Analysis of Engineering Systems – MAE 488

Homework # 2

Spring 2019

Instructions: SHOW YOUR WORK! *If insufficient work is shown, you will receive no credit (even for a correct answer). As always, be sure to include units where appropriate.*

1. Determine if the following ordinary differential equations (ODEs) are linear or nonlinear and state the reason why if nonlinear.

   a. $\ddot{x}x + 3\dot{x} = 0$
   b. $\dot{z} = \sin z$
   c. $\dot{z} = \sin(t)$
   d. $y'' + 3yy' = 0$
   e. $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = e^x$
   f. $\ddot{y} + 3t^2\dot{y} + 5y^2 = e^{-2t}$

2. Solve the following ODE with initial conditions by direct integration. Check your answer by taking the appropriate derivative(s) of the solution.

$$4\ddot{x} = 3t,\ \ x(0) = 1,\ \dot{x}(0) = 2$$

3. Solve the following ODE with initial conditions by separation of variables. Check your answer by taking the appropriate derivative(s) of the solution.

$$\frac{dy}{dx} + 4y = 8\ ,\ y(0) = 3$$

4. Use the table of Laplace transforms and properties to obtain the Laplace transform of the following functions. Specify which transform pair or property is used and write in the simplest form. For part b, do not use # 29 in Table 2.2.1.
   a. $x(t) = \cos(3t)$
   b. $y(t) = t\cos(3t)$
   c. $z(t) = e^{-2t}[t\cos(3t)]$
   d. $x(t) = 3\sin(4t) + 5\cos(4t)$
   e. $y(t) = t^3 + 3t^2$
   f. $z(t) = t^4e^{-2t}$

5. For each of the following Laplace transforms: (i) Use partial fraction expansion to obtain the inverse Laplace transform and (ii) verify your answer using the residue function in Matlab (show). If the denominator of the Laplace transform has complex roots, express the solution in terms of a sine and a cosine.

   a. $F(s) = \frac{25}{s(s+4)^2}$
   b. $F(s) = \frac{21}{s^2(s+3)}$
   c. $F(s) = \frac{2s+2}{s^2+6s+13}$
   d. $F(s) = \frac{20s+16}{s^3+6s^2+8s}$.