1. Given

$$f(x) = \begin{cases} 0 & x < 0 \\ cx & 0 \le x \le 5 \\ (10-x)c & 5 \le x \le 10 \\ 0 & x \ge 10 \end{cases}$$

Given the ture For Randem Variable x

a)

$$1 = \int_{-\infty}^{\infty} f(x)\,dx$$

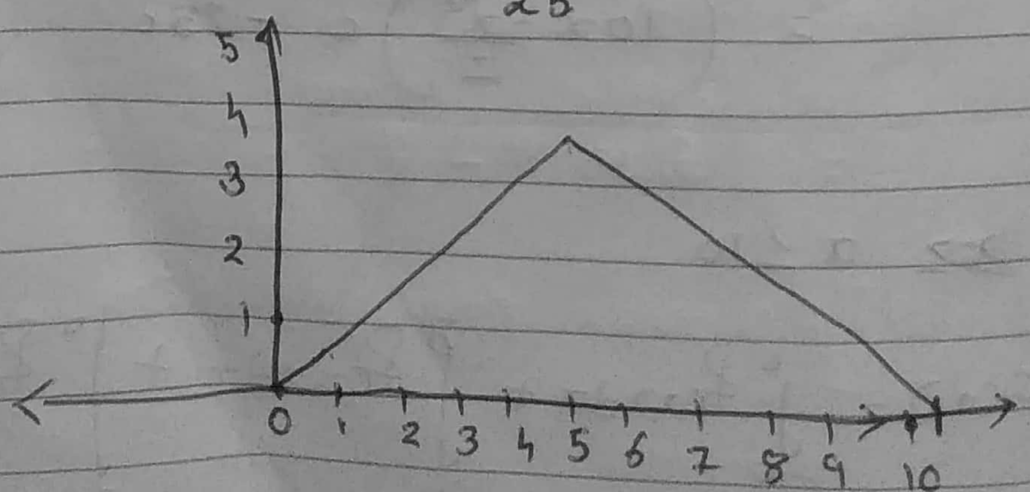
$$1 = \int_{0}^{5} cx\,dx + c\int_{5}^{10} 10 - x\,dx$$

$$1 = c\left[\frac{x^2}{2}\right]_0^5 + c\left[10x - \frac{x^2}{2}\right]_5^{10}$$

$$1 = \left[\frac{25}{2}\right]c + c\left[\cancel{100} - \cancel{\frac{100}{2}}^{50} - \cancel{50} + \frac{25}{2}\right]$$

$$1 = 25c$$

$$c = \frac{1}{25}$$



b) $x \le 0$

$$P(X \le x) = \int_{-\infty}^{x} f(x)\,dx = 0$$

$0 \le x < 5$

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$

$$= 0 + \int_{0}^{x} cx\,dx = \frac{cx^2}{2}$$

$5 \le x \le 10$

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx + \int_{5}^{x} f(x)\,dx$$

$$= \frac{25c}{2} + c\int_{5}^{x} (10 - x)\,dx$$

$$= \frac{25c}{2} + c\left(10x - \frac{x^2}{2}\right)\Bigg|_{5}^{x}$$

$$\cancel{= \frac{25c}{2} + \left(10x - \frac{x^2}{2}\right)c + \frac{25c}{2} - 50c}$$

$$= \frac{25c}{2} + 10xc - \frac{x^2c}{2} - 50c + \frac{25c}{2}$$

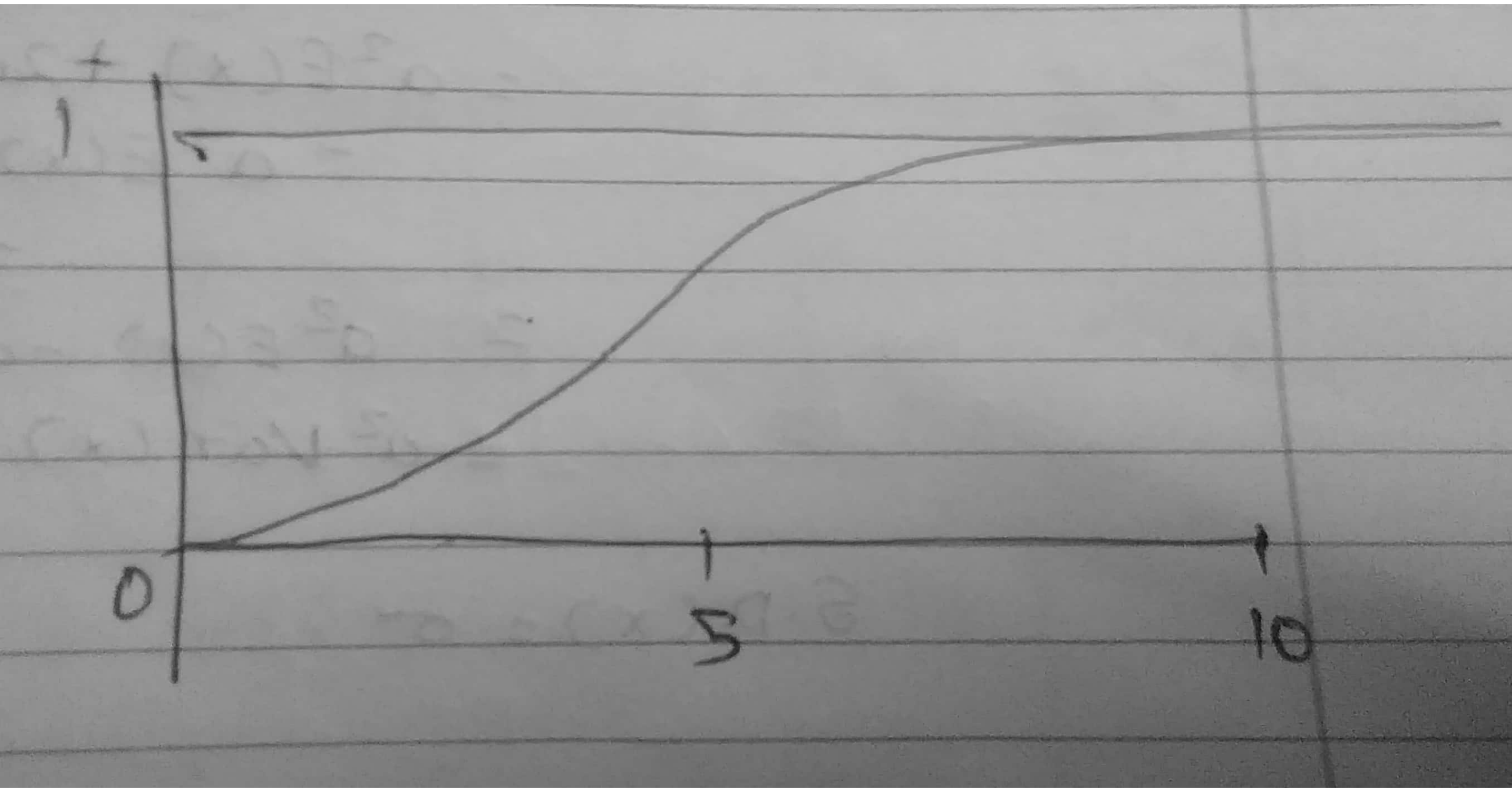
$$= \left(10x - \frac{x^2}{2}\right)c - 25c$$

$x \ge 10$

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx + \int_{5}^{10} f(x)\,dx + \int_{10}^{\infty} f(x)\,dx$$

$$= 0 + \frac{25c}{2} + \frac{25c}{2} + 0 = 25c$$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \dfrac{x^2}{50} & 0 \leq x \leq 5 \\ \dfrac{2}{5}x - \dfrac{x^2}{50} - 1 & 5 \leq x \leq 10 \\ 1 & x > 10 \end{cases}$$


1
0
5
10

c) $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

$$= \int_0^5 x^2 c\, dx + \int_5^{10} (10x - x^2)\, c\, dx$$

$$= c\left[\frac{x^3}{3}\Big|_0^5 + \frac{10x^2}{2}\Big|_{10} - \frac{x^3}{3}\Big|_{10} - \frac{10x^2}{2}\Big|_5 + \frac{x^3}{3}\Big|_5 \right]$$

$$= c\left[\frac{2\times 125}{3} + \frac{1000}{2} - \frac{1000}{3} - \frac{250}{2} \right]$$

$$= c\left[83.33 + 500 - 333.33 - 125 \right]$$

$$= 5$$

$$Var(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_0^5 x^3 c\, dx + \int_5^{10} (10x^2 - x^3)\, c\, dx$$

$$= c\left[\frac{x^4}{4}\Big|_5 + \frac{10x^3}{3}\Big|_{10} - \frac{x^4}{4}\Big|_{10} - \frac{10x^3}{3}\Big|_5 + \frac{x^4}{4}\Big|_5 \right]$$

$$= c\left[\frac{625\times 2}{4} + \frac{10000}{3} - \frac{10000}{4} - \frac{10\times 125}{3} \right]$$

$$= c\left[312.5 + 3333.33 - 2500 - 416.67 \right]$$

$$= 29.1664$$

$$Var(x) = 4.1664$$

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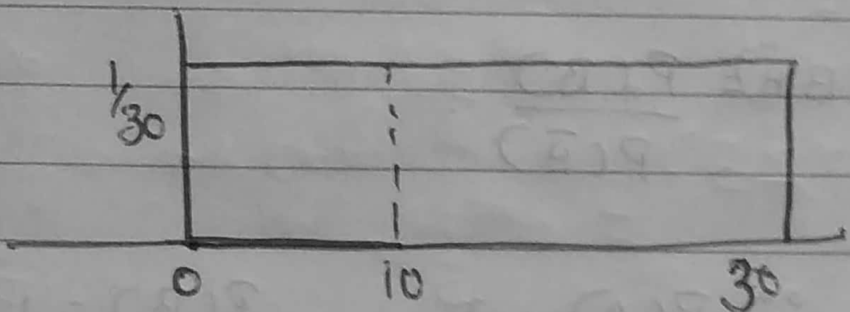
2. Y = random variable describing your waiting time.

a) $P(Y > 10) = ?$

but $P(Y > 10) = 1 - P(0 \leq Y \leq 10)$

the Graph Can be given as



$\therefore f(x) = \frac{1}{30}$.

$\therefore P(Y > 10) = 1 - P(0 \leq Y \leq 10)$

$= 1 - \frac{1}{30} \times 10$

$= \frac{30 - 10}{30} = \frac{2}{3}$

From above $\because$ bus arrive in ~~any~~ ~~in~~ next 30 i.e probability $\frac{1}{30}$ at instance.

$\therefore$ Graph.

b) ~~let~~ A Be the bus arrive in first 10 min
B be the bus arrive in next 10 min

∵ A happen bug not ~~for~~ [illegible]

2. $P(B|\bar{A}) = ?$

$$= \frac{P(\bar{A} \cap B)}{P(\bar{A})}$$

∵ A & B are two disjoints set

$$\bar{A} \cap B = B$$

$$P(B|\bar{A}) = \frac{P(B)}{P(\bar{A})}$$

$$\because P(A) = 1/3 \qquad P(B) = 1/3$$

$$P(\bar{A}) = \frac{2}{3}$$

$$P(B|\bar{A}) = \frac{1/\cancel{3}}{2/\cancel{3}} = 1/2$$

$$P(B) = P(Y \leq 20) - \overline{P(0<y<10)}$$

$$P(A) = P(0<y<10)$$

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3. $$E[X] = \int_0^\infty P[X > x]\,dx - \int_0^\infty P[X < -x]\,dx$$

R.H.S part one

$$\int_0^\infty P[X > x]\,dx = \int_0^\infty \int_x^\infty f_X(y)\,dy\,dx$$

by Tonelli's Theorem.

observe that the integrating domain is

$0 < x < \infty$ & $x < y < \infty$

which is to say.

$0 < y < \infty$ & $0 < x < y$

$$= \iint_{0 < x < y < \infty} f(xy)\,d(x,y)$$

$$= \int_0^x \int_0^y f(y)\,dx\,dy.$$

$$\therefore \int_0^\infty P[X > x]\,dx = \int_0^\infty y f(y)\,dy$$

similarly $$\int_0^\infty P[X < -x]\,dx = \int_{-\infty}^0 y f(y)\,dy$$

$$RHS = \int_{-\infty}^\infty y f(y)\,dy = E(X) = LHS.$$

1 a) $1 = \int_{-\infty}^{\infty} f(x)\,dx$

$$1 = \int_{0}^{\infty} cxe^{-x/2}\,dx$$

$$1 = c\left(-2x-4\right)e^{-x/2}\Big|_{0}^{\infty}$$

$$1 = c\,(+4)$$

$$c = 1/4$$

b) $P(x \geq 5) = \int_{5}^{\infty} f(x)\,dx = \frac{1}{4}\int_{5}^{\infty} xe^{-x/2}\,dx$

$$= \frac{1}{4}\left(-2x-4\right)e^{-x/2}\Big|_{5}^{\infty}$$

$$= \frac{1}{4}\left(10+4\right)e^{-5/2} = \frac{7}{2}e^{-5/2}$$

c) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{\infty} \frac{x^2}{4}e^{-x/2}\,dx$

$$= -2\left(x^2+4x+8\right)e^{-x/2}\Big|_{0}^{\infty}$$

$$= +2(8) = 16$$

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5. $\because E(Y) = aE(x) + b$

by linearity property.

$$Var(Y) = Var(ax+b)$$
$$= a^2 Var(x)$$

$SD[Y] = ?$

$$= \sqrt{var(y)} = a\sqrt{Var(x)}$$
$$= a\, S.D[x]$$

$$\because Var(ax+b) = E[(ax+b)^2] - (E[ax+b])^2$$

$$= a^2E[x] + 2abE[x] + b^2$$
$$- (aE(x)+b)^2$$
$$= a^2E(x) + 2abE(x) + b^2$$
$$- a^2E(x)^2 - 2abE(x)$$
$$- b^2$$
$$= a^2E(x) - a^2E(x)^2$$
$$= a^2 Var(x)$$

$S.D(x) = \sigma$

$S.D.[y] = a\sigma$

b) MGF

$$M.F.G. = M_x = E(e^{xt})$$

$$M_y = E(e^{yt})$$

$$\cancel{= E(e^{xt})}$$

$$= E(e^{(ax+b)t})$$

$$= E(e^{axt} e^{bt})$$

$$= e^{bt} E(e^{axt})$$

$$= e^{bt} M_x(bt)$$