7 Markov Chains

For the Markov chain, the past and **the future are conditionally independent**. For the special case of random walk on an undirected network, the network structure is the key to determining the stationary distribution.

We can picture a Markov chain intuitively by imagining a system with *states* and someone randomly wandering around from state to state.

For many interesting Markov chains, the *stationary* distribution of the chain helps us understand how the chain will behave in the long run.

7.1 Markov property and transition matrix

A sequence of RVs $X_0, X_1, X_2, \ldots$ evolving over time. This is called a *stochastic process*.

Markov chains have a form of one-step dependence, allowing to do beyond IIDs bust still have very convenient structure.

Markov chains widely used for simulations of complex distributions, via algorithms known as *Markov chain Monte Carlo (MCMC)*.

Markov chains live in both space and time: the set of possible states X_n is called *state time*, and index n represents evolution of the process over *time*. The state space of can be discrete or continuous, and time can also be discrete or continuous. We will focus on *discrete-state*, *discrete-time* Markov Chains with a *finite* state space.

Markov Chain

A sequence of RVs $X_0, X_1, X_2, \ldots$ taking values in *state space* $\{1, 2, \ldots, M\}$ is called *Markov chain* $\forall\ n \geq 0$,

$$P(X_{n+1} = j | X_n = i, X_{n-1} = i-1, \ldots, X_0 = i_0) = P(X_{n+1} = j | X_n = i)$$

$P(X_{n+1} = j | X_n = j)$ is called the *transition probability.* from state i to state j. This Markov chain is $time - homogeneous$, which means that

$P(X_{n+1} = j | X_n = j)$ is the same $\forall\ n$.

We can describe the probabilities of moving from state to state using a matrix called *translation matrix* whose i, j entry is probability of going from i-th to j-th state in a single step.

Translation matrix

Let $X_0, X_1, X_2, \ldots$ be a Markov chain $\{1, 2, \ldots, M\}$ and let $q_{ij} = P(X_{n+1} = j | X_n = i)$ be transition probability from state i to state j. The matrix $Q = (q_{ij})$ is the *transition matrix* of the chain. Q is nonnegative and each row sums to 1.