Worcester Polytechnic Institute
Department of Mathematical Sciences
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Summer 2019 – E1 Term

MA 2631

Probability Theory

Section E161

Assignment 11

due on Tuesday, June 25

1. Assume that the joint probability mass distribution $p_{X,Y}$ of the random variable X and Y is given by

$$\begin{aligned} p_{X,Y}(1,1) = & p_{X,Y}(1,2) = p_{X,Y}(1,3) = \frac{1}{12}; \\ p_{X,Y}(2,1) = & p_{X,Y}(2,2) = p_{X,Y}(2,3) = \frac{1}{4}. \end{aligned}$$

   a) Calculate the marginal probability mass distributions p_X and p_Y.
   b) Are X and Y independent?
   c) What is the probability mass distribution of the random variable $Z = \frac{X}{Y}$?

2. Let X and Y be two jointly distributed random variables such that their marginals are Bernoulli distributed with the same success parameter $p \in (0,1)$ such that $\mathbb{P}[X=1, Y=1] = a \in (0,1) \cap (2p-1, p)$. Calculate the covariance of X and Y, $\mathbb{C}ov(X,Y)$, and their correlation $\varrho(X,Y)$.

3. Assume that X and Y are random variables with the joint density $f_{X,Y}$ given by

$$f_{X,Y} = \begin{cases} \frac{3}{x} e^{-3x} & 0 < x < \infty,\ 0 < y < x; \\ 0 & \text{else.} \end{cases}$$

   Calculate the covariance of X and Y, $\mathbb{C}ov(X,Y)$, and their correlation $\varrho(X,Y)$.

4. Let $X_1, \ldots, X_{30}$ be Poisson random variables with parameter $\lambda = 2$.

   a) Use Markov's inequality to get an upper bound for the probability

$$\mathbb{P}\Big[\sum_{k=1}^{30} X_k > 20\Big]$$

   b) Use now the central limit theorem to give an approximation of the probability in a).

6 points per problems

Additional practice problems (purely voluntary - no points, no credit, no grading):

Standard Carlton and Devore, Section 4.2: Exercises 39, 41; Section 4.5: Exercises 93, 98, 100

Hard Prove that for independent random variables $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ and $Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$ we have

$$X + Y \sim \mathcal{N}(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$$