Matthew Sicotte
Physics 326 Section 01
September 11, 2018

Lab 1: Propagation of Error

Introduction

When we perform scientific experiments, we often have to perform multiple measurements of a certain quantity. In this case, we need to be able to calculate the error of the average of these measurements so that we can treat it as a single measurement with a higher accuracy. To do this, we calculate the standard error of our measurement, which is equal to the standard deviation (found using Eq.(1)) divided by the square root of the number of trials performed, as shown in Eq.(2).

$$(1) \qquad \sigma_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$$(2) \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Additionally, in scientific experiments, we often have to determine how the errors for each measurement influence our final results. To do this, it is necessary to use the propagation of error formula. Suppose that $\{x_1, x_2, \ldots, x_n\}$ is a set of n measurements, with errors of $\{\delta x_1, \delta x_2, \ldots, \delta x_n\}$, and we want to find the error in the value of a variable q, which is a function of $x_1, x_2, \ldots, x_n$. Then, the error contributed to q from $\{x_1, x_2, \ldots, x_n\}$ is found exactly by Eq.(3) and quickly estimated by Eq.(4). The error contributed by a single measured quantiy a is found using Eq.(5).

$$(3) \qquad \delta q = \sqrt{(\frac{\partial q}{\partial x_1}\delta x_1)^2 + (\frac{\partial q}{\partial x_2}\delta x_2)^2 + \ldots + (\frac{\partial q}{\partial x_n}\delta x_n)^2}$$

$$(4) \qquad \delta q \leq |\frac{\partial q}{\partial x_1}\delta x_1| + |\frac{\partial q}{\partial x_2}\delta x_2| + \ldots + |\frac{\partial q}{\partial x_n}\delta x_n|$$

$$(5) \qquad \delta q_a = |\frac{\partial q}{\partial a}\delta a|$$

In this lab, we will be using a torsion pendulum to determine the modulus of rigidity for a steel wire. In this case, the equation for the modulus of rigidity, M, is given by Eq.(6):

$$(6) \qquad M = \frac{32\pi}{3}\frac{Lm(a^2+b^2)}{d^4T^2}$$

where L is the length of the wire, m is the mass of the bar, a is the length of the bar, b is the width of the bar, d is the diameter of the wire, and T is the period of the torsion pendulum, all of which are measured.

The main objective of this lab is to learn about errors in measurement and how they relate to the propagation of error in our calculations.

Experimental Method

This lab consisted of the following apparatus. A long, thin steel wire was hung from a standard lab stand. The bottom of the wire was threaded through a small hole in the central vertical axis of a brass bar. A screw was threaded through a larger hole parallel to the width of the bar and was used to hold the bottom end of the wire in place.

For our measurements, we used a micrometer to measure the width of the block and the diameter of the wire. To measure the length of the wire, we used a meter ruler with markings every 1 mm. Digital calipers were used to measure the length of the bar. Additionally, to measure the mass of the bar, a standard scale was used. Finally, to measure the period of the torsion pendulum, we used a stopwatch on a cell phone.

For the first part of this lab, we determined the least count of each instrument to get a rough idea of the error of each of our measurements. As it was very hard to get a very precise measurement of the width of the bar, we took three different measurements, determined the average and the standard deviation, and then estimated the propagation of error using Eq.(4) and Eq.(6). By analyzing this result, we found that the largest errors were from our measurements of the diameter of the wire and the period of the torsion pendulum.

For the second part of the lab, we improved the measurements with the largest errors by taking multiple trials and finding the average values from them. Then, we calculated the standard error for each of these measurements. For each of these measurements, the standard deviation of the mean is considered to be the appropriate error as we performed many trials. For the measurements that were stable and did not have any significant variation from trial to trial, we used the least count of the instrument used. Finally, to find the uncertainty for our modulus of rigidity, we found the exact error by using Eq.(3) and Eq.(6). We also compared our modulus of rigidity to known values for different common types of steel.

First Part Results

In the first part of this lab, we took one measurement of each quantity and determined the least count and stability of each measurement. Table 1.1 shows our measurements for the least count and stability of our initial measurements.

Table 1.1

Measurement	L (m)	m (kg)	a (m)	b (m)	d (m)	T (s)
Least count	1×10^{-3}	1×10^{-4}	1×10^{-4}	1×10^{-6}	1×10^{-6}	0.1
Stable	Yes	Yes	Yes	No	Yes	Yes

Table 1.1: Least count and stability of initial measurements.

Since the measurement for b was not stable, we took three measurements for b (shown in Table 1.2) and calculated the average and standard error for these measurements (shown in Table 1.3).

Table 1.2

Trial m	1	2	3
$b(m)$	1.9066×10^{-2}	1.9006×10^{-2}	1.9023×10^{-2}

Table 1.2: Measurements of the width of the bar, b.

Table 1.3

Average b (m)	Standard Error $\sigma_{\bar{b}}$ (m)
19.03	0.02

Table 1.3: Calculated average value and standard error for b (using Eq.(2)).

Upon analyzing the stability and uncertainty of our measurements, we obtained the following initial values, as shown in Table 1.4.

Table 1.4

Variable	Value
L	$(8.29 \pm 0.01) \times 10^{-1}$ m
m	$(2.360 \pm 0.001) \times 10^{-1}$ kg
a	$(1.144 \pm 0.001) \times 10^{-1}$ m
b	$(1.903 \pm 0.002) \times 10^{-2}$ m
d	$(4.77 \pm 0.01) \times 10^{-4}$ m
T	4.0 ± 0.1 s

Table 1.4: Initial values and errors for the six measured quantities.

Then, by using Eq.(5) for each of our measured quantities, we estimated the error that contributed to our desired quantity, M, from each measurement. By performing the required differentiation of M, we determined the formulas for the error contributed by each quantity, as shown in Table 1.5.

Table 1.5

Quantity	L	m	a
Error due to Measurement	$\frac{32\pi}{3}\frac{m(a^2+b^2)}{d^4T^2}\delta L$	$\frac{32\pi}{3}\frac{L(a^2+b^2)}{d^4T^2}\delta m$	$\frac{32\pi}{3}\frac{2Lma}{d^4T^2}\delta a$
Error due to Measurement (Pa)	1.3×10^8	3×10^7	1.8×10^8

Quantity	b	d	T
Error due to Measurement	$\frac{32\pi}{3}\frac{2Lmb}{d^4T^2}\delta b$	$\frac{32\pi}{3}\frac{4Lm(a^2+b^2)}{d^5T^2}\delta d$	$\frac{32\pi}{3}\frac{2Lm(a^2+b^2)}{d^4T^3}\delta T$
Error due to Measurement (Pa)	6×10^6	9×10^8	5×10^9

Table 1.5: Equations and values for the error of M due to our measurement of each quantity. All equations were directly found using Eq. (5), partially differentiating the expression for M (Eq. (6)) with respect to the appropriate variable.

Using Eq. (4), we obtained a total error of 7×10^9 Pa, or 7 GPa.
In comparison, our estimated value for the modulus of rigidity, M, was 106 GPa, or 1.06×10^{11} Pa.

Based on the analysis of these errors, the most important quantities to be measured again multiple times were the diameter of the wire and the period of the torsion pendulum. The period, T, was especially important to measure multiple times because our measurements may have been influenced by our reaction times as well as other variables relating to each trial such as air resistance.

Second Part Results

Upon determining the quantities to re-measure, we took 24 different measurements for the period, T, and 6 different measurements for the wire diameter, d. All other quantities measured appeared to be stable, so we used our values from the first part of the lab. The average value and associated error of each measured quantity is shown in Table 2.1.

Table 2.1

Quantity	Value
L	$(8.29\pm0.01)\times10^{-1}$ m
m	$(2.360\pm0.001)\times10^{-1}$ kg
a	$(1.144\pm0.001)\times10^{-1}$ m
b	$(1.9032\pm0.0018)\times10^{-2}$ m
d	$(5.072\pm0.005)\times10^{-4}$ m
T	(3.76 ± 0.03) s

Table 2.1: Final values for the average value and error of each measured quantity. For l, m, and a, due to stable measurements, the error is the least count of the instrument used. For all other values, the error was calculated using Eq. (2).

Analysis

Using Eq. (3) and Eq. (6), we obtained a value for M of $(9.4 \pm 0.2) \times 10^{10}$ Pa, which is 94 ± 2 GPa. This is about a 2-percent error, well within our target of 5 percent. As a comparison, the handbook value for the modulus of rigidity for steel is generally about 80 GPa, but it varies from 70 to 100 GPa. Thus, our measurements for M are well within the possible range of values.

Conclusions

In conclusion, this lab helped us learn about errors in measured quantities and the propagation of error. Although the most commonly stated modulus of rigidity of steel is 80 GPa, which is about 7 standard errors lesss than our experimental value, there is significant variation in different types of steel. However, there were also many possible sources of error in our lab. For example, there was a hole in the bar that housed a screw made of a different metal. This could have significantly altered the bar's moment of inertia, which would have made Eq. (6) slightly inaccurate. Also, the brass bar wobbled slightly as it was spinning, and the wire was of varying thickness, which could have also made Eq. (6) inaccurate. However, despite these errors, we still got a fairly accurate value for the modulus of rigidity of our steel wire.

Appendix A: Raw Data for Part 1

Table A.1

Variable	L	m	a	b	d	T
Value	8.29E-01	2.36E-01	1.14E-01	1.90E-02	4.77E-04	4.00E+00
Error	1.00E-03	1.00E-04	1.00E-04	2.00E-05	1.00E-06	0.1
Error From	1.28E+08	3.24E+07	1.81E+08	6.02E+06	8.93E+08	5.32E+09

M	Total Estimated Error
1.06E+11	7.E+09

Table A.1: Excel spreadsheet with measurements from part 1.

Table A.2

	A	B	C
1	Variable	L	m
2	Value	0.829	0.236
3	Error	0.001	0.0001
4	Error From	=32*PI()/3*C2*(D2^2+E2^2)/(F2^4*G2^2)*B3	=23*PI()/3*B2*(D2^2+E2^2)/(F2^4*G2^2)*C3
5			
6		M	Total Estimated Error
7		=32*PI()/3*B2*C2*(D2^2+E2^2)/(F2^4*G2^2)	=SUM(B4:G4)

	D	E	F	G
1	a	b	d	T
2	0.1144	0.01903	0.000477	4
3	0.0001	0.00002	0.000001	0.1
4	=32*PI()/3*2*B2*C2*D2/(F2^4*G2^2)*D3	=32*PI()/3*2*B2*C2*	=32*PI()/3*4*B2*C2*	=32*PI()/3*2*B2*C2*
5				
6				
7				

Table A.2: Excel spreadsheet with formulas from part 1.

Appendix B: Raw Data for Part 2

Table B.1

Period T (s)	Mass m (kg)	Block Length a (m)	Block Width b (m)	Wire Diameter d (m)	Wire Length L (m)
3.7	2.360E-01	1.144E-01	1.9006E-02	5.07E-04	8.29E-01
3.8			1.9023E-02	5.07E-04	
3.8			1.9066E-02	5.07E-04	
3.7				5.09E-04	
4.1				5.05E-04	
3.8				5.08E-04	
4.0					
3.6					
4.0					
3.6					
3.7					
3.9					
4.0					
3.7					
3.7					
3.9					
3.7					
3.6					
3.8					
3.7					
3.7					
3.7					
3.6					
3.4					

Table B.1: Excel spreadsheet with trial-by-trial measurements from part 2.

Table B.2

	Period T (s)	Mass m (kg)	Block Length a (m)	Block Width b (m)	Wire Diameter d (m)	Wire Length L (m)
Average	-	-	-	-	-	-
	3.76	2.360E-01	1.144E-01	1.9032E-02	5.072E-04	8.29E-01
Error	-	-	-	-	-	-
	3.E-02	1.E-04	1.E-04	1.8E-05	5.E-07	1.E-03
Total Error Contributed						
	1.7E+09	4.E+07	2.E+08	5.E+06	4.E+08	1.E+08

Final M	Total Error
9.4E+10	2.E+09

Table B.2: Excel spreadsheet with measurements from part 2.

Table B.3

	A
1	Period T (s)
2	Average
3	=AVERAGE('Torsion Pendulum Data'!A2:A25)
4	Error
5	=STDEV.S('Torsion Pendulum Data'!A2:A25)/SQRT(COUNT('[Book2]Torsion Pendulum Data'!A2:A25))
6	
7	Total Error Contributed
8	=32*PI()/3*2*F3*B3*(C3^2+D3^2)/(E3^4*A3^3)*A5
9	
10	Final M
11	=32*PI()/3*F3*B3*(C3^2+D3^2)/(E3^4*A3^2)

	B	C
1	Mass m (kg)	Block Length a (m)
2	-	-
3	0.236	0.1144
4	-	-
5	0.0001	0.0001
6		
7		
8	=32*PI()/3*(F3*(C3^2+D3^2)/(E3^4*A3^2)*B5)	=32*PI()/3*2*F3*B3*C3/(E3^4*A3^2)*C5
9		
10	Total Error	
11	=SQRT(A8^2+B8^2+C8^2+D8^2+E8^2+F8^2)	

	D
1	Block Width b (m)
2	-
3	=AVERAGE('Torsion Pendulum Data'!D2:D4)
4	-
5	=STDEV.S('Torsion Pendulum Data'!D2:D4)/SQRT(COUNT('[Book2]Torsion Pendulum Data'!D2:D4))
6	
7	
8	=32*PI()/3*2*B3*D3*F3/(E3^4*A3^2)*D5
9	
10	
11	

	E
1	Wire Diameter d (m)
2	-
3	=AVERAGE('Torsion Pendulum Data'!E2:E7)
4	-
5	=STDEV.S('Torsion Pendulum Data'!E2:E7)/SQRT(COUNT('[Book2]Torsion Pendulum Data'!E2:E7))
6	
7	
8	=32*PI()/3*4*F3*B3*(C3^2+D3^2)/(E3^5*A3^2)*E5
9	
10	
11	

	F
1	Wire Length L (m)
2	-
3	0.829
4	-
5	0.001
6	
7	
8	=32*PI()/3*B3*(C3^2+D3^2)/(E3^4*A3^2)*F5
9	
10	
11	

Table B.3: Excel spreadsheet with formulas from part 2.