Analysis of Engineering Systems – MAE 488

Homework # 3

Spring 2019

Instructions: SHOW YOUR WORK! *If insufficient work is shown, you will receive no credit (even for a correct answer). As always, be sure to include units where appropriate. All plots should have labels on each axis (with units), a title (e.g. "MAE 488, Homework 1, Problem 1, Part a"), and a legend if more than one plot is in the same figure (except for subplots). The text book has compiled tables with the solution of various differential equations (e.g. Table 2.3.1, 2.3.2). While you can use these to check your homework, do not use them for your solution. Problems solved using such formulas will receive zero credit. The same is true for quizzes and exams.*

1. Use the initial and final value theorems to determine $x(0^+)$ and $x(\infty)$ for the following:
    a. $X(s) = \frac{4}{2s+5}$ b. $X(s) = \frac{9s+2}{3s^2+7s+4}$ c. $X(s) = \frac{4}{s^2+25}$

2. Solve the following problems using the Laplace transform method

    a. $\ddot{x}(t) + 9\dot{x}(t) + 14x(t) = 0, \quad x(0) = 3, \quad \dot{x}(0) = 2$
    b. $\ddot{x}(t) + 6\dot{x}(t) + 9x(t) = 0, \quad x(0) = 4, \quad \dot{x}(0) = 3$
    c. $\ddot{x}(t) + 4\dot{x}(t) + 13x(t) = 0, \quad x(0) = 2, \quad \dot{x}(0) = 5$

3. Solve the following problem with the trial solution method. Identify the free, forced, transient, and steady-state responses.

$$\ddot{x}(t) + 10\dot{x}(t) + 34x(t) = 0, \quad x(0) = 1, \quad \dot{x}(0) = 1$$

4. Determine if the following ODEs correspond to a system that is stable, unstable, or neutrally stable.

    a. $\ddot{x}(t) + 8\dot{x}(t) + 15x(t) = 10$
    b. $\ddot{x}(t) + 10\dot{x}(t) + 25x(t) = 10$
    c. $2\dot{x}(t) - 9x(t) = 5$
    d. $\dot{x}(t) = 5$
    e. $\ddot{x}(t) + 15x(t) = 10$
    f. $\ddot{x}(t) + 2\dot{x}(t) = 10$

5. If applicable, for each of the following characteristic equations calculate ζ, ω_n, τ, and ω_d. Recall that τ can be defined in terms of ζ and ω_n for 2nd order systems. If not, state the reason why for each parameter.

    a. $s^2 + 10s + 74 = 0$
    b. $s^2 - 4s + 13 = 0$
    c. $s^2 + 16s + 64 = 0$
    d. $s + 5 = 0$

6. Obtain the transfer function for the following ODEs (or system of ODEs) with input $f(t)$

   a. $\ddot{x}(t) + \quad 10\dot{x}(t) + 21x(t) = 17f(t)$
   b. $\ddot{x}(t) + \quad 6\dot{x}(t) + 15x(t) = 3\dot{f}(t) + 7f(t)$
   c. $\dot{y} + 3y + 15x = \ f(t) \qquad \dot{x}(t) = \frac{y}{3}$

7. For the following ODE
   i. Solve for $x(t)$
   ii. Compare the values of $x(0^+)$ and $x(0^-)$
   iii. Compare the values of $\dot{x}(0^+)$ and $\dot{x}(0^-)$

$$2\ddot{x}(t) + 30\dot{x}(t) + 112x(t) = 5\delta(t), \quad x(0^-) = \dot{x}(0^-) = 0$$

8. For the following ODE
   i. Calculate the response
   ii. Use the appropriate function in MATLAB to solve for the response
   iii. Verify your answer by plotting the results from both methods (solid blue line for calculated, solid red line for Matlab).

$$4\ddot{x}(t) + 28\dot{x}(t) + 24x(t) = 3u(t)$$