Midterm Practice Exam

M. Q 1 let A & B be independent events.

Show that E & F also independent

when $E = A$ $F = B^c$

we have two fact about independent events.

$$P(A) + P(A^c) = 1$$
$$P(A \cap B^c) + P(A \cap B) = P(A)$$

$\because$ A & B are independent event

$$P(A \cap B) = P(A)\,P(B)$$
$$= P(A)\,(1 - P(B^c))$$
$$= P(A) - P(A)\,P(B^c)$$

$$P(A)\,P(B^c) = P(A) - P(A \cap B)$$
$$= P(A \cap B^c)$$

$\therefore$ $P(A \cap B^c) = P(A)\,P(B^c)$

hence if $E = A$ & $F = B^c$

then its proove that they also

are independent events.

M.2. Given PMF

$$P[X=n] = (1-P)^n P. \qquad P \in (0,1)$$
$$n \in I^+$$

a) Show that $\sum_{n=1}^{\infty} P[X=n] = 1$

$$\therefore \sum_{n=0}^{\infty} P[X=n] = P + (1-P)P + (1-P)^2 P + (1-P)^3 P + \dots$$

$$= P\left[1 + (1-P) + (1-P)^2 + (1-P^3)^3 + \dots\right]$$

$$= P\left[\frac{1}{1-(1-P)}\right]$$

$$= \cancel{P}\frac{1}{\cancel{P}} = 1$$

hence prove.

b) $$P[X \geq 3] = 1 - P[X<3]$$
$$= 1 - [\{P[X=0] + P[X=1] + P[X=2]\}]$$
$$= 1 - \left[P + P(1-P) + P(1-P)^2\right]$$
$$= 1 - P\left[1 + 1 - P + 1 + P^2 - 2P\right]$$
$$= 1 - \left[3 - 3p + p^2\right]p$$
$$= 1 - 3p + 3p^2 - p^3 = (1-P)^3$$

c) $n, i \in I^+$

$$P[X \geq n+i \mid X \geq n] = P[X \geq i]$$

$$RHS = P[X \geq i] = \sum_{k=i}^{\infty} (1-p)^k p$$

$$= p(1-p)^i \frac{1}{1-(1-p)}$$

$$= (1-p)^i$$

$$LHS = P(X \geq i+n \mid X \geq n)$$

$$= \frac{P(X \geq i+n, X \geq n)}{P(X \geq n)}$$

$$= \frac{P(X \geq i+n)}{P(X \geq n)} = \frac{(1-p)^{i+n}}{(1-p)^n}$$

$$= (1-p)^i$$

$$= RHS.$$

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Midterm

Q.3)

a) Boole's Inequility is given as

$$P\left[\bigcup_{n=1}^{\infty} E_n\right] \leq \sum_{n=1}^{\infty} P[E_n]$$

Proof by induction:

hold for $n=1$ (trivial case)

for $n=m$.

$$P\left[\bigcup_{n=1}^{m} E_n\right] \leq \sum_{n=1}^{m} P[E_n]$$

show hold true for $n=m+1$

$$P\left(\bigcup_{n=1}^{m+1} E_n\right) = P\left[\left(\bigcup_{n=1}^{m} E_n\right) \cup E_{m+1}\right]$$

$$= P\left[\bigcup_{n=1}^{m} E_n\right] + P\left[E_{m+1}\right]$$

$$- P\left[\left(\bigcup_{n=1}^{m+1} E_n\right) \cap E_{m+1}\right]$$

$$\leq P\left(\bigcup_{n=1}^{m} E_n\right) + P(E_{m+1})$$

$$\leq \sum_{n=1}^{m} P(E_n) + P(E_{m+1})$$

$$\leq \sum_{n=1}^{m+1} P(E_n).$$

hence proved.

b) By De morgan's law

$$\left(\bigcap_{n=1}^{\infty} E_n\right)^c = \bigcup_{n=1}^{\infty} E_n^c$$

$$\therefore P\left(\bigcap_{n=1}^{\infty} E_n\right) = 1 - P\left(\left(\bigcap_{n=1}^{\infty} E_n\right)^c\right)$$

$$= 1 - P\left(\bigcup_{n=1}^{\infty} E_n^c\right)$$

$\because$

$$P\left(\bigcup_{n=1}^{\infty} E_n^c\right) \le \sum_{n=1}^{\infty} P(E_n^c)$$

$$1 + P\left(\bigcup_{n=1}^{\infty} E_n^c\right) \le 1 + \sum_{n=1}^{\infty} P(E_n^c)$$

$$1 - \sum_{n=1}^{\infty} P(E_n^c) \le 1 - P\left(\bigcup_{n=1}^{\infty} E_n^c\right)$$

$$\therefore P\left(\bigcap_{n=1}^{\infty} E_n\right) = 1 - \sum_{n=1}^{\infty} P(E_n^c)$$

$$= 1 - \sum_{n=1}^{\infty} (1 - P(E_n))$$

$\because$ all $P(E_n) = 1$

hence

$$P\left(\bigcap_{n=1}^{\infty} E_n\right) = 1$$

hence proved.

M.4

a) There 65 student

then unique combination can be given as

$\frac{(65+1)}{2} = 33$ unique ways where 64 is total.

∴ there are 33 ways to split students into two class.

Similarly

b) If capacity of class is 34

∴ Combination are

34	31
33	32
32	33
31	34

only two ways to split class.

~~which~~ 34, 31 Unique ways 2 & 4 total.

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