuous but not differentiable. A continuous RV is a RV with a continuous distribution.



For a continuous RV X with CDF F, the PDF of X is derivative f of the CDF: $f(x) = F'(x)$

The support of X: all x where $f(x) > 0$.

The PDF is kinda similar to PMF, but for PDF quantity of $f(x)$ **is not a probability**. To obtaion the probability, we need to **integrate** PDF.

We can be carefree about including or excluding endpoints as above for continuous RVs, but we must not be careless about this for discrete RVs.

Valid PDF of a continuous RV:

1. Nonnegative: $f(x) \geq 0$
2. Integrates to 1: $\int_{-\infty}^{\infty} f(x)dx = 1$

Example: logistic distribution.

$X \sim$ Logistic.

CDF:

$$F(x) = \frac{e^x}{1+e^x}, x \in \mathbb{R}$$

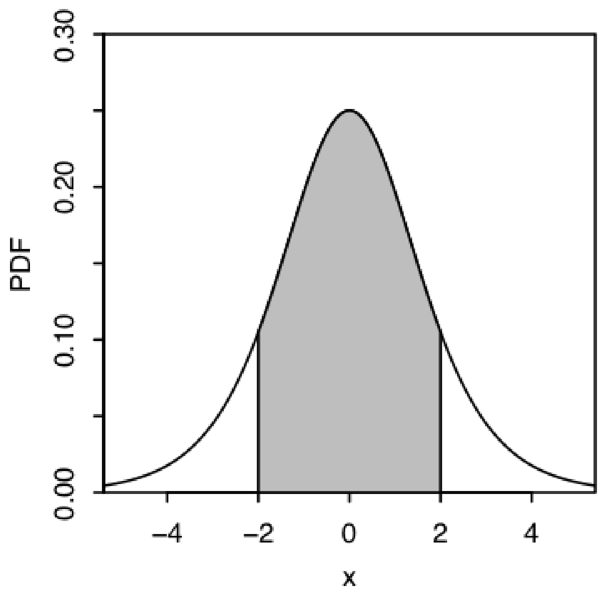
PDF:

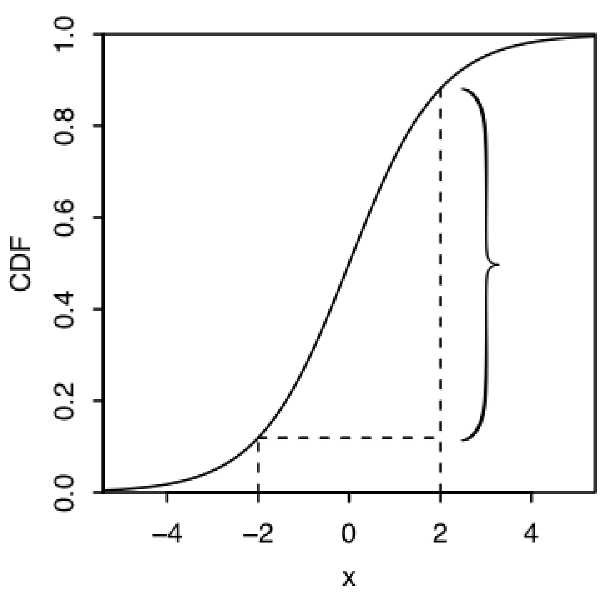
$$F(x) = \frac{e^x}{(1+e^x)^2}, x \in \mathbb{R}$$

To find $P(-2 < X < 2)$, we need to integrate PDF from -2 to 2:

$$P(-2 < X < 2) = \int_{-2}^{2} \frac{e^x}{(1+e^x)^2} = F(2) - F(-2) \approx 0.76$$

Or $P(-2 < X < 2)$ is indicated by the shaded area under the PDF and the height of the curly brace on the CDF.





4.2 Uniform distribution