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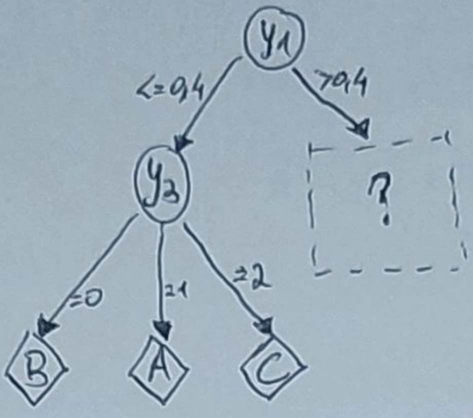
Homework I

1. $IG(y_i) = H(y_{out}) - H(y_{out} \mid y_i)$

$$H(y) = \sum_{i=1}^{n} -p(y_i) \log_2[p(y_i)]$$



- $\boxed{y_1 > 0{,}4}$:

D	y_1	y_2	y_3	y_4	y_{out}
x_5	0,45	1	0	2	C
x_6	0,52	0	0	0	B
x_7	0,58	2	1	2	C
x_8	0,62	1	0	1	A
x_9	0,71	1	2	1	A
x_{10}	0,83	1	2	1	B
x_{11}	0,90	2	1	2	B
x_{12}	0,95	2	2	2	C

- $p(A) = \frac{2}{8} = \frac{1}{4}$
- $p(B) = \frac{3}{8}$
- $p(C) = \frac{3}{8}$

- $H(y_{out}) = -p(A)\log_2[p(A)] - p(B)\log_2[p(B)] - p(C)\log_2[p(C)] =$

$$= -\frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{3}{8}\log_2\left(\frac{3}{8}\right) - \frac{3}{8}\log_2\left(\frac{3}{8}\right) \simeq 1{,}561$$

- $H(y_{out} \mid y_2) = p(0)\,H(y_{out} \mid y_2=0) + p(1)\,H(y_{out} \mid y_2=1) + p(2)\,H(y_{out} \mid y_2=2) =$

$$= \frac{1}{8}\times 0 + \frac{1}{2}\times\left[\overbrace{-\frac{1}{2}\log_2\left(\frac{1}{2}\right)}^{A} \overbrace{-\frac{1}{4}\log_2\left(\frac{1}{4}\right)}^{B} \overbrace{-\frac{1}{4}\log_2\left(\frac{1}{4}\right)}^{C}\right] + \frac{3}{8}\times\left[\overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{B} \overbrace{-\frac{2}{3}\log_2\left(\frac{2}{3}\right)}^{C}\right] \simeq 1{,}094$$

- $H(y_{out} \mid y_3) = p(0)\,H(y_{out} \mid y_3=0) + p(1)\,H(y_{out} \mid y_3=1) + p(2)\,H(y_{out} \mid y_3=2) =$

$$= \frac{3}{8}\times\left[\overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{A} \overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{B} \overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{C}\right] + \frac{1}{4}\times 1 + \frac{3}{8}\times\left[\overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{A} \overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{B} \overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{C}\right] \simeq 1{,}439$$

- $H(y_{out} \mid y_4) = p(0)\,H(y_{out} \mid y_4=0) + p(1)\,H(y_{out} \mid y_4=1) + p(2)\,H(y_{out} \mid y_4=2) =$

$$= \frac{1}{8}\times 0 + \frac{3}{8}\times\left[\overbrace{-\frac{2}{3}\log_2\left(\frac{2}{3}\right)}^{A} \overbrace{-\frac{1}{3}\log_2\left(\frac{1}{3}\right)}^{B}\right] + \frac{1}{2}\times\left[\overbrace{-\frac{1}{4}\log_2\left(\frac{1}{4}\right)}^{B} \overbrace{-\frac{3}{4}\log_2\left(\frac{3}{4}\right)}^{C}\right] = 0{,}75$$

- $IG(y_2) = H(y_{out}) - H(y_{out} \mid y_2) = 1{,}561 - 1{,}094 = 0{,}467$

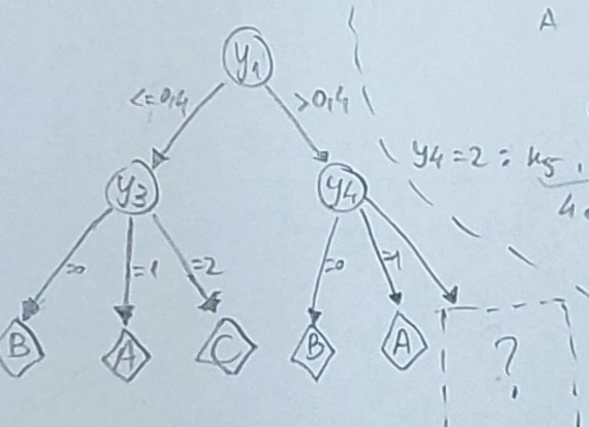
$IG(y_4) > IG(y_2) > IG(y_3)$

- $IG(y_3) = H(y_{out}) - H(y_{out} \mid y_3) = 1{,}561 - 1{,}439 = 0{,}122$
- $IG(y_4) = H(y_{out}) - H(y_{out} \mid y_4) = 1{,}561 - 0{,}75 = 0{,}811$ → highest IG

$y_4 = 0$: $x_6 \rightarrow$ Ⓑ

$y_4 = 1$: x_8 , x_9 , x_{10}
A A B
⇓
Ⓐ

$y_4 = 2$: $\underbrace{x_5, x_7, x_{11}, x_{12}}_{\text{4 observations}}$
⇓
split



- $\boxed{y_1 > 0{,}4 \, , \, y_4 = 2}$:

D	y_1	y_2	y_3	y_4	y_{out}
x_5	0,45	1	0	2	C
x_7	0,58	2	1	2	C
x_{11}	0,90	2	1	2	B
x_{12}	0,95	2	2	2	C

- $p(B) = \frac{1}{4}$
- $p(C) = \frac{3}{4}$

- $H(y_{out}) = -p(A)\log_2[p(A)] - p(B)\log_2[p(B)] - p(C)\log_2[p(C)] =$

  $= 0 - \frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{3}{4}\log_2\left(\frac{3}{4}\right) \simeq 0{,}811$

- $H(y_{out} \mid y_2) = p(0)\, H(y_{out} \mid y_2 = 0) + p(1)\, H(y_{out} \mid y_2 = 1) + p(2)\, H(y_{out} \mid y_2 = 2) =$

  $= 0 + \frac{1}{4} \times 0 + \frac{3}{4} \times \left[-\frac{1}{3}\log_2\left(\frac{1}{3}\right) - \frac{2}{3}\log_2\left(\frac{2}{3}\right)\right] \simeq 0{,}689$

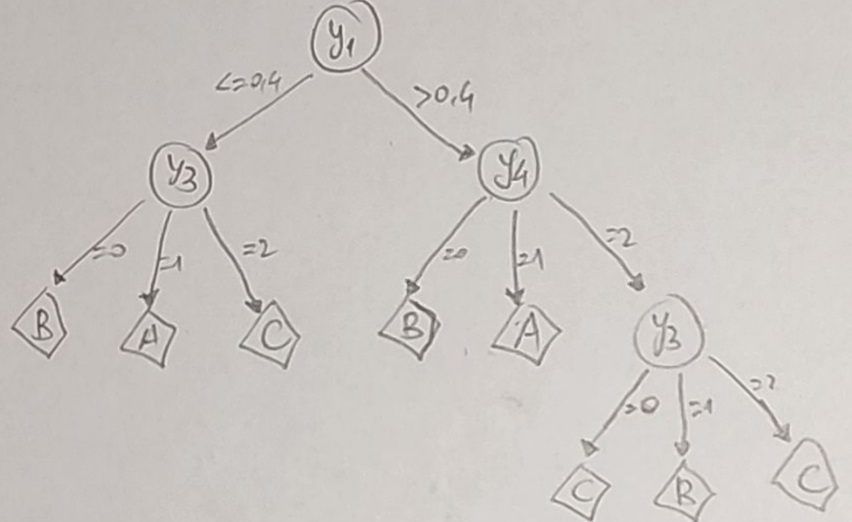
- $H(y_{out} \mid y_3) = p(0)\, H(y_{out} \mid y_3 = 0) + p(1)\, H(y_{out} \mid y_3 = 1) + p(2)\, H(y_{out} \mid y_3 = 2) =$

  $= \frac{1}{4} \times 0 + \frac{1}{2} \times 1 + \frac{1}{4} \times 0 = \frac{1}{2} = 0{,}5$

- $IG(y_2) = H(y_{out}) - H(y_{out} \mid y_2) = 0{,}811 - 0{,}689 = 0{,}122$

  $IG(y_3) > IG(y_2)$

- $IG(y_3) = H(y_{out}) - H(y_{out} \mid y_3) = 0{,}811 - 0{,}5 = 0{,}311$ → highest IG



$y_3 = 0$: x_5 → C

$y_3 = 1$: x_7, x_{11}
C B ⇒ B ↗ by ascending alphabetic

$y_3 = 2$: x_{12} → C

2.

D	Yout (true)	yout (predict)	
x_1	A	A	✓
x_2	B	B	✓
x_3	C	C	✓
x_4	A	A	✓
x_5	C	C	✓
x_6	B	B	✓
x_7	C	B	×
x_8	A	A	✓
x_9	A	A	✓
x_{10}	B	A	×
x_{11}	B	B	✓
x_{12}	C	C	✓

Training Confusion Matrix

		TRUE		
		A	B	C
Predict	A	4	1	0
	B	0	3	1
	C	0	0	3

3. $F_1 = \dfrac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$ Recall $= \dfrac{TP}{P} = \dfrac{TP}{TP + FN}$ Precision $= \dfrac{TP}{TP + FP}$

A: $TP = 4$

$TN = 3 + 1 + 3 = 7$

$FP = 1$

$FN = 0$

B: $TP = 3$

$TN = 4 + 3 = 7$

$FP = 1$

$FN = 1$

C: $TP = 3$

$TN = 4 + 1 + 3 = 8$

$FP = 0$

$FN = 1$

P - precision R - recall

(A)

$$P = \frac{TP}{TP+FP} = \frac{4}{4+1} = \frac{4}{5}$$

$$R = \frac{TP}{TP+TN} = \frac{4}{4} = 1$$

$$F1 = \frac{2 \cdot \frac{4}{5} \cdot 1}{\frac{4}{5}+1} = \frac{\frac{8}{5}}{\frac{9}{\cancel{8}}} = \frac{8}{9}$$

(B)

$$P = \frac{TP}{TP+FP} = \frac{3}{3+1} = \frac{3}{4}$$

$$R = \frac{TP}{TP+TN} = \frac{3}{3+1} = \frac{3}{4}$$

$$F1 = \frac{2 \cdot \frac{3}{4} \cdot \frac{3}{4}}{\frac{3}{4}+\frac{3}{4}} = \frac{\frac{18}{16}}{\frac{6}{4}} = \frac{18 \cdot 4}{16 \cdot 6} = \frac{3}{4}$$

(C)

$$P = \frac{TP}{TP+FP} = \frac{3}{3} = 1$$

$$R = \frac{TP}{TP+TN} = \frac{3}{3+1} = \frac{3}{4}$$

$$F1 = \frac{2 \cdot 1 \cdot \frac{3}{4}}{1+\frac{3}{4}} = \frac{\frac{6}{\cancel{4}}}{\frac{7}{\cancel{4}}} = \frac{6}{7}$$

class (B) has the lowest training F1 score. $\left(\frac{3}{4} < \frac{6}{7} < \frac{8}{9}\right)$

4.

D	y_1	y_{out}	
x13	0,02	A	3
x1	0,12	A	
x2	0,18	B	
x3	0,25	C	3
x14	0,27	C	
x4	0,33	A	
x5	0,45	C	3
x6	0,52	B	
x7	0,58	C	
x8	0,62	A	2
x9	0,71	A	
x10	0,83	B	3
x11	0,90	B	
x12	0,95	C	

5 equally spaced bins in [0,1]:

↳ [0 ; 0,2] : 3

↳]0,2 ; 0,4] : 3

↳]0,4 ; 0,6] : 3

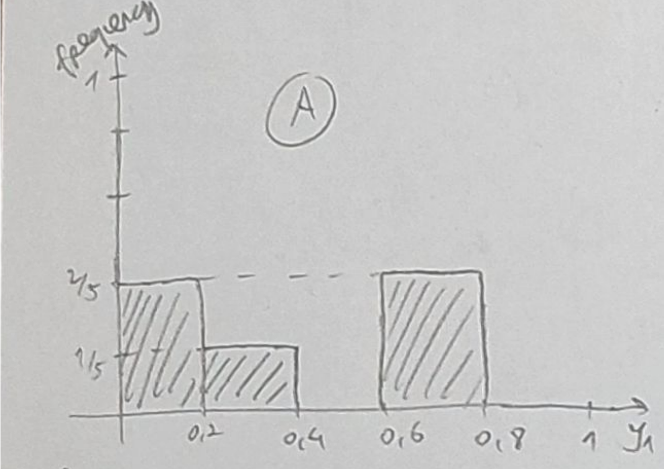
↳]0,6 ; 0,8] : 2

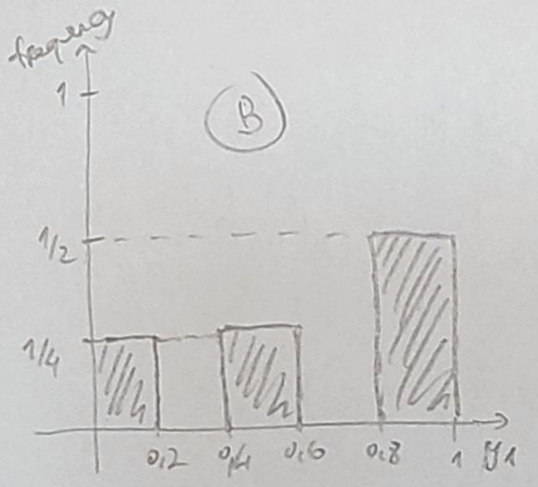
↳]0,8 ; 1] : 3

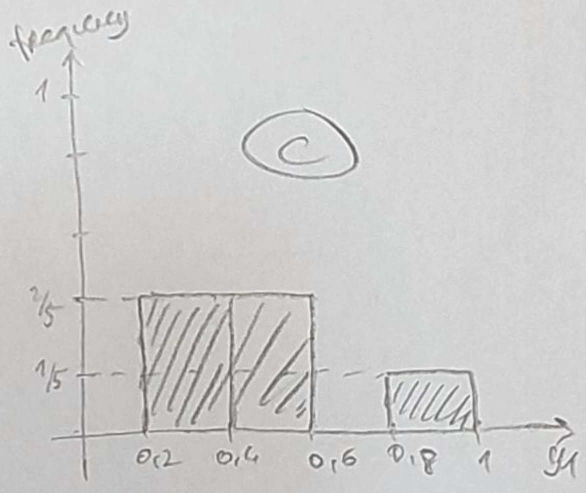
n = 14

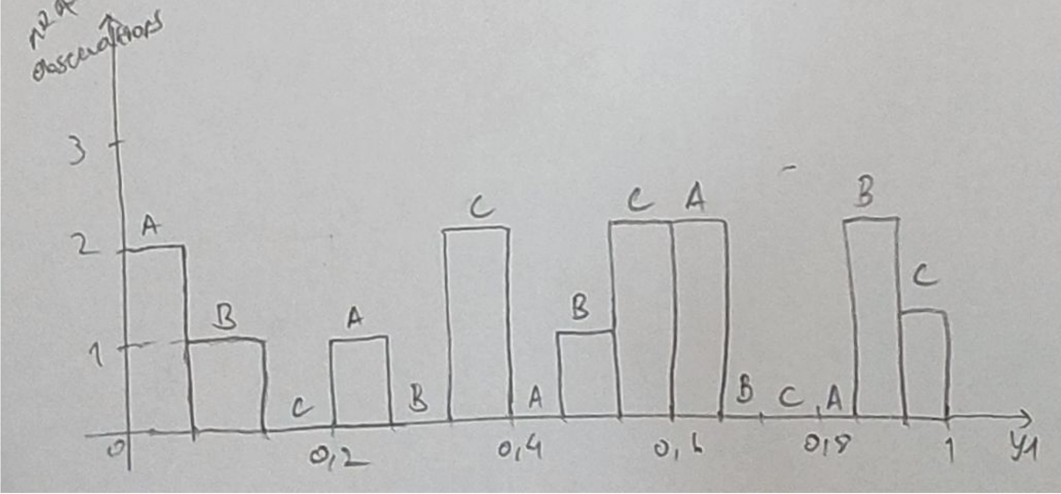
A (5)	B (4)	C (5)
2/5	1/4	0
1/5	0	2/5
0	1/4	2/5
2/5	0	0
0	1/2	1/5

(frequency of the classes in each bin)



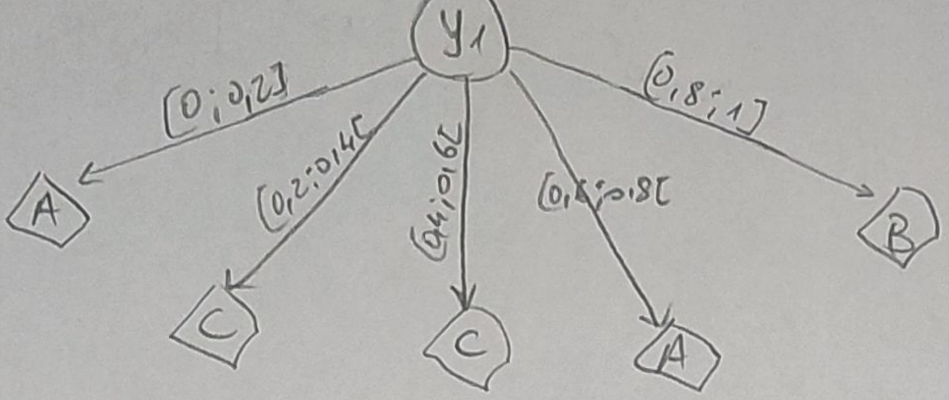






classes per bin:

[A ; C ; C ; A ; B]



5.

$$y_1 : \underbrace{0{,}12 \quad 0{,}18 \quad 0{,}25 \,\|\, 0{,}33 \quad 0{,}45 \quad \underbrace{0{,}52 \,\|\, 0{,}58} \quad 0{,}62 \quad 0{,}71 \,\|\, 0{,}83 \quad 0{,}90 \quad 0{,}95}$$

$$Q_2 = \frac{0{,}52+0{,}58}{2} = 0{,}55$$

$$Q_1 = \frac{0{,}25+0{,}33}{2} = 0{,}29$$

$$IQR = Q_3 - Q_1 = 0{,}77 - 0{,}29 = 0.48$$

$$Q_3 = \frac{0{,}71+0{,}83}{2} = 0{,}77$$

$$\text{Bounds} = [Q_1 - 1{,}5 \times IQR \,;\, Q_3 + 1{,}5 \times IQR] =$$

$$= [0{,}29 - 1{,}5 \times 0{,}48 \,,\, 0{,}77 + 1{,}5 \times 0{,}48] =$$

$$= [-0{,}43 \,;\, 1{,}49] \longrightarrow \text{No ootliers}$$