Assignment 6

6.1 Given PMD $P[y=i] = c \cdot 0.2^i$ on $i \in I^+$

a) Calculate c

$\because$ For PMD

$$\sum_{i=0}^{\infty} P[y=i] = 1$$

$$\therefore \sum_{i=0}^{\infty} P[y=i] = c(0.2)^0 + c(0.2)^1 + c(0.2)^2 + \ldots$$

$\because$ it is G.P. hence sum

$= \frac{a}{1-r}$ for infinite G.P.

$$\therefore \sum_{i=0}^{\infty} P[y=i] = c\left[\frac{1}{1-0.2}\right]$$

$$= \frac{1}{0.8} \times c$$

$$\because \sum_{i=0}^{\infty} P[y=i] = 1.$$

hence $c = 0.8$

$$\therefore P[y=i] = 0.8 * 0.2^i$$

b) Calculate $P[Y=0]$ & $P[Y>3]$

$\because P[Y=0] = 0.8*(0.2)^0$
$= 0.8$

$\because P[Y>3] = 1 - P[Y \leq 3]$

$\because P[Y\leq 3] = P[Y=0] + P[Y=1] + P[Y=2] + P[Y=3]$

$= 0.8 [0.2 + 1 + 0.04 + 0.008]$
$= 0.8 [1.248]$
$= 0.9984$

$\therefore P[Y>3] = 1 - 0.9984$
$= 0.0016$

6. 2) let X be a random Variable taking value in N show that

$$E[X] = \sum_{n=1}^{\infty} P[X \geq n]$$

Ans as per defination of expactation of random variable ~~E[X]~~ x can be given as

$$E[X] = \sum_{n=1}^{\infty} n \cdot P[X = n]$$

Scanned by CamScanner

Expanding above we get

$$E[x] = P[x=1] + 2P[x=2] + 3 \cdot P[x=3] + \dots + n \cdot P[x=n] + \dots$$

Rearrange the above sequence

$$= P[x=1] + P[x=2] + P[x=3] + \dots$$
$$+ P[x=2] + P[x=3] + \dots$$
$$+ P[x=3] + \dots$$

Can be rewritten as

$$= P[x \geq 1] + P[x \geq 2] + P[x \geq 3] + \dots$$

hence

$$E[x] = \sum_{n=1}^{\infty} P[x \geq n]$$

6.3. Give PMD for random variable x

$P[x=-1] = 0.3$, $P[x=2] = 0.5$
$P[x=5] = 0.1$, $P[x=10] = 0.1$

Expected value of $x = ?$

$$\because E[x] = \sum_{n=-\infty}^{\infty} n \cdot P[x=n]$$

$$= -1 \times 0.3 + 0.5 \times 2 + 0.1 \times 5 + 10 \times 0.1$$

$$= -0.3 + 1 + 0.5 + 1 = 2.2$$

Q.3 The given Relation of $y = e^{2x}$

$E[y] = ?$

$$E[f(x)] = \sum_{x=-\infty}^{\infty} f(x)\, P[X = x]$$

$$= 0.3e^{-2} + 0.5e^{4} + 0.1e^{10} + 0.1e^{20}$$

6.4

$E(x^2) = ?$

For given PMF for random variable x

$$P[x=n] = \frac{e^{-\lambda}\lambda^{n}}{n!} \qquad n \in N, \quad \lambda > 0.$$

$$E(x^2) = E((x)(x-1) + x)$$

$$= E((x)(x-1)) + E(x)$$

$\because$ given function is poisson distribution & $E(x) = \lambda$

$$\therefore E(x(x-1)) = \sum_{x=0}^{\infty} x(x-1)\frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=2}^{\infty} x(x-1)\frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{(x-2)!}$$

$$= \lambda^2 e^{-\lambda}\sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}$$

$$= \lambda^2 e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \cdots\right]$$

$$= \lambda^2 e^{-\lambda} e^{\lambda} = \lambda^2.$$

$$\therefore E(x^2) = \lambda^2 + \lambda$$