Collatz conjecture

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Abstract:

ENG/ Finding useful relationships in the Collatz hypothesis.
This publication will be about the Collatz conjecture (1). This problem is also known as the 3n + 1 problem, Ulam's problem (after Stanisław Ulam), Kakutan's problem (after Shizu Kakutani), Thwaite's problem (after Sir Bryan Thwaites), Hasse's algorithm (after Helmut Hasse), or also as the Syracuse problem. So far, no one has satisfactorily proved the Collatz hypothesis mathematically. Jeffrey Lagarias stated in 2010 that Collatz's conjecture "is an extremely complex problem that is completely beyond the reach of modern mathematics. Before him, Paul Erdős made a similar statement (roughly: mathematics is not ready for such problems).

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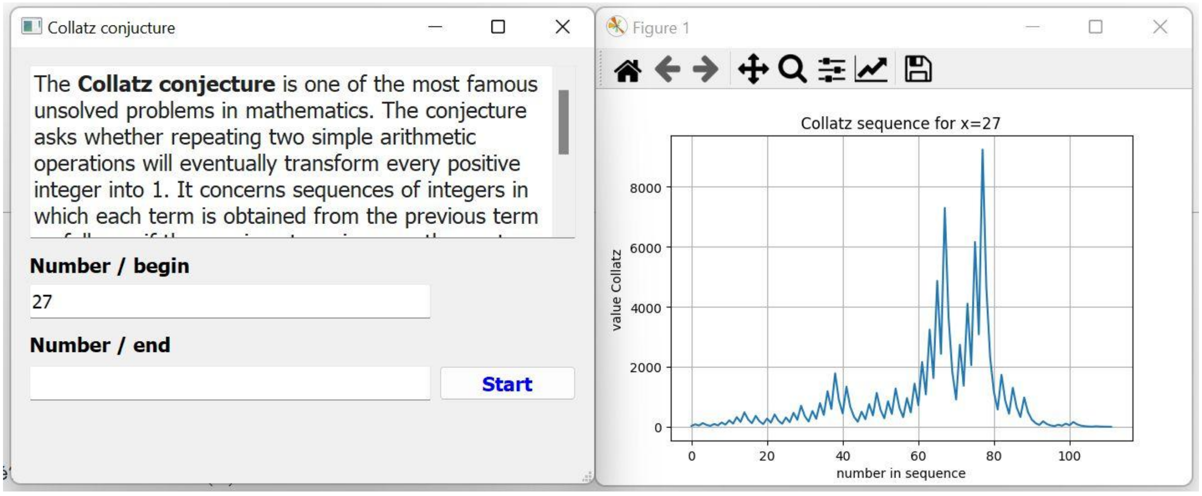
def:

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \equiv 0 \pmod{2} \\ 3n+1 & \text{if } n \equiv 1 \pmod{2}. \end{cases} \tag{1.0}$$

Zdrojový kód / Source Code



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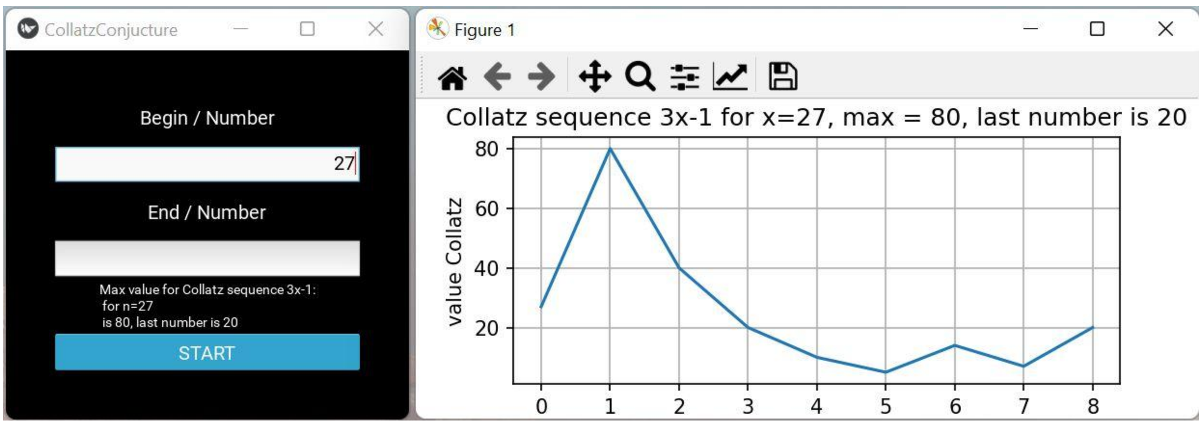
version 1.0 :
download the code from GitHub: [collatz.py]
or download ".exe" file: [collatz.rar] (compressed WinRar, size: 72MB)
version 1.1: (also lists maxima, kivy app)
download the code from GitHub: [collatz1.py]
or download ".exe" file: [collatz1.rar] (compressed WinRar, size: 80MB)

Postupnosť 3n-1 /sequence 3n-1

def:
for : $f(n) = 3n - 1,\ if\ n \equiv 1\ (mod\ 2),\ f(n) = n/2,\ if\ n \equiv 0\ (mod\ 2)$ (1.1)

Zdrojový kód / Source Code



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version 1.1: (vypíše aj maxima, kivy app)

download the code from GitHub: collatz1_3x-1.py
or download ".exe" file: collatz1_3x-1.rar (compressed WinRar, size: 80MB)

Program vypíše maxima aj posledné číslo postupnosti, teda >1, =1. Pokiaľ narazí na opakujúcu sa sekvenciu skončí pri >1. Pri preskúmavaní intervalu zobrazí len maxima na grafe. Pokiaľ by sa graf nezobrazil znamená to (okrem výpočtovej náročnosti vstupu), že program nenašiel periódy pre všetky čísla z intervalu, resp. neskončil na 1. To by znamenalo, že sa program zacyklil. To sa však z krátkych testov intervalov čísiel nestalo. / The program prints the maximum and the last number of the sequence, i.e. >1, =1. If it encounters a repeating sequence, it ends up being >1. When examining the interval, it will only display highs on the chart. If the graph is not displayed, it means (in addition to the computational complexity of the input) that the program did not find periods for all numbers from the interval, or did not end at 1. This would mean that the program looped. However, this did not happen from the short interval tests.

Algoritmus našiel 2 opakujúce sa sekvencie, okrem zostupnej línie 2^n , zapíšme si to nasledovne: / The algorithm found 2 repeating sequences, except for the descending line 2^n , let's write it as follows:

(1) -is the descending line of 2^n [...32,16,8,4,2,1]
(2) -is a repeating sequence of numbers [5,14,7,20,10]
(3) -is a repeating sequence of numbers
[17,50,25,74,37,110,55,164,82,41,122,61,182,91,272,136,68,34]

Krátky test pre interval <3,1000> výpis nižšie: / Short test for interval <3.1000> listing below:

[[3, 1], [4, 1], [5, 2], [6, 1], [7, 2], [8, 1], [9, 2], [10, 2], [11, 1], [12, 1], [13, 2], [14, 2], [15, 1], [16, 1], [17, 3], [18, 2], [19, 2], [20, 2], [21, 3], [22, 1], [23,
3], [24, 1], [25, 3], [26, 2], [27, 2], [28, 2], [29, 1], [30, 1], [31, 3], [32, 1], [33, 3], [34, 3], [35, 2], [36, 2], [37, 3], [38, 2], [39, 1], [40, 2], [41, 3], [42, 3], [43,
1], [44, 1], [45, 3], [46, 3], [47, 2], [48, 1], [49, 3], [50, 3], [51, 2], [52, 2], [53, 1], [54, 2], [55, 3], [56, 2], [57, 1], [58, 1], [59, 1], [60, 1], [61, 3], [62, 3], [63,
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103, 1], [104, 2], [105, 1], [106, 1], [107, 2], [108, 2], [109, 3], [110, 3], [111, 3], [112, 2], [113, 1], [114, 1], [115, 1], [116, 1], [117, 3], [118, 1], [119, 2], [120, 1],
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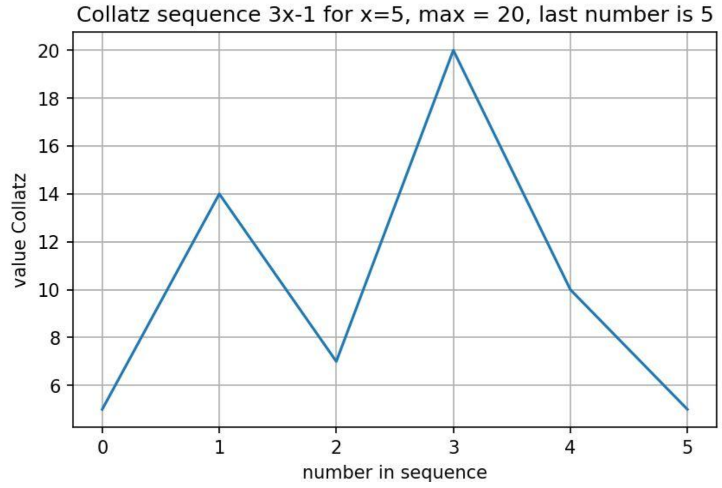
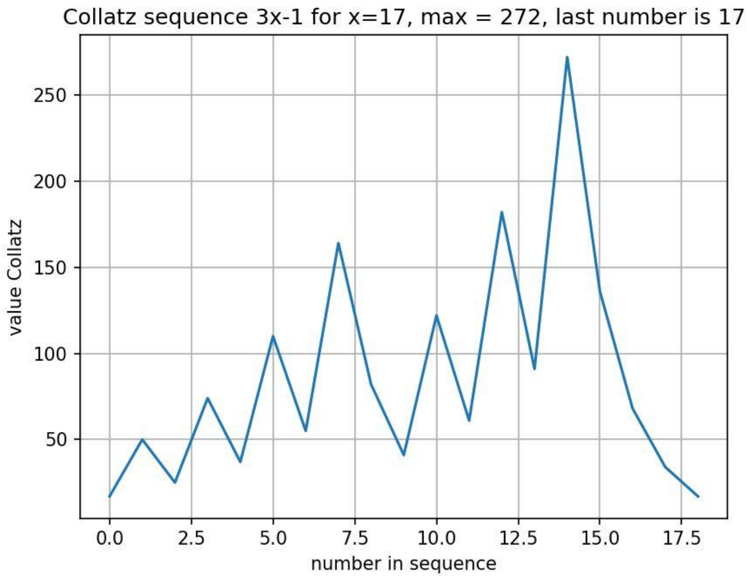
Pre (1.1) existujú dve opakujúce sa sekvencie čísiel /For (1.1) there are two repeating sequences of numbers.

Fractal (1): $seq_1 = [5, 14, 7, 20, 10]$ (1.2)



Fractal(2):

$seq_2 = [17, 50, 25, 74, 37, 110, 55, 164, 82, 41, 122, 61, 182, 91, 272, 136, 68, 34]$ (1.3)



Vytvorme rovnicu pre seq_1: / Let's create an equation for seq_1:

$$n = \frac{1}{2}\left(\frac{1}{2}\left(3\left(\frac{1}{2}(3n - 1)\right) - 1\right)\right) \quad (1.4)$$

upravme:

$$n = \frac{1}{4}\left(\frac{3}{2}(3n - 1) - 1\right)$$

$$n = \frac{9n}{8} - \frac{5}{8}$$

$$n = 5$$

Riešeniu rovnice (1.4) vyhovuje iba seq_1 / Only seq_1 satisfies the solution of equation (1.4).

Pre rovnicu (1.0) vznikne rovnica: / For the equation (1.0), the equation will be:

$$n = \frac{1}{2}\left(\frac{1}{2}\left(3\left(\frac{1}{2}(3n + 1)\right) + 1\right)\right) \qquad (1.5)$$

upravme:

$$\frac{n}{8} + \frac{5}{8} = 0$$

$$n =- 5$$

Toto riešenie je však neprípustné, keďže sa jedná o záporné číslo. / However, this solution is inadmissible, as it is a negative number.

Vytvorme rovnicu pre seq_2: / Let's create an equation for seq_2:

$$\frac{1}{16}(3(\frac{1}{2}(3(\frac{1}{2}(3(\frac{1}{2}\frac{1}{2}(3(\frac{1}{2}(3(\frac{1}{2}(3(\frac{1}{2}(3n - 1)) - 1)) - 1)) - 1)) - 1)) - 1)) - 1)$$
$$= n \qquad (1.6)$$

upravme:

$$\frac{139n}{2048} - \frac{2363}{2048} = 0$$

$$n = 17$$

Riešeniu rovnice (1.6) vyhovuje iba seq_2 / Only seq_2 satisfies the solution of equation (1.6).

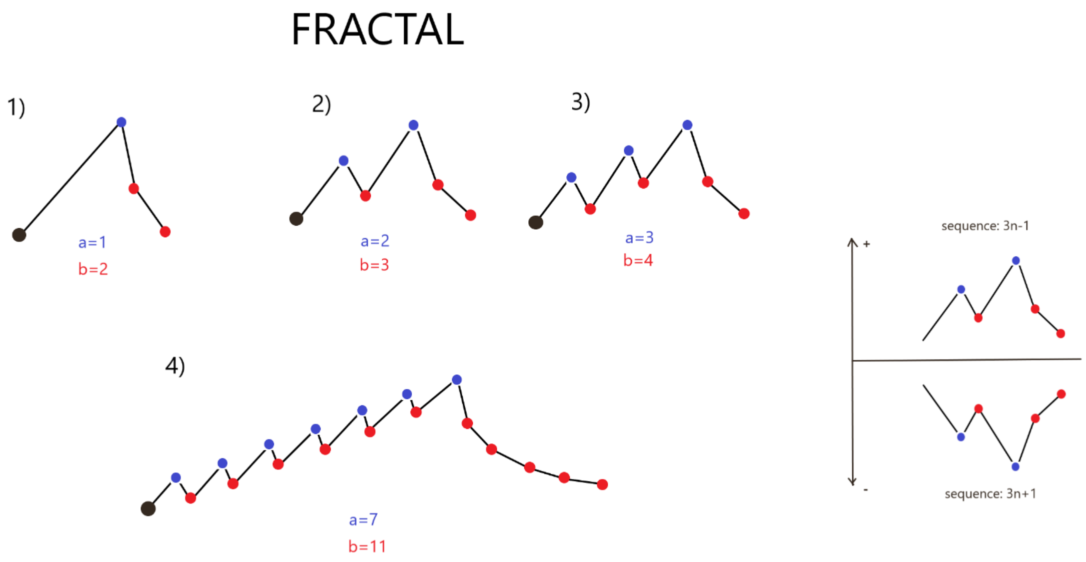
Pre rovnicu (1.0) vznikne rovnica: / For the equation (1.0), the equation will be:

$$\frac{1}{16}(3(\frac{1}{2}(3(\frac{1}{2}(3(\frac{1}{2}\frac{1}{2}(3(\frac{1}{2}(3(\frac{1}{2}(3(\frac{1}{2}(3n + 1)) + 1)) + 1)) + 1)) + 1)) + 1)) + 1)$$
$$= n \qquad (1.7)$$

$$n =- 17$$

Toto riešenie je však neprípustné, keďže sa jedná o záporné číslo. / However, this solution is inadmissible, as it is a negative number.

Základné fraktály / Basic fractals



Pic. No.1 Basic fractals for sequence 3n-1

Odvoďme rovnice pre fraktály z obr. 1 / Let us derive the equations for fractals from fig. 1

1) $\frac{(3^1-2^2)\,n}{2^2} - \frac{1}{2^2} = 0;\ -n - 1 = 0;\quad n = -1$

2) $\frac{(3^2-2^3)\,n}{2^3} - \frac{3+2}{2^3} = 0;\ n - 5 = 0;\quad n = 5$

3) $\frac{(3^3-2^4)\,n}{2^4} - \frac{3\cdot3+3\cdot2+2\cdot2}{2^4} = 0;\ \ 11\,n - 19 = 0;\quad n \neq integer$

4) $\frac{(3^7-2^{11})\,n}{2^{11}} - \frac{3^6+3^5\cdot2+3^4\cdot2^2+3^3\cdot2^3+3^2\cdot2^4+3\cdot2^5+2^6}{2^{11}} = 0;\ 139\,n - 2059 = 0;\quad n \neq integer$

Dá sa odvodiť všeobecná rovnica pre N- fractal: / A general equation for the N-fractal can be derived:

for a=integer, b=integer (see pic. no.1)

$$A\,n - B = 0;\quad n = B/A;\quad If\ B\ mod\ A = 0;\quad \Rightarrow n\ \ is\ integer\ solution$$
(1.8)

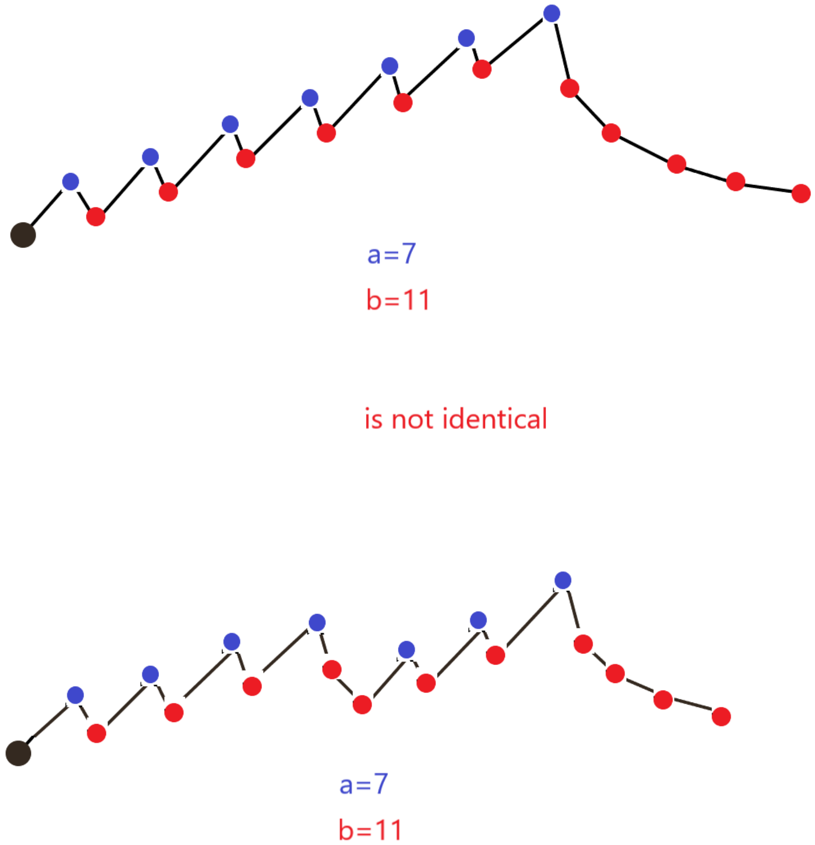
$$A = 3^a - 2^b$$
(1.9)

$$B = \sum_{i=1}^{a} 3^{a-i} \cdot 2^{i-1}$$

(1.10)

$$n = \frac{\sum_{i=1}^{a} 3^{a-i} \cdot 2^{i-1}}{3^{a}-2^{b}}$$

(1.11)



pic. No.2 Porovnanie fraktálov / comparing fractals

Výpočet pre dve varianty v zmysle obr.2 / Calculation for two variants according to fig.2

$$\frac{(3^{7}-2^{11})\,n}{2^{11}} - \frac{3^{6}+3^{5}\cdot 2+3^{4}\cdot 2^{2}+3^{3}\cdot 2^{3}+3^{2}\cdot 2^{4}+3\cdot 2^{5}+2^{6}}{2^{11}} = 0;\ 139\,n - 2059 = 0; \quad n \neq integer$$

$$\frac{(3^{7}-2^{11})\,n}{2^{11}} - \frac{3^{6}+3^{5}\cdot 2+3^{4}\cdot 2^{2}+3^{3}\cdot 2^{3}+3^{2}\cdot 2^{5}+3\cdot 2^{6}+2^{7}}{2^{11}} = 0;\ 139\,n - 2363 = 0; \quad n = 17$$

Pre B prípad sa na konci rovnice zvýšila mocnina dvojky. / For case B, the power of two was increased at the end of the equation.

Tvrdenie /statement:

To znamená, že rôzne variácie fraktálov vedu na kombinátorickú explóziu. / This means that different variations of fractals lead to a combinatorial explosion.

Rovnica (1.10) sa dá upraviť na jednoduchšiu rovnicu: / Equation (1.10) can be modified to a simpler equation:

$$B = \sum_{i=1}^{a} 3^{a-i} \cdot 2^{i-1} = 3^a - 2^a$$

(1.12)

$$n = \frac{3^a - 2^a}{3^a - 2^b}$$

(1.13)

For $n \in Z,\ a < b:\ \left(3^a - 2^a\right) mod \left(3^a - 2^b\right) = 0$

(1.14)

solution (1.14) is:

$a = 1,\ b = 2\ and\ a = 2,\ b = 3$

Test:

Ak má začiatok a koniec fraktálu (obr.2) skončiť na rovnakej úrovni potom existuje priama závislosť medzi "a" a "b". Táto závislosť je nasledovná: / If the beginning and end of the fractal (Fig. 2) should end at the same level, then there is a direct dependence between "a" and "b". This dependency is as follows:

$$3^a > 2^b \quad \Rightarrow b = int(\frac{a\ ln\ 3}{ln\ 2})$$

(1.15)

into the equation (1.13):

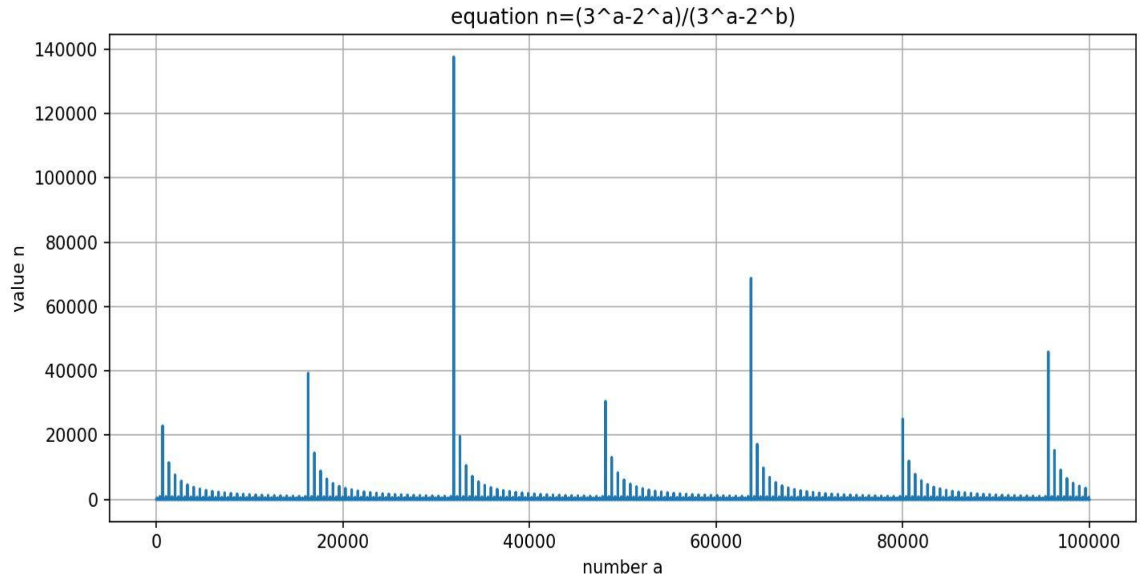
$$n = \frac{3^a - 2^a}{3^a - 2^b}$$

(1.16)

example:

$a = 7;\quad b = int(7\ ln\ 3/ln\ 2) = 11$

Dostali sme rovnicu s jednou premennou. Pozrime sa na priebeh tejto rovnice / We got an equation with one variable. Let's look at the progression of this equation.



pic. No.3 graph of the equation (1.16)

<u>Dôkaz / proof.</u>

Upravme rovnicu (1.14) do iného požadovaného tvaru: / Let's modify the equation (1.14) into another desired form:

$$3^a - 2^a \equiv 0 \; mod \left(3^a - 2^{int(a \, ln \, 3/ln \, 2)}\right)$$
(1.17)

potom/ next:

$$3^a \equiv 2^a \, (mod \left(3^a - 2^{int(a \, ln \, 3/ln \, 2)}\right))$$
(1.18)

2^a je možné rozložiť iba na súčin dvoch párnych čísiel okrem čísla 1. / 2^a can only be factored into the product of two even numbers except the number 1.

example:
$2^8 = 16 \cdot 16 = 32 \cdot 8 = 2 \cdot 128 = 4 \cdot 64 = 1 \cdot 256$

<u>Prípad (1) / case (1):</u>

člen $3^a - 2^{int(a \, ln \, 3/ln \, 2)}$ je však vždy nepárne číslo. Pretože platí: / however, the term $\left(3^a - 2^{int(a \, ln \, 3/ln \, 2)}\right)$ is always an odd number. Because:

$3^a \; - \; is \; always \; odd; \;\; 2^{int(a \, ln \, 3/ln \, 2)} \; - \; is \; always \; even$

$odd\,number\ -\ even\,number\ =\ always\,odd\,number$

takže $2^a\,mod\left(3^a - 2^{int(a\,ln\,3/ln\,2\,)}\right)$ musí mať vždy nenulový zvyšok, okrem mod (1). / so $2^a\,mod\left(3^a - 2^{int(a\,ln\,3/ln\,2\,)}\right)$ must always have a non-zero remainder, except for mod (1).

Z toho teda vyplýva, že kongruencia nie je možná: / It therefore follows that congruence is not possible

$0\ \equiv\ 0\ (mod\left(3^a - 2^{int(a\,ln\,3/ln\,2\,)}\right))$ - congruence is not possible.

<u>Prípad (2) / case (2):</u>
Nenulové zvyšky /non-zero remainder

Pre zjednodušenie urobme substituciu: / For simplicity, let's make a substitution:

$int(a\,ln\,3/ln\,2\,)\ =\ b$
(1.19)
dosadíme do (1.18): / we substitute in (1.18):
$3^a\ \equiv\ 2^a\ \ \left(mod\left(3^a - 2^b\right)\right)$

urobme malý trik: / let's do a little trick:
$3^a\ - 2^b + 2^b \equiv\ 2^a\ \ \left(mod\left(3^a - 2^b\right)\right)$
(1.20)
$3^a\ - 2^b\ mod\left(3^a - 2^b\right) =\ 0$

potom dostaneme: / then we get:
$2^b \equiv\ 2^a\ \ \left(mod\left(3^a - 2^b\right)\right)$
(1.21)

zavedme substitúciu: / let's make a substitution:
$\gamma\ =\ 3^a - 2^b$
(1.22)

potom dostaneme: / then we get:
$2^b \equiv\ 2^a\ \ (mod\,\gamma)$
(1.23)
next:
$b\ =\ a\ +\ \phi$ (substitution)
(1.24)
$2^{a+\phi} \equiv\ 2^a\ \ (mod\,\gamma)$
(1.25)

upravme na tento požadovaný tvar: // let's adjust to this desired shape:

$$2^{a}2^{\phi} \equiv \ 2^{a} \quad (mod\ \gamma)$$
(1.26)

Táto kongruencia môže platiť ak $\gamma \ = \ 1$ (uvedené jednoduché riešenia), ďalej iba ak platí: / This congruence can hold if $\gamma \ = \ 1$ (given simple solutions), then only if:

$$2^{\phi} \equiv 1 \quad (mod\ \gamma)$$

Táto podmienka však nie je splnená, pretože : / However, this condition is not met because:

$$2^{\phi} \neq 1$$

$$2^{\phi} \neq 1 + j\left(3^{a} - 2^{a}2^{\phi}\right); \ \ j \in integer$$

$$2^{\phi} < 3^{a} - 2^{a}2^{\phi}; \quad for:\ a > 2$$

$$2^{\phi} \neq 1 + j\left(3^{a} - 2^{a}2^{\phi}\right); \ \ j = 1$$
(1.27)

Všeobecný fraktál / General fractal

Pokúsim sa zovšeobecniť vzťahy, ktoré platia pre všeobecný vzor/fraktál. / I will try to generalize the relationships that apply to the general pattern/fractal.
pic.No.4 ukážka všeobecných vzorov Collatz conjecture / an example of general Collatz conjecture patterns.

Pre všeobecný fraktál v zmysle rovnice (1.11) (1.16) platí všeobecný vzťah: / For a general fractal in the sense of equation (1.11) (1.16) the general relation applies:

$$n \ = \frac{3^{a}-2^{a}+\varepsilon_{k}}{3^{a}-2^{b}}$$
(2.0)

$\varepsilon_{k} \ - \ \ is\ corrective\ term,\ \varepsilon_{k} \in integer$

$\varepsilon_{k} = \ \varepsilon \ + \ k(3^{a} - 2^{b}); \ \ \varepsilon < 3^{a} - 2^{b}$

Všeobecný fraktál musí spĺňať kongruenciu: / A general fractal must satisfy the congruence:

$$3^{a} - 2^{a} + \varepsilon_{k} \equiv 0 \qquad mod\ (3^{a} - 2^{b})$$
(2.1)

úpravou dostaneme aj: / modification we also get:

$2^b + \varepsilon_k \equiv 2^a \ \ mod\ (3^a - 2^b)$

(2.2)

resp. / respectively:

$2^b + \varepsilon_0 \equiv 0 \ \ mod\ (3^a - 2^b) \ \ or\ \ 2^b \equiv - \varepsilon_0 \ \ mod\ (3^a - 2^b)$

(2.3)

$\varepsilon_0 = \varepsilon_k - 2^a$

$\varepsilon_0 = \varepsilon + k(3^a - 2^b); \ \ \varepsilon < 3^a - 2^b$

Example:
variant B pic. No. 2
$a = 7,\ b = 11,\ \varepsilon_k = 2363 - 2059 = 304$

$n = \frac{3^7 - 2^7 + 304}{3^7 - 2^{11}} = 17$

$\varepsilon_0 = \varepsilon_k - 2^a = 304 - 2^7 = 176; \quad \varepsilon = 176 - 1(139) = 37$

$2^{11} + 176 \equiv 0 \ \ mod\ (139) : True$

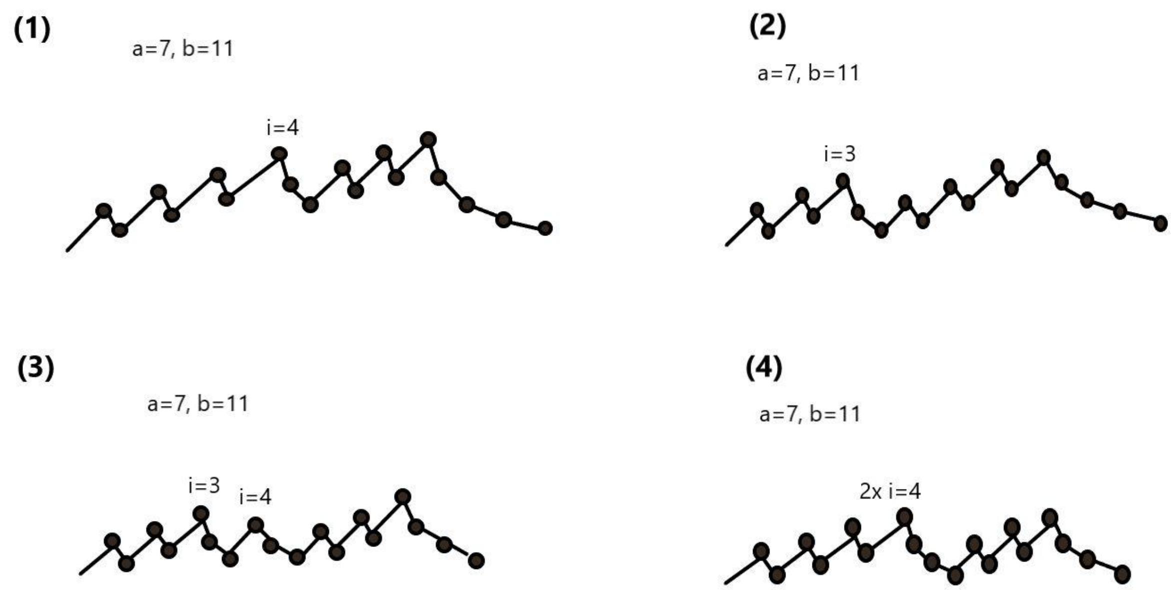
$2^{11} + 37 \equiv 0 \ \ mod\ (139) : True$

Statement:

Pre všeobecný fractál s premennými - a, b vždy existuje $\varepsilon < 3^a - 2^b$*, ktorá spĺňa kongruenciu (2.2) alebo (2.3). / for a general fractal with variables - a, b there always exists* $\varepsilon < 3^a - 2^b$ *which satisfies congruence (2.2) or (2.3).*

Hľadanie všeobecných vzťahov pre epsilon / Finding general relations for epsilon

Najskôr urobím analýzu vzťahov pre varianty z obr. 4 /First, I will analyze the relationships for the variants from fig. 4



pic. No.4

variant (1):

$a = 7,\ b = 11,\ \varepsilon_k = 2363 - 2059 = 304$

$3^6 + 3^5 \cdot 2 + 3^4 \cdot 2^2 + 3^3 \cdot 2^3 + 3^2 \cdot 2^5 + 3 \cdot 2^6 + 2^7 = 2363 = 3^a - 2^b + \varepsilon_k$

$\varepsilon_k = 3^2 \cdot 2^4 + 3^1 \cdot 2^5 + 3^0 \cdot 2^6 = 304$

$\varepsilon_k = 2^4 \cdot 3^{7-4} - 2^7 = 2^i \cdot 3^{a-i} - 2^a;\ i = 4$

variant (2):

$a = 7,\ b = 11,\ i = 3$

$3^6 + 3^5 \cdot 2 + 3^4 \cdot 2^2 + 3^3 \cdot 2^4 + 3^2 \cdot 2^5 + 3 \cdot 2^6 + 2^7 = 2579 = 3^a - 2^b + \varepsilon_k$

$\varepsilon_k = 3^3 \cdot 2^3 + 3^2 \cdot 2^4 + 3^1 \cdot 2^5 + 3^0 \cdot 2^6 = 520$

$\varepsilon_k = 2^3 \cdot 3^{7-3} - 2^7 = 2^i \cdot 3^{a-i} - 2^a;\ i = 3$

variant (3):

je superpozíciou variantu (1) a (2) /is a superposition of variant (1) and (2)

$a = 7,\ b = 11,\ i = 3,\ i = 4$

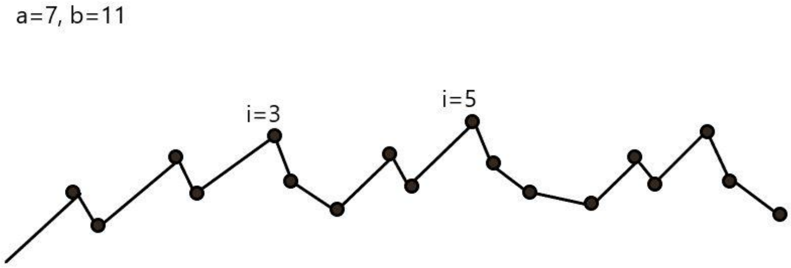
$\varepsilon_k = \left(2^3 \cdot 3^{7-3} - 2^7\right) + 2 \cdot \left(2^4 \cdot 3^{7-4} - 2^7\right) = 520 + 2 \cdot 304 = 1128$

variant (4):

$a = 7,\ b = 11,\ \ i = 4$

$\varepsilon_k = \left(2^4 \cdot 3^{7-4} - 2^7\right) + 2 \cdot \left(2^4 \cdot 3^{7-4} - 2^7\right) = 304 + 2 \cdot 304 = 912$

(5) variant



pic. No.5

variant (5):

$a = 7,\ b = 11,\ i = 3,\ i = 5$

$\varepsilon_k = \left(2^3 \cdot 3^{7-3} - 2^7\right) + 6 \cdot \left(2^5 \cdot 3^{7-5} - 2^7\right) = 1480$

rozdiel medzi variantom (5) a (3) je v tom, že pri i=5 vrchole dôjde o jeden pokles naviac, preto tam je 6 x násobok. / the difference between variant (5) and (3) is that at peak i=5 there is one more drop, therefore it is a 6 x multiple.

Tieto vzťahy je možné zovšeobecniť na : / These relations can be generalized to:

$\varepsilon_k = \left(2^{i_1}3^{a-i_1} - 2^a\right) + 2\left(2^{i_2}3^{a-i_2} - 2^a\right) + 6\left(2^{i_3}3^{a-i_3} - 2^a\right) + 14\left(2^{i_4}3^{a-i_4} - 2^a\right) + 30\left(2^{i_5}3^{a-i_5} - 2^a\right)...$

$2 = 2 \cdot 1,\ 6 = (2 + 1) \cdot 2,\ 14 = (6 + 1) \cdot 2,\ 30 = (14 + 1) \cdot 2 ...$

(2.4)

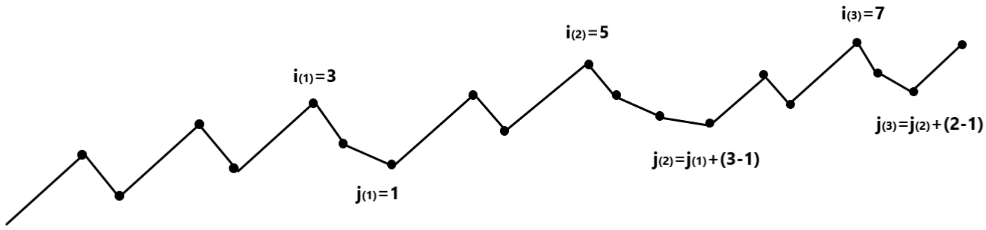
V rovnici (2.4) nemusia byť postupne všetky členy, ak ide o viac ako 2 násobný pokles (napr.3), potom tam nebude člen $2\left(2^{i_2}3^{a-i_2} - 2^a\right)$ a miesto neho bude člen $6\left(2^{i_2}3^{a-i_2} - 2^a\right)$. / In the equation (2.4) not all terms have to be successively, if it is a more than 2-fold decrease (e.g. 3), then the term $2\left(2^{i_2}3^{a-i_2} - 2^a\right)$ will not be there and the term$6\left(2^{i_2}3^{a-i_2} - 2^a\right)$ will be there instead.

všeobecne formálne by bola zapísaná rovnica (2.4) nasledovne: / in general, equation (2.4) would be written formally as follows:

$\varepsilon_k = \left(2^{i_1}3^{a-i_1} - 2^a\right) + \sum_{i=2}^{<b}\left(2^{j_k} - 2\right)\left(2^{i_k}3^{a-i_k} - 2^a\right)$

$i_k -\ peak,\ j_k -\ decline$

(2.5)



pic. No.6 Význam premenných i_k, j_k / Meaning of variables i_k, j_k

Treba mať na pamäti, že rovnica nemusí obsahovať všetky členy postupne, ani v súčte radu. Preto je to len formálny zápis (nie presný). Príklad variant (5) ukazuje chýbajúce členy. / It should be remembered that the equation does not have to contain all the terms in sequence, even in the sum of the series. Therefore, it is only a formal notation (not exact). variant example (5) shows the missing members.

Conclusion

Bohužiaľ pre všeobecný fraktál to vedie na zložité rovnice (2.4)(2.5). Pre splnenie kongruencie (2.2) alebo (2.3) nebude asi možné všeobecne niečo vylúčiť, pretože vzťahy pre ε_k vedú na zložité vzťahy. Vzťahy však je možné využiť pre algoritmus, ktorý preverí spočítateľné kombinácie fraktálov.

/ Unfortunately for the general fractal this leads to complicated equations (2.4)(2.5). In order to fulfill the congruence (2.2) or (2.3), it will probably not be possible to exclude something in general, because the relations for ε_k lead to complex relations. However, the relationships can be used for an algorithm that checks countable combinations of fractals.
