

Worcester Polytechnic Institute
 Department of Mathematical Sciences
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Summer 2019 – E1 Term

MA 2631

Probability Theory

Section E161

Assignment 8

due on Friday, May 14

1. (**5 points**) Assume that a random variable X has a density of the form $f_X(x) = cg(x)$ for some real constant c and

$$g(x) = \begin{cases} 0 & x < 0, \\ x & 0 \leq x < 5, \\ 10 - x & 5 \leq x < 10, \\ 0 & x \geq 10. \end{cases}$$

- a) Determine the value of the constant c , sketch the function f_X .
 - b) Calculate an sketch F_X the cumulative distribution function of X .
 - c) Calculate expectation and variance of the random variable X .
2. (**5 points**) You arrive at a random time at a bus stop, and you know that the bus is arriving every 30 minutes. Denote with Y the random variable describing your waiting time.
 - a) What is the probability that you will have to wait longer then 10 minutes.
 - b) Assume that you waited already 10 minutes, what is the probability that the bus will arrive in the next 10 minutes?

Describe every of the statements of a) and b) in terms of Y and calculate the probabilities explicitly using the density f_Y .

3. **(5 points)** Prove that for an arbitrary continuous random variable X with density f we have

$$\mathbb{E}[X] = \int_0^\infty \mathbb{P}[X > x] dx - \int_0^\infty \mathbb{P}[X < -x] dx.$$

4. **(5 points)** The lifetime of an electrical device (in months) is given by the continuous random variable X with density

$$f(x) = \begin{cases} cxe^{-\frac{x}{2}} & \text{if } x > 0; \\ 0 & \text{if } x \leq 0. \end{cases}$$

- a) What is c ?
 - b) What is the probability that the device functions more than 5 months?
 - c) What is the expected lifetime of the device?
5. **(4 points)** Let X be a continuous random variable with density f , expectation $\mathbb{E}[X] = \mu$ and variance $\mathbb{V}ar[X] = \sigma^2$. Define a new random variable $Y := aX + b$ for some $a, b \in \mathbb{R}$.
- a) Calculate the standard deviation $SD[Y]$!
 - b) Express the moment generating function m_Y in terms of m_X .

Additional practice problems (purely voluntary - no points, no credit, no grading):

Standard Carlton and Devore, Section 3.1: Exercises 8, 17, 18; Section 3.2: Exercises 23, 24, 26