Worcester Polytechnic Institute
Department of Mathematical Sciences
Professor: Stephan Sturm
Online Success Coach: Patchara Santawisook

Summer 2019 – E1 Term

## MA 2631

# Probability Theory

Section E161

## Assignment 6

due on Tuesday, May 7

1. Consider the probability mass distribution $\mathbb{P}[Y = i] = c \cdot 0.2^i$ on the non-negative integers for some constant $c$.

   a) Calculate $c$.

   b) Calculate $\mathbb{P}[Y = 0]$ and $\mathbb{P}[Y > 3]$.

2. Let $X$ be a random variable taking values in $\mathbb{N}$. Show that

$$\mathbb{E}[X] = \sum_{n=1}^{\infty} \mathbb{P}[X \geq n].$$

3. Consider the random variable $X$ with the probability mass distribution

$$\mathbb{P}[X = -1] = 0.3, \qquad \mathbb{P}[X = 2] = 0.5, \qquad \mathbb{P}[X = 5] = 0.1, \qquad \mathbb{P}[X = 10] = 0.1.$$

   Calculate the expected value of $X$ and $Y$ with $Y = e^{2X}$.

4. Let $X$ be a random variable with probability mass function

$$p(n) = \mathbb{P}[X = n] = e^{-\lambda}\frac{\lambda^n}{n!} \qquad \text{for } nn \in \mathbb{N}.$$

   for some parameter $\lambda > 0$. Calculate $\mathbb{E}\big[X^2\big]$.

6 points per problems

**Additional practice problems (purely voluntary - no points, no credit, no grading)**:

**Standard** Carlton and Devore, Section 2.2: Exercises 11, 12, 16; Section 2.3: 32, 36