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Summer 2019 – E1 Term

MA 2631

Probability Theory

Section E161

Midterm Practice Exam

This exam consists of four (4) problems on two (2) pages. You have fifty (50) minutes. **Good luck!**

Exercise	1	2	3	4	Σ
Points					

1. Let A and B be independent events. Show that also the events E and F, given by $E = A$ and $F = B^c$, are independent.

2. Consider a discrete random variable X given by the probability mass distribution
$$p(n) = P[X = n] = (1-p)^n p$$
for non-negative integers n and some $p \in (0,1)$.

   a) Prove that the probability mass distribution describes indeed a probability, i.e. show that
$$\sum_{n=0}^{\infty} p(n) = 1.$$

   b) Calculate the probability of $\mathbb{P}[X \geq 3]$.

   c) Prove that it holds for non-negative integers n, i that
$$\mathbb{P}[X \geq n+i \,|\, X \geq n] = \mathbb{P}[X \geq i].$$

3. Let E_1, E_2, $\ldots$, E_n, $\ldots$ countably many events.

   a) Prove that

   $$\mathbb{P}\left[\bigcup_{n=1}^{\infty} E_n\right] \leq \sum_{n=1}^{\infty} \mathbb{P}\big[E_n\big].$$

   b) Prove that if $\mathbb{P}\big[E_n\big] = 1$ for all $n \geq 1$, it follows that

   $$\mathbb{P}\left[\bigcap_{n=1}^{\infty} E_n\right] = 1.$$

   Hint: Remember that F_1, F_2, $\ldots$, F_n, $\ldots$ with $F_1 = E_1$ and $F_n = E_n \cap \left(\bigcup_{j=1}^{n-1} E_j\right)^c$ for $n \geq 2$ are disjoint events satisfying

   $$\bigcup_{n=1}^{\infty} E_n = \bigcup_{n=1}^{\infty} F_n.$$

4. 65 students are registered for MA 2631, which will be held in two sections.

   a) In how many ways can the students be split into two sections?

   b) Due to the size of the classroom at most 34 students can be in each section. In how many ways can the two sections be organized under this constraint?