Assignment - 11

Q1

Given table

x \ y	1	2	3	total
1	$1/12$	$1/12$	$1/12$	$3/12$
2	$1/4$	$1/4$	$1/4$	$3/4$
total	$\frac{4}{12}$	$4/12$	$4/12$	1

a) Marginal probability mass distribution

for P_x & P_y is as follow

x	P_x
1	$3/12$
2	$3/4$

y	P_y
1	$4/12$
2	$4/12$
3	$4/12$

b) is x & y are independent

if x & y are independent
the for all x & y
$P_{xy} = P_x P_y$

∴ from above table it true
∴ x & y are independe varia b lo
for Eg $P_{1,1} = 1/12$ $P_{x=1} = 3/12$ $P_{y=1} = 4/12$

$P_{x=1} P_{y=1} = \frac{3}{12} \times \frac{4}{12} = 1/12$

hence true

$z = x/y$

For $x = 1, 2$
$y = 1, 2, 3$

z	0.33	0.5	0.66	1		2
x	1	1	2	1	2	2
y	3	2	3	1	2	1
P_z	1/12	1/12	1/4	$\frac{1}{12} + \frac{1}{4}$		1/4

$\frac{1}{3}$

hence probability distribution.

z	P_z
$\frac{1}{3} = 0.33$	1/12
$\frac{1}{2} = 0.5$	1/12
$\frac{2}{3} = 0.66$	1/4
1	1/3
2	1/4

Q2 $\because$ $Var(x) = E[(x - E(x))^2]$

let $E(x)$ be P

$= E[(x - P)^2]$

$= P(1-P^2) + (1-P)(-P)^2$

$= P(1-P)$

$\sigma_x = \sqrt{var(x)} = \sqrt{P(1-P)}$

$\because$ $E(x) = a$

$= \sqrt{a(1-a)}$

$\sigma_y = \sqrt{a(1-P)}$

$Cov(x,y) = E[x,y] - E[x]E[y]$

$= a - pq$

$= a - a^2$

Finaly by substitution ρ_{xy}

$$\rho_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$

$$= \frac{a(1-a)}{\sqrt{a(1-a)}\sqrt{a(1-a)}}$$

$= 1$

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Q3

$$f_{xy} = \begin{cases} \frac{3}{x} e^{-3x} & 0 < y < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

$Cov(x, y) = ?$

Correlation $(x, y) = ?$

$$f_x(x) = \int_{-\infty}^{\infty} f_{xy}\, dy = \int_0^x \frac{3}{x} e^{-3x}\, dy = 3e^{-3x}$$

$$E_{f_x}[x] = \int_{-\infty}^{\infty} x f_{xy}(x)\, dx$$

$$= \int_0^{\infty} 3x e^{-3x}\, dx$$

$$= -\frac{(3x+1)e^{-3x}}{3} \Big|_0^{\infty}$$

$$= +\frac{1}{3}$$

$$Var_{f_x}[x] = E_{f_x}[x^2] - [E_{f_x}(x)]^2$$

$$E_{f_x}[x^2] = \int_0^{\infty} 3x^2 e^{-3x}\, dx$$

$$= -\frac{(9x^2 + 6x + 2)}{9} e^{-3x} \Big|_0^{\infty}$$

$$= 2/9$$

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$$Var_{f_x}[x] = \frac{2}{9} - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{9}$$

$$f_y(y) = \int_y^{\infty} \frac{3}{x} e^{-3x}\, dx$$

= Non integradable.

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Q4

a) Give poission (2) random variable with $X_1, \ldots, X_{30}$

a) Markov's inequality to get upper bound is as follows

$$P\left[\sum_{k=1}^{30} X_k > 20\right] \leq \frac{\mu}{a}$$

$\because$ given $\lambda = 2$

$\therefore$ $E(X) = \mu = \lambda$ in poission distribut

$$E(X) = \mu = 2$$

$$a = 20$$

$$P\left[\sum_{k=1}^{30} X_k > 20\right] = \frac{2}{20} = \frac{1}{10}$$

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ii) CLT can be given as

$$P\left(\frac{\sum_{1}^{n} X_n - \mu}{\sigma/\sqrt{n}} > a\right) \simeq \phi(a)$$

$$\phi(a) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

$$\therefore P\left(\frac{\sum^{1} X_k - 2}{2/\sqrt{30}} > \frac{20-2}{2/\sqrt{30}}\right) \simeq \cancel{40}\ \phi(49.29)$$

$$\simeq \underline{1}$$