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Linea Algebra 2016

1) The basis is 1) Linearly independent
2) Their combination must produce every vector in M

$\therefore$ Basis of M is

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}$$

$$\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

Dimension $= |\text{Basis } M| = 9$

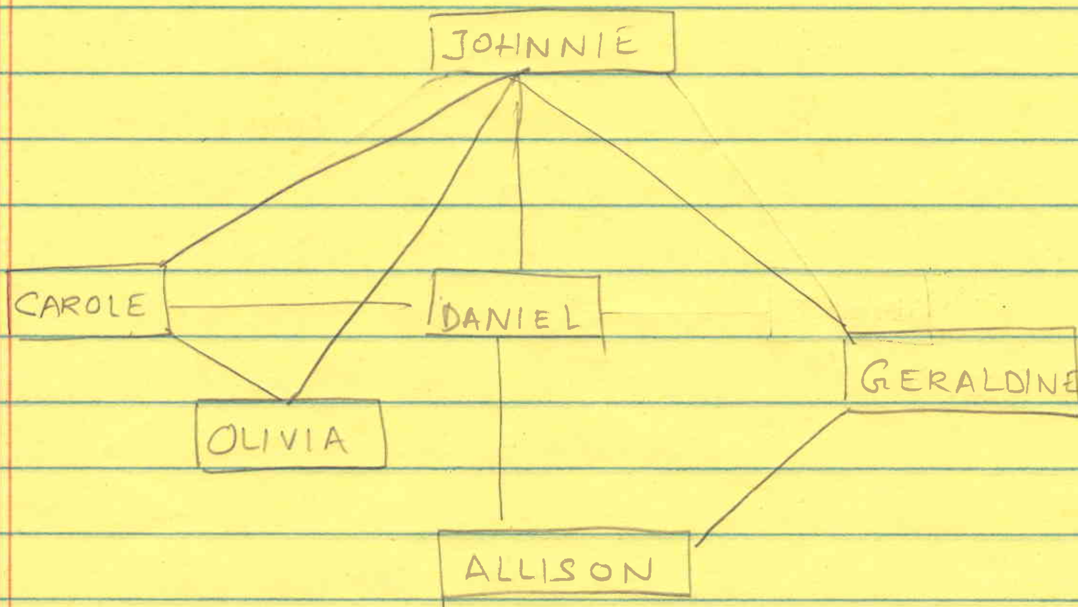
2) Using the same logic as above,
Basis S =

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

Dimension $S = |\text{Basis } S| = 6$

3) $S \cap M = S$ $\therefore$ Dimension $S \cap M = 6$

4)



	CAROLE	OLIVIA	JOHNNIE	DANIEL	ALLISON	GERALDINE
CAROLE	1	1	1	1	0	0
OLIVIA	1	1	1	0	0	0
JOHNNIE	1	1	1	1	0	1
DANIEL	1	0	1	1	1	0
ALLISON	0	0	0	1	1	1
GERALDINE	0	0	1	0	1	1

Instead of having only binary (Fried / Not Friend) relation, we would have 1 for acquintaces, 2 for friendship, 3 for colleagues in the Matrix. Likewise, the graph edges would have weights.

5) Let $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ & $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$

Now x & y are othogonal $\Rightarrow$ dot product of x & $y = 0$

ie $\sum_{i=1}^{n} \bar{x}_i y_i = 0$ But $\sum_{i=1}^{n} \bar{x}_i y_i = x^T y = 0$ (Since x_i is real $\bar{x}_i = x_i$)

Likewise $y^T x = 0$

$\therefore x^T y = y^T x = 0$

6) $(A^T A)^T = A^T (A^T)^T = A^T A$

$\therefore$ if $M = A^T A \Rightarrow M^T = M \Rightarrow A^T A$ is symmetric

7) $QQ^T = I$. Let $Q = \begin{bmatrix} q_{11} & \cdots & q_{1m} \\ \vdots & & \\ q_{m1} & \cdots & q_{mm} \end{bmatrix}$ $Q^T = \begin{bmatrix} q_{11} & \cdots & q_{m1} \\ \vdots & & \vdots \\ q_{1m} & & q_{mm} \end{bmatrix}$

$\therefore (QQ^T)_{ij} = \sum_{k=1}^{m} q_{ik} q_{kj}$

1) $\therefore \|q_i\|^2 = \sum_{j=1}^{m} (QQ^T)_{ij} = \sum_{j=1}^{m} \sum_{k=1}^{m} q_{ik} q_{kj} = \sum_{k=1}^{m} q_{ik} \left(\sum_{j=1}^{m} q_{kj}\right)$

$= \sum_{k=1}^{m} q_{ik} \cdot 1$ ($\because QQ^T = 1$

$= 1 . \sum_{k=1}^{m} q_{ik} = 1$

2) $q_i \neq q_j \quad \therefore q_i^T q_j = \sum_{k=1}^{m} q_{ik} q_{kj} = (QQ^T)_{ij} = I_{ij}$

$\because i \neq j \Rightarrow I_{ij} = 0 . \therefore q_i^T q_j = 0$

7.3) $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow Q^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore QQ^T = \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore QQ^T = \underline{I} \Rightarrow Q^T = Q^{-1}$ $\left(\because \det(Q) = 1 \neq 0 \text{ So } Q^{-1} \text{ exists}\right)$

8) 1) $A = I_4$ $A^T = \underline{I}_4 \Rightarrow AA^T = I_4$ is symmetrical

2) $A = I_4 \Rightarrow A^{-1} = I_4$

$\therefore A + A^{-1} = 2I_4 = 2A$

3) $A = I_4$ $\min(\lambda_i) = 1$

4) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ $\det(A) = 0$

5) *) $(A^TA)^T = A^TA \Rightarrow A^TA = A^TA \Rightarrow$ any 4x4 matrix A would suffice

*) If $\det(A) = 0$ A^{-1} does not exist $\therefore$ We can't have both 2 & 4.

*) $\det(A) = \prod_{i=1}^{n} \lambda_i$ Now if $\min(\lambda_i) > 0 \Rightarrow \det(A) \neq 0$

$\therefore$ 8.5) $\therefore$ $A = I_4$ satisfies 1), 2) 3)

9) $$E \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 3 & 3 & 1 \end{bmatrix}$$

$$\underset{A}{\parallel} \qquad\qquad \underset{B}{\parallel}$$

$\therefore$ $E \cdot A = B$

$\Rightarrow E A \cdot A^{-1} = B \cdot A^{-1} \Rightarrow E(A \cdot A^{-1}) = B \cdot A^{-1} \Rightarrow E = B \cdot A^{-1}$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix} \Rightarrow B \cdot A^{-1} = E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$