Assignment - 7

Q.1 Success probability p & unsuccess probability $(1-p) = q$

$$P[Z=x] = p(1-p)^{x} \qquad x = 0, 1, 2, \ldots \infty$$

$$\because \ Var[z] = E[z^2] - (E[z])^2$$

$$E[z] = \sum_{n=0}^{\infty} n\, P[Z=n]$$

$$= \sum_{n=0}^{\infty} n\, p(1-p)^{n}$$

$$= p \sum_{n=0}^{\infty} \frac{q^n}{n} = p \frac{1}{p^2} = \frac{1}{p}$$

$$E[z^2] = \sum_{n=0}^{\infty} n^2 p[Z=n] = \sum_{n=0}^{\infty} n^2 p(1-p)^n$$

$$= p \sum_{n=0}^{\infty} n^2 q^n$$

$$= p \frac{(q+1)}{(1-q)^3}$$

$$= \frac{p(2-p)}{p^3} = \frac{2-p}{p^2}$$

$$Var[z] = \frac{2-p}{p^2} - \frac{1}{p^2}$$

$$= \frac{1-p}{p^2}$$

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7 Q2. let y be binomial distribution random variable with n trials of success probability p

$Var[y] = np(1-p)$ ~~$y: P(y=1) = p$~~ ~~$P(y=0) = 1-p$~~

$\because$ $Var[y] = E[y^2] - (E[y])^2$

$\because$ $y: P(y=1) = nP$
$P(y=0) = n(1-P)$

$\because$ $E(y) = P.n + (1-P).0 = n.E(x)$
$= nP$

$Var[y] = n\,Var[x]$

where $x: P(x=1) = P$
$P(x=0) = 1-P$

$Var(x) = P(1-P)^2$
$+ (1-P)(0-P)^2$

~~$= (1-P(P$~~
$= P(1-P)(1\cancel{-P}\cancel{+P})$
$= P(1-P)$

$Var(y) = nP(1-P)$

7. 3. Given Memoryless Property.

let $x \sim Geo(P)$

we assume that x is Geometric distribution random variable

then $x: P(1-P)^n$ where $n = 0, 1, 2, 3 \ldots$

$$LHS = P(X \geq i)$$

$$= \sum_{j=i}^{\infty} P(1-P)^j$$

$$= P(1-P)^i \sum_{j=i}^{\infty} P(1-P)^{j-i}$$

$$= \cancel{P}(1-P)^i \frac{1}{\cancel{P}}$$

$$= (1-P)^i$$

$$RHS = P(X \geq n+i \mid X \geq n)$$

$$= \frac{P(X \geq n+i)}{P(X \geq n)} = \frac{\cancel{P}(1-P)^{n+i}}{(1-P)^n}$$

$$= (1-P)^i$$

hence this Property true for x as geometric distribution.

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where for p is the probability of correctness of an event.

$P \in (0, 1)$

7.8. 4 Given Poisson Distribution

$$P[x=x] = \frac{e^{-\lambda}\lambda^{x}}{x!} \quad \lambda > 0.$$

$$Var[x] = E(x^2) - [E(x)]^2$$

as Calculate before

$$E(x^2) = \lambda^2 + \lambda$$
$$E(x) = \lambda$$

$$\because E(x) = \sum_{x=1}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!} = \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{(x-1)!}$$
$$= e^{-\lambda}\lambda e^{\lambda}$$
$$= \lambda$$

$$Var[x] = \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$E(x^3) = E(x^3 - 3x^2 + 2x + 3x^2 - 2x)$$
$$= E[x(x-1)(x-2)] + 3E(x^2) - 2E(x)$$

$$= \lambda^3 + 3(\lambda^2 + \lambda) - 2(\lambda)$$
$$= \lambda^3 + 3\lambda^2 + 3\lambda - 2\lambda$$
$$= \lambda^3 + 3\lambda^2 + \lambda$$

$$E(x(x-1)(x-2)) = \lambda^3 e^{-\lambda} \sum_{x=3}^{\infty} \frac{\lambda^{x-3}}{(x-3)!}$$

$$= \lambda^3 e^{-\lambda} e^{\lambda} = \lambda^3$$