베타분포 $Beta(\alpha,\beta)$, $X_1 \sim \Gamma(\alpha,1)$ $X_2 \sim \Gamma(\beta,1)$ 독립.

$Y_1 = X_1 + X_2$, $Y_2 = \frac{X_1}{X_1+X_2}$ 독립, $Y_2 \sim Beta(\alpha,\beta)$, $f(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}$, $0<y<1$

$\mu = \frac{\alpha}{\alpha+\beta}$, $\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$

t분포 $T = \frac{Z}{\sqrt{V/r}}$, $Z \sim N(0,1)$, $V \sim \chi^2(r)$, $Z \perp\!\!\!\perp V$

$f(x) = \frac{\Gamma(\frac{r+1}{2})}{\sqrt{\pi r}\,\Gamma(\frac{r}{2})}\left(1+\frac{x^2}{r}\right)^{-\frac{r+1}{2}}$, $E(T)=0$, $V(T) = \frac{r}{r-2}$

$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ 증명: $V = \Sigma\left(\frac{X_i-\mu}{\sigma}\right)^2 \sim \chi^2(n)$, $V = \Sigma\left(\frac{X_i-\bar{X}}{\sigma}\right)^2 + n\left(\frac{\bar{X}-\mu}{\sigma}\right)^2 = \frac{(n-1)S^2}{\sigma^2} + \left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}\right)^2$

mgf: $(1-2t)^{-\frac{n}{2}} = E\left(\exp\left(\frac{(n-1)S^2}{\sigma^2}t\right)\right)\cdot(1-2t)^{-\frac{1}{2}} \Rightarrow \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

$T = \frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \Big/ \sqrt{\frac{(n-1)S^2}{\sigma^2}/(n-1)} \sim t(n-1)$

F분포: $\frac{U/r_1}{V/r_2}$, $f(x) = \frac{\Gamma(\frac{r_1+r_2}{2})}{\Gamma(\frac{r_1}{2})\Gamma(\frac{r_2}{2})}\left(\frac{r_1}{r_2}\right)^{\frac{r_1}{2}} \cdot x^{\frac{r_1}{2}-1} \cdot \left(\frac{1}{\frac{r_1 w}{r_2}+1}\right)^{\frac{r_1+r_2}{2}}$, $0<x<\infty$, $E(W) = \frac{r_2}{r_2-2}$

순서통계량. $f_{Y_k}(y_k) = \frac{n!}{(n-k)!(k-1)!} f_X(y_k) F_X(y_k)^{k-1}[1-F_X(y_k)]^{n-k}$

가장 작은 order: $\underbrace{P(Y_1>y_1)}_{1-F_{Y_1}(y_1)} = P(X_1>y_1, X_2>y_1, \cdots, X_n>y_1) \overset{ind}{=} P(X_1>y_1)P(X_2>y_1)\cdots P(X_n>y_1)$

$= (1-F_X(y_1))(1-F_X(y_1))\cdots(1-F_X(y_1)) = (1-F_X(y_1))^n$

가장 큰 order: $\underbrace{P(Y_n<y_n)}_{F_{Y_n}(y_n)} = P(X_1<y_n, X_2<y_n, \cdots, X_n<y_n) \overset{ind}{=} P(X_1<y_n)P(X_2<y_n)\cdots P(X_n<y_n)$

$= F_X(y_n)\cdot F_X(y_n)\cdots F_X(y_n) = F_X(y_n)^n$

지수족: $f(x;\theta) = h(x)\exp(\eta(\theta)T(x) - B(\theta))$

$B(n,p)$: $\binom{n}{x}p^x(1-p)^{n-x} = \underbrace{\binom{n}{x}}_{h(x)}\exp\left(\underbrace{\log\frac{p}{1-p}}_{\eta(p)}\cdot\underbrace{x}_{T(x)} + \underbrace{\log(1-p)^n}_{-B(p)}\right)$

$poi(\lambda)$: $\frac{\lambda^x e^{-\lambda}}{x!} = \underbrace{\frac{1}{x!}}_{h(x)}\exp\left(\underbrace{\log\lambda}_{\eta(\lambda)}\cdot\underbrace{x}_{T(x)} - \underbrace{\lambda}_{B(\lambda)}\right)$ / $N(\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = \underbrace{\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right)}_{h(x)}\exp\left(\underbrace{\frac{\mu}{\sigma^2}}_{\eta(\mu)}\overbrace{x}^{T(x)} - \underbrace{\frac{\mu^2}{2\sigma^2}}_{B(\mu)}\right)$

$E\left(\frac{\partial\eta(\theta)}{\partial\theta}T(X)\right) = \frac{\partial B(\theta)}{\partial\theta}$, $V\left(\frac{\partial\eta(\theta)}{\partial\theta}T(X)\right) = \frac{\partial^2 B(\theta)}{\partial\theta^2} - E\left(\frac{\partial^2\eta(\theta)}{\partial\theta^2}T(X)\right)$

canonical form: $f(x;\theta) = h(x)\exp(\theta T(x) - A(\theta))$ / $M_T(s) = \exp(A(\theta+s) - A(\theta))$

$E_\theta(T(X)) = \dot{A}(\theta)$, $Var_\theta(T(X)) = \ddot{A}(\theta)$

multiparameter: $f(x;\theta) = h(x)\exp\left(\sum_{i=1}^k \eta_i(\theta)T_i(x) - B(\theta)\right) = h(x)\exp\left(\sum_{i=1}^k \theta_i T_i(x) - A(\theta)\right)$

$E\left(\sum_{k=1}^k \frac{\partial\eta_k(\theta)}{\partial\theta_j}T_k(X)\right) = \frac{\partial B(\theta)}{\partial\theta_j}$, $Var\left(\sum_{k=1}^k \frac{\partial\eta_k(\theta)}{\partial\theta_j}T_k(X)\right) = \frac{\partial^2 B(\theta)}{\partial\theta_j^2} - E\left(\sum_{k=1}^k \frac{\partial^2\eta_k(\theta)}{\partial\theta_j^2}T_k(X)\right)$

정규분포: $f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = \underbrace{1}_{h(x)}\exp\left(\underbrace{-\frac{1}{2\sigma^2}}_{\eta_1(\mu,\sigma^2)}\overbrace{x^2}^{T_1(x)} + \underbrace{\frac{\mu}{\sigma^2}}_{\eta_2(\mu,\sigma^2)}\overbrace{x}^{T_2(x)} \underbrace{- \log\sqrt{2\pi\sigma^2} - \frac{\mu^2}{2\sigma^2}}_{-B(\mu,\sigma^2)}\right)$

$logit\, p = \log\frac{p}{1-p} = \theta \Rightarrow p = \frac{e^\theta}{1+e^\theta}$

$\log\frac{f(x|y=1)}{f(x|y=0)} = \beta_0 + \beta_1 x \Rightarrow P(y=1|x) = \frac{\exp(\log p_0 + \beta_0 + \beta_1 x)}{1+\exp(\log p_0 + \beta_0 + \beta_1 x)}$, $p_0 = \frac{P(Y=1)}{P(Y=0)}$

$logit\{P(y=1|x)\} \simeq \sum_{k=0}^{8}\beta_k x^k$

$E(X_2) = E[E(X_2|X_1)]$, $Var(X_2) = E[Var(X_2|X_1)] + Var[E(X_2|X_1)]$