Analysis of Engineering Systems – MAE 488

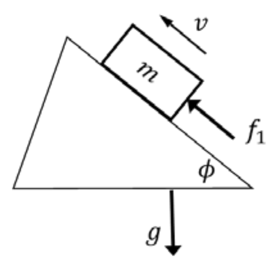
Homework # 4

Spring 2019

Instructions: SHOW YOUR WORK! *If insufficient work is shown, you will receive no credit (even for a correct answer). As always, be sure to include units where appropriate. All plots should have labels on each axis (with units), a title (e.g. "MAE 488, Homework 1, Problem 1, Part a"), and a legend if more than one plot is in the same figure (except for subplots).*

Ex. 3.1.2
Ex. 3.1.3

1. Consider the block on an incline in the figure below.

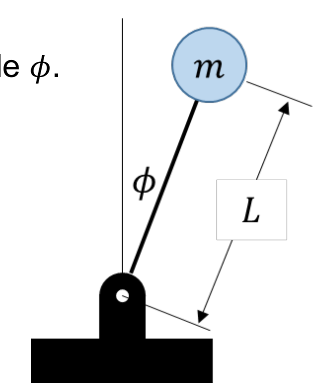


   a. Draw the free body diagram assuming the velocity is positive
   b. Derive the equations of motion in the form: $ma = \sum forces$
   c. Draw the free body diagram assuming the velocity is negative
   d. Derive the equations of motion in the form: $ma = \sum forces$
   e. Suppose $m = 10$ kg, $\phi = 25°$, $v_o = 2$ m/s, and $\mu = 0.3$. Determine if the mass comes to rest if
      i. $f_1 = 100$ N
      ii. $f_1 = 50$ N

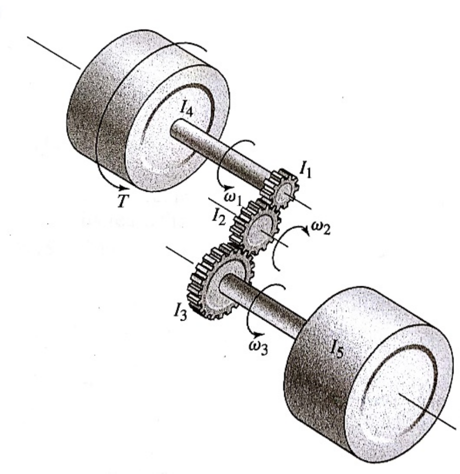
      In either case, if the mass comes to a rest, compute the time at which it stops.

P3.9

2. Consider the inverted pendulum to the right.
   a. Derive the nonlinear equations of motion in terms of the angle ϕ.
   b. Linearize the equations of motion assuming that ϕ is small.



P3.24 3. Consider the geared system shown below.



The inertias are

$I_1 = 10^{-3}\ \mathrm{kg \cdot m^2}$ $\quad I_2 = 3.84\ \times 10^{-3}\ \mathrm{kg \cdot m^2}$ $\quad I_3 = 0.0148\ \mathrm{kg \cdot m^2}$

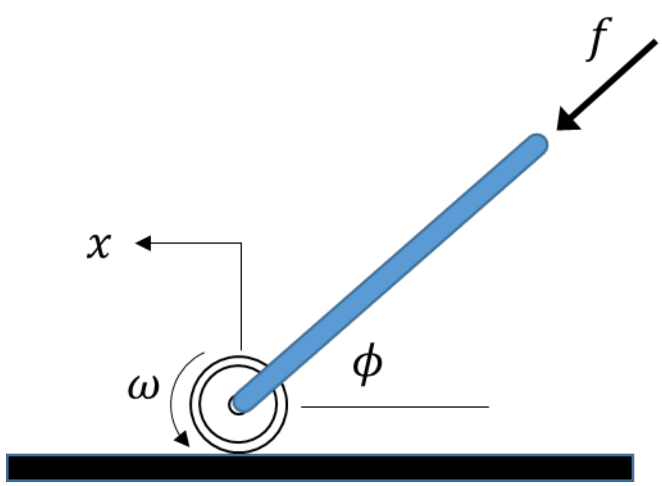
$I_4 = 0.03\ \mathrm{kg \cdot m^2}$ $\quad I_5 = 0.15\ \mathrm{kg \cdot m^2}$

The speed ratios are

$$\frac{\omega_1}{\omega_2} = \frac{\omega_2}{\omega_3} = 1.6$$

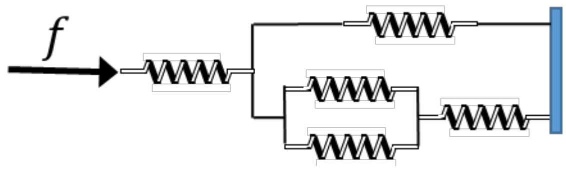
Assume the shaft inertias are negligible. Derive the system model in terms of the speed ω_3 with the applied torque, T, as the input.

P3.33 4. Consider the roller and handle shown below.



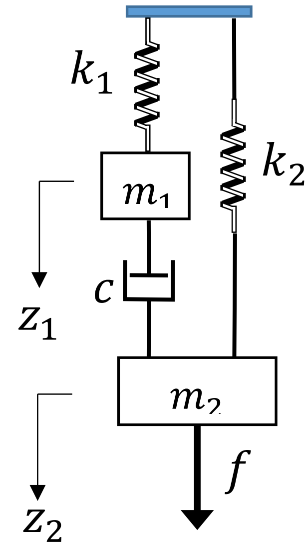
The roller has a radius, R, and inertia $\frac{mR^2}{2}$. A person pushes the handle with a force f applied at an angle ϕ to the horizontal. The roller weighs 800 N and has a diameter of 0.4 m. Assuming the roller does not slip; derive the equations of motion in terms of (a) the rotational velocity ω and (b) the linear displacement x.

P4.9 5. Determine the equivalent spring constant for the arrangement shown below. All of the springs have the same spring constant, k.

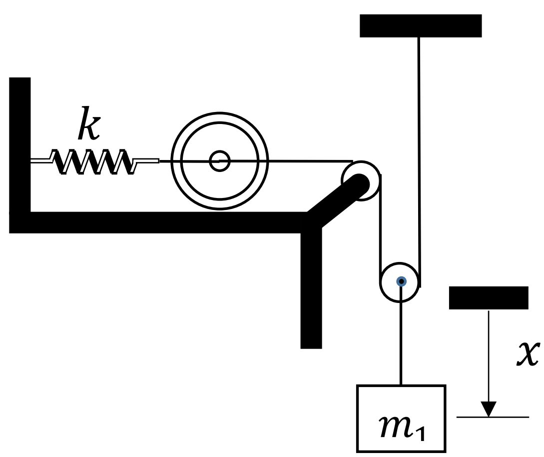


6. Consider the mass spring damper system shown to the right. Suppose $k_1 = k,\ k_2 = 3k$, and $m_1 = m_2 = m$. Obtain the equations of motion in terms of the displacements from the equilibrium position, z_1 and z_2.

P4.28
With Damper



P4.32 7. Consider the system shown below. The spring is connected via a cable to the non-rotating shaft of the roller with mass m_2 and radius R. A second cable connected to the shaft passes through the two pulleys.



Use conservation of energy to derive the equations of motion. The equilibrium position corresponds to $x = 0$. Neglect the masses of the pulleys and assume the cable is inextensivle.

P4.90 8. (a) Obtain the equations of motion for the system shown below. The masses are $m_1 = 20$ kg and $m_2 = 60$ kg. The spring constants are $k_1 = 3 \times 10^4$ N/m and $k_2 = 6 \times 10^4$ N/m.

(b) Derive the transfer function(s)

(c) Obtain a plot of the unit step response of x_1.

(d) Obtain a plot of the unit step response of x_2.

(e) Obtain a plot of both unit step responses in one figure.