마르코프 : $u(X)>0$, $E(u(X))$ 존재 , $P[u(X)\ge c] \le \frac{E[u(X)]}{c}$

체비셰프 : X : r.v , 분산 $\sigma^2<\infty$, $E(X)=\mu$, 모든 $k>0$에 대해 $P(|X-\mu|\ge k\sigma)\le \frac{1}{k^2}$

Convergence in probability : $X_n \xrightarrow{P} X \Leftrightarrow \forall\epsilon>0,\ \lim_{n\to\infty} P(|X_n-X|\ge\epsilon)=0$.

· $\bar X_n = \frac{1}{n}\sum X_i \Rightarrow \bar X_n \xrightarrow{P}\mu$ 증명 : $P(|\bar X_n-\mu|\ge\epsilon) = P(|\bar X_n-\mu|\ge \underbrace{(\frac{\sqrt{n}\,\epsilon}{\sigma})}_{k}\cdot\underbrace{(\frac{\sigma}{\sqrt{n}})}_{\sigma}) \le (\frac{\sigma}{\sqrt{n}\,\epsilon})^2 \xrightarrow{n\to\infty} 0$

Convergence in distribution : $X_n\xrightarrow{D}X \Leftrightarrow \lim_{n\to\infty} F_{X_n}(x)=F_X(x)\ \ \forall x\in C(F_X)$ F_X가 연속인 모든 점의 집합

Slutsky th. : X_n, X, A_n, B_n : r.v , a,b : 상수, $X_n\xrightarrow{D}X$, $A_n\xrightarrow{P}a$, $B_n\xrightarrow{P}b$

$\Rightarrow A_n+B_nX_n \xrightarrow{D} a+bX$.

Delta method : $\sqrt{n}(X_n-\theta)\xrightarrow{D}N(0,\sigma^2)$, $g(x)$: θ에서 미분가능 , $g'(\theta)\ne0$.

$\Rightarrow \sqrt{n}(g(X_n)-g(\theta)) \xrightarrow{D} N(0,\sigma^2\{g'(\theta)\}^2)$ / $\frac{X_n-\theta}{\sigma/\sqrt{n}}\xrightarrow{D}N(0,1) \Rightarrow \frac{g(X_n)-g(\theta)}{\frac{\sigma}{\sqrt{n}}g'(\theta)}\xrightarrow{D}N(0,1)$

$g'(\theta)=0,\ g''(\theta)\ne0 \Rightarrow n(g(X_n)-g(\theta)) \xrightarrow{D} \sigma^2\frac{g''(\theta)}{2}\chi_1^2 = \Gamma(\frac{1}{2}, \sigma^2 g''(\theta))$

증명 idea : $g(X_n)=g(\theta)+g'(\theta)(X_n-\theta)+g''(\theta)\frac{(X_n-\theta)^2}{2}+o_p(|X_n-\theta|^2)$

CLT : $X_1,\cdots,X_n$: iid r.v , μ,σ^2 , $Y_n=\frac{\sum X_i - n\mu}{\sqrt{n}\,\sigma} = \frac{\bar X_n-\mu}{\sigma/\sqrt{n}}\xrightarrow{D}N(0,1)$

증명: $Z_i=\frac{X_i-\mu}{\sigma}$, $\sum Z_i = \frac{\sum X_i - n\mu}{\sigma}$ $Y_n=\sum Z_i\cdot\frac{1}{\sqrt{n}}$, $M_{Y_n}(t) = \cdots = M_{\sum Z_i}(\frac{t}{\sqrt{n}}) \overset{iid}{=} \cdots = (M_Z(\frac{t}{\sqrt{n}}))^n$

$= (\sum_{k=0}^{\infty} \frac{M_Z^{(k)}(0)}{k!}(\frac{t}{\sqrt{n}})^k)^n = \cdots \xrightarrow{n\to\infty} e^{\frac{t^2}{2}}$

이항분포 : $p(x)=\binom{n}{x}p^x(1-p)^{n-x}$, $M(t)=[(1-p)+pe^t]^n$, $\mu=np$, $\sigma^2=np(1-p)$

음이항분포 : $p(x)=\binom{x+r-1}{r-1}p^{r-1}(1-p)^x$, $M(t)=p^r(1-(1-p)e^t)^{-r}$, $\mu=r\frac{1-p}{p}$, $\sigma^2=r\frac{1-p}{p^2}$

다항분포 : $p(x_1,\cdots,x_k)=\frac{n!}{x_1!\cdots x_k!}p_1^{x_1}\cdots p_{k-1}^{x_{k-1}}p_k^{x_k}$, $M(t_1,\cdots,t_k)=(p_1e^{t_1}+p_2e^{t_2}+\cdots+p_{k-1}e^{t_{k-1}}+p_k)^n$

초기하분포 : $p(x)=\frac{\binom{N-D}{n-x}\binom{D}{x}}{\binom{N}{n}}$ $\mu=\frac{Dn}{N}$ $\sigma^2=n\cdot\frac{D}{N}\cdot\frac{N-D}{N}\cdot\frac{N-n}{N-1}$

확률정의
1. $P(A)\ge0\ \forall A\in\mathcal{F}$
2. $P(\Omega)=1$
3. A_m, A_n 서로소 $P(\bigcup_{n=1}^{\infty} A_n)=\sum_{n=1}^{\infty} P(A_n)$

포아송분포 : $p(x)=\frac{\lambda^x e^{-x}}{x!}$, $M(t)=e^{\lambda(e^t-1)}$, $\mu=\lambda$, $\sigma^2=\lambda$

정규분포 : $f(x)=\frac{1}{\sqrt{2\pi}\,\sigma}\exp(-\frac{(x-\mu)^2}{2\sigma^2})$, $M(t)=\exp(\mu t+\frac{1}{2}\sigma^2t^2)$

$\Gamma(\alpha)=\int_0^\infty y^{\alpha-1}e^{-y}dy$, $\Gamma(1)=1$, $\Gamma(\alpha)=(\alpha-1)\Gamma(\alpha-1)=(\alpha-1)!$, $\Gamma(\frac{1}{2})=\sqrt{\pi}$.

감마분포 : $\Gamma(\alpha,\beta)$, $f(x)=\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-\frac{x}{\beta}}$, $M(t)=(1-\beta t)^{-\alpha}$, $\mu=\alpha\beta$, $\sigma^2=\alpha\beta^2$

$X_i \overset{iid}{\sim}\Gamma(\alpha_i,\beta)$ 이면 $\sum X_i\sim\Gamma(\sum\alpha_i,\beta)$ 이고, $kX\sim\Gamma(\alpha,k\beta)$ 이다

W_k : Poi(λ)에서 k번째 사건이 일어나기까지 걸린 시간, $W_k\sim\Gamma(k,\frac{1}{\lambda})$

지수분포: Exp(λ), $\Gamma(1,\frac{1}{\lambda})$, $f(x)=\lambda e^{-\lambda x}$ $F(x)=1-e^{-\lambda x}$, $\mu=\frac{1}{\lambda}$, $\sigma^2=\frac{1}{\lambda^2}$

카이제곱분포 : $\chi^2(r)=\Gamma(\frac{r}{2},2)$, $f(x)=\frac{1}{\Gamma(\frac{r}{2})2^{\frac{r}{2}}}x^{\frac{r}{2}-1}e^{-\frac{x}{2}}$, $M(t)=(1-2t)^{-\frac{r}{2}}$ $(t<\frac{1}{2})$

$\mu=r$, $\sigma^2=2r$ / $X\sim N(\mu,\sigma^2) \Rightarrow V=\frac{(X-\mu)^2}{\sigma^2}\sim\chi^2(1)$