Mechanics Modeling of Origami Robots

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***Abstract*—Origami robots are characterized by their compact design, quasi-2D manufacturing process, and folding joint-based transmission kinematics. The constituent of engineering materials and integration of core robotic components, make it unrealistic to rely on conventional mathematical models to model the mechanics of these robots. Hence, origami robots require a comprehensive mechanics model to accelerate the structure design and provide a guide to the actuation strategy. Currently, there is no such mechanics model that can achieve this goal. We propose a nonlinear lattice-and-plate model to simulate the mechanics of origami robots, including the localized bending on flexible hinges, global displacements of rigid panels, and trajectory of predefined outputs. Moreover, this proposed model is of high accuracy and efficiency, i.e., the accuracy characterized on folding joints shows that the deviations are within ±6.2%, and the runtime is within several seconds regardless of the structural complexity of the robots. We further validate the capability of the model on a gripper, a bellow actuator, and a parallel platform. To conclude, the computational model provides a universal tool to analyze the mechanics of any origami robot.**

I. Motivation and Background

ORIGAMI design approach of folding planar sheets into three-dimensional (3D) objects offers customizability, scalability, and adaptability in autonomous robots [1]. Currently, origami structures, ranging from microscale to macroscale, are proved to be applicable in various scenarios, e.g. medical devices [2], robotic manipulators and grippers [3], and locomotive robots [4, 5]. To accelerate the development of new functional origami devices including origami robots, we develop a comprehensive model to synthesize their mechanics [6-8], i.e. the response to the internal actuation and/or external stimuli.

Traditional origami models are built by creasing thin flexible mediums. While in origami robots, we are required to use thick engineering materials to build foldable structures. To replicate the folding motion, flexible materials are used to achieve a localized reduction of stiffness along the fold regions. This flexible fold region is named flexible hinge [6]. In origami robots, the flexible hinges are normally built by smart materials, which enables the integration of strain sensing and self-folding. Based on the constitutive material, the folding stiffness of flexible hinges can be linear and nonlinear. Moreover, the hinges also induce a geometrical constraint to the maximal folding angles. Hence, it is more challenging to analyze their mechanics.

The current research on origami modeling focuses mainly on the thin sheet prototypes by ignoring the facet thickness and folding width, e.g. the rigid-facet model, lattice framework approaches [9, 10], and finite element method. However, these models are unable to efficiently predict the mechanics of origami robots, which are composed of nonzero-thickness panels and nonzero-width flexible hinges. To model thick origamis, previous researchers either use a ruling surface [11] or discrete bars [12] to model the localized bending on the flexible hinge. Nevertheless, both approaches ignore the semi-rigid connection between material interfaces. Inspired by the previous work [9, 10], our new mechanics model triangulates the structure into lattices to model the in-plane deformation and uses a plate model to analytical capture the out-of-plane bending and folding. For clarity, we name this model the ***lattice-and-plate model***. The proposed model can apply to a large spectrum of origami robots published before [3-5], whose models are either too simple or limited to specific structures.

II. The Lattice-and-Plate Mechanics Model

The fundamental idea of our computational model is shown in Fig. 1 by taking a bellow actuator as an example, which is discretized into truss and spring elements. Herein, the truss elements model the in-plane stretching and shearing of panels and the spring elements capture the out-of-plane bending and folding. The folding stiffness is obtained from the analytical model of flexible hinges. With consideration of semi-rigid connection on material interfaces, the flexible hinges are modeled by elastically supported thin plates [6]. Suppose the plate is elastically supported on $\zeta_1 = 0, a$, and subject to internal moment $\boldsymbol{T}_{spr}^{r}$ on $\zeta_1 = a$, and free of load on $\zeta_2 = \pm \frac{L_0^e}{2}$,

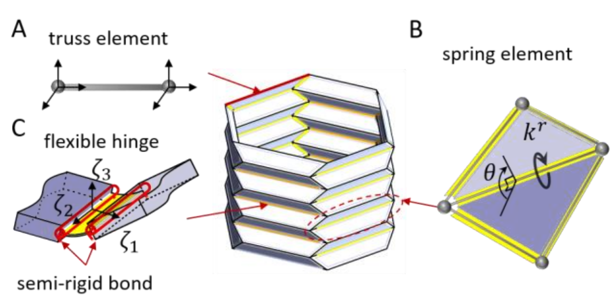

Fig. 1 Schematic of the lattice-and-plate model. Taking a bellow actuator as an example, it is discretized into (a) truss elements and (b) spring elements. Specifically (c) the folding stiffness and deflection of the hinges are modeled by the plate model.

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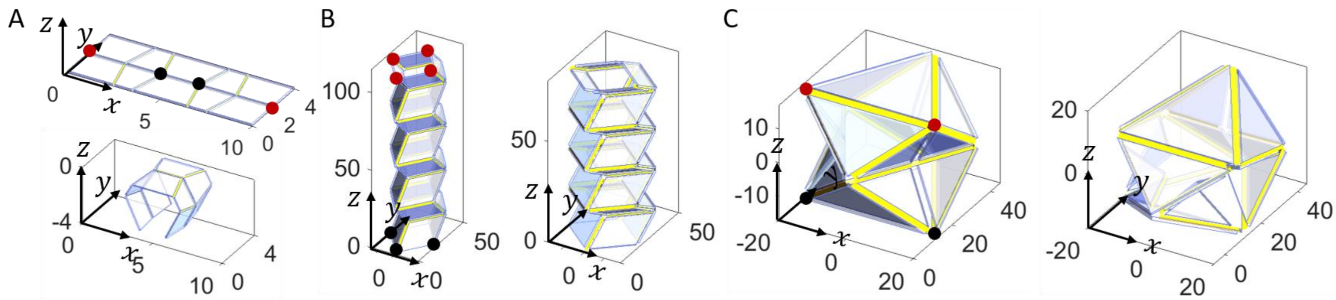

Fig. 2 Boundary conditions and simulation results for (A) a origami gripper, (B) bellow actuator, and (C) parallel platform. Herein, the black dots are fixed, and red dots are subject to uniform force along -z direction.

its deflection w is given by,

$$w = -\frac{\|\boldsymbol{T}_{spr}^r\|}{2\boldsymbol{D}(1-v^2)}\zeta_1^2 + \frac{v\|\boldsymbol{T}_{spr}^r\|}{2\boldsymbol{D}(1-v^2)}\zeta_2^2 - \zeta_1 \tan\frac{L_0^e\|\boldsymbol{T}_{spr}^r\|}{k_{spr}} - \frac{v(L_0^e)^2\|\boldsymbol{T}_{spr}^r\|}{2\boldsymbol{D}(1-v^2)}\sum_{m=1}^{\infty}\left[\frac{1}{m\pi} - \left(\frac{2}{m\pi}\right)^3\right](-1)^{\frac{m-1}{2}}\cos\frac{m\pi y}{L_0^e} \quad (1)$$

where k_{spr} describes the semi-rigid connection between thick panels and flexible hinges, for $k_{spr} = 0, 0 < k_{spr} < \infty, k_{spr} = \infty$ means simply supported, elastically supported, and ideal clamp, respectively; $\boldsymbol{D}$ is the flexural rigidity of the flexible hinge; v is Poisson's ratio, m is an odd integer.

Regarding the discrete trusses and springs, we adopt the virtual work principle (VWP) to establish the governing equation between the nodal force $\boldsymbol{P}$ and nodal displacement $\boldsymbol{u}$, which is given as [7, 8],

$$\sum_e \int_{V^e} S^e : \delta\varepsilon_G^e \, dV^e + \sum_r k^r \theta^r \delta\theta^r = \delta\boldsymbol{u}^T\boldsymbol{P}, \quad (2)$$

where the superscripts e and r are the index of the truss and spring elements, respectively; V^e is the volume of truss e, S^e is the second Piola-Kirchhoff stress tensor, k^r is the stiffness of the rotational spring r, θ^r is bending or folding angle, and $\delta\theta^r$ is variation of θ^r. To be noted, k^r describes both bending and folding stiffness, while the latter is obtained from Eq.1 by $\|\boldsymbol{T}_{spr}^r\|/\frac{dw}{d\zeta_1}|\zeta_1 = a$. In this work, we use the Newton-Raphson method to solve the nonlinear equilibrium in Eq. 2 to accurately capture the mechanics of origami robots under various boundary conditions.

III. Numerical Examples

The capability and efficiency of the lattice-and-plate model are validated through various thick origami structures under different boundary conditions, including folding, unfolding, and loading. In our work [6-8], we evaluated the accuracy of the model based on origami folding joints. The deviation is within ±6.2% compared to the experimental tests and ±12.6% with regards to finite element method simulations, which indicates the high accuracy of our model. As indicated in Fig. 2, the lattice-and-plate model is adopted to analyze the mechanics of three widely used origami structures, i.e. a Miura-Ori-based gripper, a bellow actuator, and a multiple degree-of-freedom platform. The model is implemented in Matlab code, and all simulations are run in an i7 2.20 GHz Windows 10 platform, and the runtime in Table 1 is provided to show the efficiency of the lattice-and-plate model.

Table 1 Profiles of the origami robots and runtime of the simulations.

Origami patterns	Lattice-and-plate discretization			Runtime (s)
	No. of trusses	No. of springs	No. of hinges	
Gripper	27	23	13	2.85
Actuator	100	92	60	5.97
Platform	36	24	24	1.94

The profile of the three origami structures and the runtime are listed in Table 1, and the semi-rigid connection between rigid panels and flexible hinges is set as $k_{spr} = 0.1$. The runtime is almost proportional to the number of discretized elements regardless of the geometry complexity of origami structures. Moreover, all the runtime is within several seconds, revealing the high efficiency of our computational model.

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