

Homework 3 (due at the start of class September 15, 2022)

1 Problem

Convert the following continuous-time system $\dot{x} = Ax + Bu$ with

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

into an exact discrete-time system

$$x_k = Fx_{k-1} + Gu_{k-1}$$

with sampling time Δt . You can check your answer using computer tools but should derive the discrete-time system by hand and show all of your work. Hint: The identity $e^X e^{-X} = I$ (for an arbitrary matrix X) is useful for computing G .

2 Problem

Repeat the previous problem using the approximate discretization described in Lecture 4 with $\Delta t = 0.25$. Evaluate both system matrices F, G and $F_{\text{approx}}, G_{\text{approx}}$ (where the former is the exact and the latter is the approximate system). Submit any MATLAB code you use.

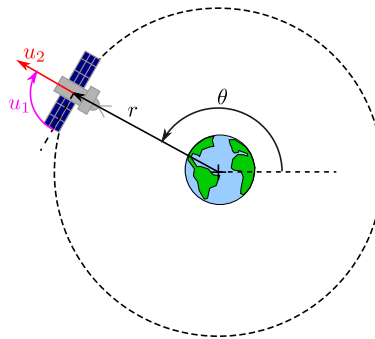
3 Problem

The motion of a satellite in orbit around the earth can be approximately described by the equations:

$$\ddot{r} = r\dot{\theta}^2 - \frac{\mu}{r^2} + 2u_2 \sin u_1 \quad (1)$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} + \frac{u_2}{2r} \cos u_1 \quad (2)$$

where r is the distance from the center of the earth, θ is the angle of the orbit (we're assuming the orbit is in a 2D plane), μ is the standard gravitational parameter, u_1 is the thrust angle and u_2 is a thrust magnitude.



A nominal trajectory for this system is a circular orbit where $r = r_0$ (a constant radius so that $\dot{r} = 0$) and $\dot{\theta} = \omega$ (a constant angular rate) where the radius and angular rate satisfy $\mu = r_0^3 \omega^2$. In this nominal trajectory no thrust is needed $u_2 = 0$ and the nominal angle of the thrust can be assumed $u_1 = \pi/2$. Linearize the system around this nominal trajectory to give a system of the form:

$$\Delta \dot{x} = A \Delta x + B \Delta u \quad (3)$$

4 Problem

Use MATLAB's `lsim` tool to simulate the system from the previous problem assuming:

- An orbit 4,000 km above the Earth's surface. That is, $r_0 = h + r_{\text{Earth}}$ where $h = 4E6$ meters is the altitude of the orbit and $r_{\text{Earth}} = 6371000$ meters is the radius of the Earth.
- The gravitational parameter for the earth is $\mu = GM$ where $G = 6.674E-11 \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$ is the gravitational constant and $M = 5.97219E24 \text{ kg}$ is the mass of the Earth.

Simulate the system starting along the reference trajectory while applying a thruster force of 0.25 Newtons for $T = 14400$ seconds (i.e., 4 hours). Plot the actual radius $r(t)$ and angle $\theta(t)$ (not deviations) during the simulation. Your plot should look something like the two left-most panels below (the right panel is optional).

