Homework 2 (due at the start of class September 08, 2022)

Homework should be submitted as a LaTeXdocument, see template on Canvas for details.

1 Problem

Use the Peano-Baker series to prove that the state transition matrix $\boldsymbol{\Phi}(t,0)$ for the system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ with

$$A = \begin{bmatrix} t & t \\ 0 & t \end{bmatrix} \qquad \text{is} \qquad \boldsymbol{\Phi}(t,0) = \begin{bmatrix} e^{\frac{1}{2}t^2} & \frac{1}{2}t^2 e^{\frac{1}{2}t^2} \\ 0 & e^{\frac{1}{2}t^2} \end{bmatrix}$$

Hint: Use the series definition of the exponential function.

2 Problem

Consider the system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ with initial condition $\boldsymbol{x}_0 = [1,0]^{\mathrm{T}}$. Find the solution $\boldsymbol{x}(t)$ using the Laplace inverse approach with partial fractions.

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

3 Problem

For the same system as given in the problem above, find the solution $\boldsymbol{x}(t)$ using the eigenvalue approach (with Cayley-Hamilton Theorem).

4 Problem

Consider the following system of coupled second-order equations

$$\ddot{y} - a(\dot{z} - \dot{y}) - b(z - y) = cu_1 + du_2$$
$$\ddot{z} - e\dot{z} - f(y + z) = gu_1$$

where (a, b, c, d, e, f, g) are all constants and $z(t)$ and $y(t)$ are states and $u_1(t)$ and $u_2(t)$ are control inputs. Define an appropriate state vector $\boldsymbol{x} \in \mathbb{R}^n$ and control vector $\boldsymbol{u} \in \mathbb{R}^m$ and then re-write this system as n first-order differential equations in the form $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$.

5 Problem

An equilibrium point for a nonlinear system $\dot{\boldsymbol{x}} = f(\boldsymbol{x})$ is a state $\boldsymbol{x}^*$ for which the state-rate is zero (i.e., $\dot{\boldsymbol{x}} = f(\boldsymbol{x}^*) = 0$). For the following nonlinear system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 - 2x_1x_2 \\ -x_1 + x_1^2 + x_2^2 \end{bmatrix}$$

determine the four equilibrium points $\boldsymbol{x}_a^*$, $\boldsymbol{x}_b^*$, $\boldsymbol{x}_c^*$ and $\boldsymbol{x}_d^*$. (Hint: one of them is $[1/2, -1/2]^{\mathrm{T}}$). Then state the linearized dynamics $\Delta\dot{\boldsymbol{x}} = \boldsymbol{A}\Delta\boldsymbol{x}$ around each nominal equilibrium. Define exactly what you mean by $\Delta\boldsymbol{x}$ for each linearized system.