노범환 (BeomHwan Roh)

최소제곱법 $\hat{y}_i = \beta_0 + \beta_1 x_i$

$$Q = \Sigma (y_i - \hat{y}_i)^2 = \Sigma (y_i - \beta_0 + \beta_1 x_i)^2$$

$$\left.\frac{\partial Q}{\partial \beta_0}\right|_{\substack{\beta_0 = b_0 \\ \beta_1 = b_1}} = 2\Sigma (y_i - b_0 - b_1 x_i)\cdot(-1) = 0 \quad \cdots ①$$

$$\left.\frac{\partial Q}{\partial \beta_1}\right|_{\beta_0 = b} = 2\Sigma (y_i - b_0 - b_1 x_i)(-x_i) = 0 \quad \cdots ②$$

$$① : \Sigma (y_i - b_0 - b_1 x_i) = n\bar{y} - nb_0 - b_1 n\bar{x} = 0$$

$$\Rightarrow b_0 = \bar{y} - b_1 \bar{x}$$

$$② \quad \Sigma (y_i - b_0 - b_1 x_i) x_i = \Sigma (y_i - \bar{y} + b_1 \bar{x} - b_1 x_i) x_i = 0.$$

$$\Rightarrow \Sigma x_i (y_i - \bar{y}) = b_1 \Sigma x_i (x_i - \bar{x})$$

$$\Rightarrow b_1 = \frac{\Sigma x_i (y_i - \bar{y})}{\Sigma x_i (x_i - \bar{x})} = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

$$(\because \Sigma \bar{x}(y_i - \bar{y}) = 0, \ \Sigma \bar{x}(x_i - \bar{x}))$$

↳ constant

$$\therefore b_0 = \bar{y} - b_1 \bar{x}, \quad b_1 = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}}.$$

$$\begin{cases} S_{xx} = \Sigma (x_i - \bar{x})^2 \\ S_{yy} = \Sigma (y_i - \bar{y})^2 \\ S_{xy} = \Sigma (x_i - \bar{x})(y_i - \bar{y}) \end{cases}$$

상관계수 (R) 의 제곱은 결정계수 (R^2)

상관계수 $R = \dfrac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}} = \dfrac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}}$

결정계수 $R^2 = 1 - \dfrac{SSE}{SST} = \dfrac{SSR}{SST}$

$$\begin{cases} SST = \sum (y_i - \bar{y})^2 \\ SSR = \sum (\hat{y}_i - \bar{y})^2 \\ SSE = \sum (y_i - \hat{y}_i)^2 \end{cases}$$

$b_0 = \bar{y} - b_1 \bar{x}, \quad b_1 = \dfrac{S_{xy}}{S_{xx}}$

$$SSR = \sum (\hat{y}_i - \bar{y})^2 = \sum (b_0 + b_1 x_i - \bar{y})^2 = \sum (\cancel{\bar{y}} - b_1 \bar{x} + b_1 x_i - \cancel{\bar{y}})^2$$

$$= b_1^2 \sum (x_i - \bar{x})^2 = b_1^2 S_{xx}.$$

$$SST = \sum (y_i - \bar{y})^2 = S_{yy}.$$

$$\frac{SSR}{SST} = \frac{b_1^2 \cdot S_{xx}}{S_{yy}} = \frac{S_{xy}^2 \cdot \cancel{S_{xx}}}{S_{xx}^{\cancel{2}} \cdot S_{yy}} = \left(\frac{S_{xy}}{\sqrt{S_{xx} \cdot S_{yy}}} \right)^2 = R^2$$

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