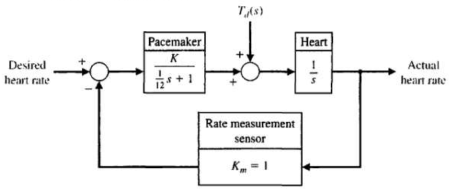
P5. 17

P5.17 Electronic pacemakers for human hearts regulate the speed of the heart pump. A proposed closed-loop system that includes a pacemaker and the measurement of the heart rate is shown in Figure P5.17 [2, 3]. The transfer function of the heart pump and the pacemaker is found to be

$$G(s) = \frac{K}{s(s/12 + 1)}.$$

Design the amplifier gain to yield a system with a settling time to a step disturbance of less than 1 second. The overshoot to a step in desired heart rate should be less than 10%. (a) Find a suitable range of K. (b) If the nominal value of K is $K = 10$, find the sensitivity of the system to small changes in K. (c) Evaluate the sensitivity of part (b) at DC (set $s = 0$). (d) Evaluate the magnitude of the sensitivity at the normal heart rate of 60 beats/minute.



(a)

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K}{s\left(\frac{s}{12}+1\right)} \cdot \frac{1}{1+\frac{K}{s\left(\frac{s}{12}+1\right)}}$$

$$= \frac{12K}{s^2+12s+12K} = \frac{\omega_n^2}{s^2+2s\omega_n\zeta+\omega_n^2}$$

So

$$\omega_n = \sqrt{12k}, \qquad \zeta = \frac{6}{\sqrt{12k}}$$

If we want P.O. under 10% and

$$P.O. = 100 \cdot e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

Then

$$0.5912 < \zeta \rightarrow K < 8.5833$$

And settling time

$$T_s = \frac{4}{\zeta\omega_n} = \frac{2}{3}s$$

which is less than the requirement of 1 second

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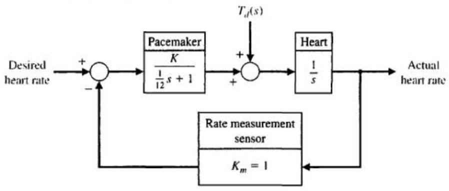

(b)

$$S_K^T(s) = S_G^T \cdot S_K^G$$

$$S_K^G(s) = \frac{\Delta G}{G} / \frac{\Delta K}{K}$$

If k=10 and let's say $\frac{\Delta K}{K} = 10\% = \frac{1}{K}$ Then

$$\Delta G = \left(\frac{K \cdot \left(1 + \frac{1}{K}\right)}{s(\frac{s}{12} + 1)}\right) - \left(\frac{K}{s\left(\frac{s}{12} + 1\right)}\right) = \frac{1}{s\left(\frac{s}{12} + 1\right)} = \frac{G}{K} \rightarrow \frac{\Delta G}{G} = \frac{1}{K}$$

And $S_K^G(s) = \frac{1/K}{1/K} = 1.$

And so

$$S_K^T(s) = S_G^T \cdot 1$$

For a closed loop system

$$S_G^T = \frac{1}{1 + G(s)} = \frac{s\left(\frac{s}{12} + 1\right)}{K + s\left(\frac{s}{12} + 1\right)} = \frac{s^2 + 12s}{120 + s^2 + 12s} = S_K^T(s)$$

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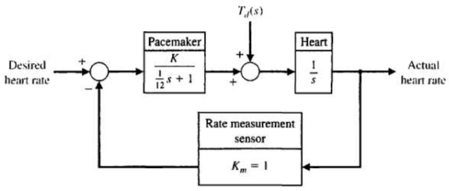


(c)

Setting s=0 gives sensitivity to K

$$S_K^T(s=0) = \frac{s^2 + 12s}{120 + s^2 + 12s} = \frac{0}{120} = 0.$$

(d)

60 beats/minute = 1 beat/second
So the heart has a frequency of 2π
In Laplace domain then $s = \omega j = 2\pi j$

$$|S_K^T(s = 2\pi j)| = 0.77$$

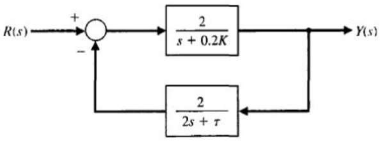
P5. 22

P5.22 Consider the closed-loop system in Figure P5.22, where

$$G_c(s)G(s) = \frac{2}{s + 0.2K} \quad \text{and} \quad H(s) = \frac{2}{2s + \tau}.$$

(a) If $\tau = 2.43$, determine the value of K such that the steady-state error of the closed-loop system response to a unit step input, $R(s) = 1/s$, is zero.

(b) Determine the percent overshoot P.O. and the time to peak T_p of the unit step response when K is as in part (a).



(a)

$$T(s) = \frac{Y(s)}{R(s)} = \frac{2}{s + 0.2k} \cdot \left(1 + \frac{2}{2s + \tau} \cdot \frac{2}{s + 0.2k}\right)^{-1} = \frac{2 \cdot (2s + \tau)}{4 + (s + 0.2k) \cdot (2s + \tau)}$$

When R(s)=1/s

$$\mathrm{E}(s) = R(s) - Y(s) = \left(\frac{1}{s}\right) - \left(\frac{1}{s} \cdot \frac{2 \cdot (2s+\tau)}{4+(s+0.2k) \cdot (2s+\tau)}\right)$$

We can then use the final value theorem to find the steady state error

$$\mathrm{E}(\infty) = \lim_{s \to 0}\big(s \cdot E(s)\big) = \lim_{s \to 0}\left(1 - \frac{2 \cdot (2s+\tau)}{4+(s+0.2k) \cdot (2s+\tau)}\right) = 1 - \frac{2 \cdot (\tau)}{4+(0.2k) \cdot (\tau)}$$

If we want $\mathrm{E}(\infty) = 0$ then we must select k such that

$$\frac{2 \cdot (\tau)}{4 + (0.2k) \cdot (\tau)} = 1 \quad \rightarrow \quad 2 = \frac{4}{\tau} + 0.2k$$

Hence:

$$k = 10 - \frac{20}{\tau} = 1.77$$

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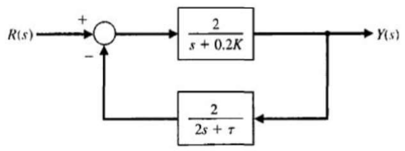

(b)

$$T(s) = \frac{Y(s)}{R(s)} = 2 \cdot \frac{(2s + \tau)}{4 + \left(s + \left(2 - \frac{4}{\tau}\right)\right) \cdot (2s + \tau)}$$

Rearrange to the right form:

$$= \frac{(2s + \tau)}{s^2 + 2s\sqrt{\tau}\left(\frac{4\tau^1 + \tau^2 - 8}{4\tau^{3/2}}\right) + \tau}$$

So:

$$\omega_n = \sqrt{\tau}, \qquad \zeta = \frac{4\tau^1 + \tau^2 - 8}{4\tau^{3/2}}$$

And hence:

$$Y(s) = \frac{(2s + \omega_n^2)}{s^2 + 2s\omega_n\zeta + \omega_n^2} \mathrm{R(s)}$$

Peak time is given by

$$T_p = \frac{\pi}{\omega_n\sqrt{1 - \zeta^2}} = 2.33$$

Percentage overshoot P.O. is given by

$$P.O. = 100 \cdot e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 16.05$$