Lecture 1: Introduction to System Dynamics

What is a System?

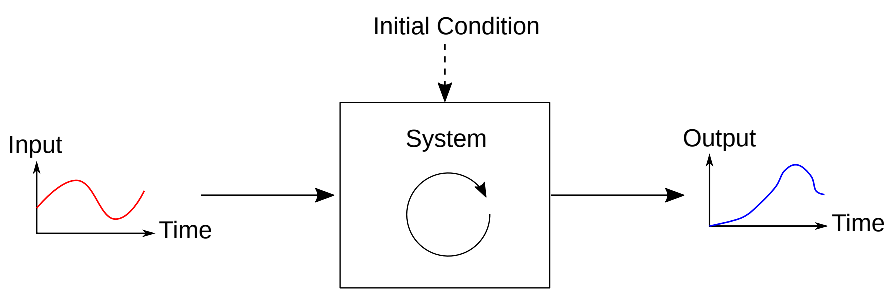
A *system* is a collection of components that act together to perform a specific task. Systems have inputs that produce a response observed in the output. As engineers, we decide how to define our system — we typically choose the the inputs and outputs to be the most pertinent variables that are relevant to our analysis or design. Then, using physics and other modeling principles, we develop the governing equations to relate them. Consider the following examples:

- e.g., a car suspension is a mechanical system.
  Input: bump in the road. Output: oscillation of the car frame.
- e.g., a servomotor is an electromechanical system.
  Input: Pulse-width-modulated (PWM) signal. Output: Servo arm position and velocity.
- e.g., a water boiler is a thermal system.
  Input: Voltage to heating element. Output: Water temperature.

The components of a system are the individual functioning units of a system

- e.g., springs, gears, motors, resistors, and pumps may be a partial list of components found in a car suspension, a servomotor, or a water boiler system

A *block diagram* graphically depicts the system (or a component of a system) as a block with input and output arrows, as shown below. The left-hand side of the above diagram depicts the time-history of an input signal and the right-hand side shows the output signal. Input signals can have different shapes, such as ramps, sinusoids, step inputs, or impulses, that determine how the output signal (system response) for a given initial condition. The outputs from one block can form the inputs to another block to create more complex systems.



A *static system*'s output or *state* depends only on the current input only. For example, if the input is $u(t)$ then for a static input-output system model the output state $x(t)$ would be expressed at any given time t as some function $x = F(g)$. On the other hand, for a *dynamic system*, the state evolves according to an ordinary differential equations (ODEs) of the form $F(t, x, \dot{x}, \ddot{x}, \cdots) = u(t)$ that relates the state x, its derivatives (e.g., $\dot{x}$ and $\ddot{x}$), and a possible external input $u(t)$. Even without an external input (i.e., ignoring $u(t)$) the system state changes over time depending on the initial conditions.

What is a Differential Equation?

An ordinary differential equation (ODE) is an equation that contains one or more derivative of a unknown function.

- Suppose $x(t)$ is a unknown function, where t is time and x is the system state.
- We use the following "dot" notation for a first derivative

$$\dot{x} = \frac{d}{dt}x$$

  and for a second derivative

$$\ddot{x} = \frac{d^2}{dt^2}x$$

- Suppose $u(t)$ is a known input or forcing function that involves constants and possibly time-varying terms (but does not involve x, $\dot{x}$)
- Then, by moving all the terms involving x and its derivative to the left-hand side, and moving lumping all constants and other time-varying terms (not involving x, $\dot{x}$, etc. to the right-hand side, an ODE can be written in the form as

$$F(t, x, \dot{x}, \ddot{x}, \dddot{x}, \cdots) = u(t)$$

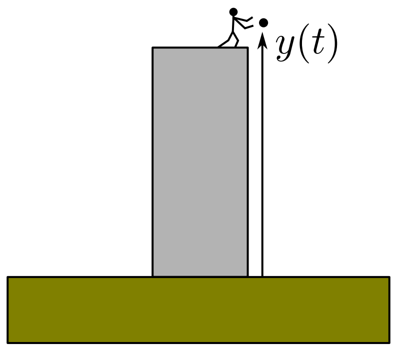
A differential equation is "ordinary" because the derivatives are ordinary e.g., dx/dt, as opposed to partial derivatives $\partial x/\partial t$. ODEs describe the time evolution of systems with just a few variables (i.e., they describe finite-dimensional systems). For example, the state of a rigid body, such as an airplane, can be described by six position/velocity variables and six orientation/angular rate variables. Equations with partial derivatives $\partial x/\partial t$ are referred to as partial differential equations (PDEs) and are used to model continuous systems such as continuous properties of fluids or bending beams. Such systems are referred to as infinite-dimensional because it would take an infinite number of variables to describe the continuous state of the system. In the example of the displacement of a beam it can be divided into an infinite number of positon variables along its length.

Examples of ODEs

Consider the following examples of ODEs:

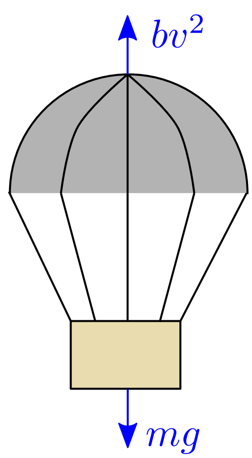
Example : A stone is thrown from a tower (neglecting drag). The state of the system is $y(t)$ (height above the ground) and the stone accelerates at a rate g downwards due to gravity (neglecting drag). The ODE describing the motion is:

$$F(t, y, \dot{y}, \ddot{y}) = \ddot{y} = g$$



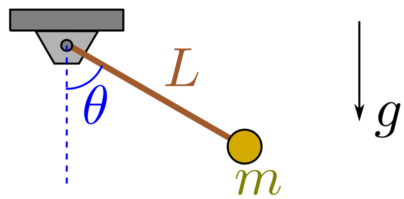
Example A parachute is deployed to slow down the descent of an aid package of mass m dropped from an airplane. The state of the system is $v(t)$ (velocity) and the package is subject to the downward force of gravity mg and upward force of drag bv^2 (assumed to be proportional to the square of the velocity), where b is a drag coefficient. Using Newton's Second Law, the ODE describing the motion is:

$$m\dot{v} = mg - bv^2 \qquad \text{or} \qquad F(t, v, \dot{v}) = m\dot{v} + bv^2 = mg$$



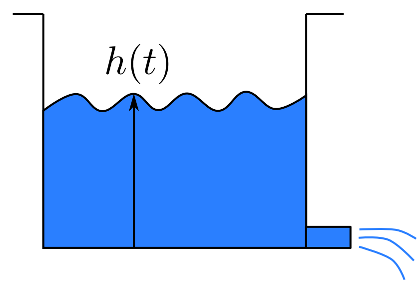
Example : A massless rod of length L forms an angle θ with the vertical. At the end of this rod is suspended a particle of mass m. The combined rod-mass system is called a pendulum and will oscillate back and forth when released from an arbitrary initial condition under the influence of gravity (and in the absence of friction). Using Newton's Second Law (e.g., the angular momentum form, or drawing a free body diagram with the internal tension force in the rod) the ODE describing the motion is:

$$F(t,\theta,\dot{\theta},\ddot{\theta}) = L\ddot{\theta} + g\sin\theta = 0$$



Example: Consider a tank that is draining water through a small hole at its base. The instantaneous water level at time t is $h(t)$. Using Bernoulli's Law, the rate of water that leaves the tank is proportional to the square root of h and a constant k that depends on gravity and tank geometry. The ODE describing these system dynamics is:

$$\dot{h} = -k\sqrt{h} \qquad \text{or} \qquad F(t,h,\dot{h}) = \dot{h} + k\sqrt{h} = 0$$



Aims of This Course

After completing this course, you should be able to:

1. Solve first and second order linear differential equations using Laplace transforms.
2. Obtain differential equation motion models of mechanical systems based on free-body diagrams.
3. Obtain the response of first and second order systems due to initial conditions and various forcing functions, such as a step, ramp, and sinusoid.
4. Develop lumped parameter models of dynamic systems.
5. Use MATLAB for dynamic simulation.

References and Further Reading

- Davies and Schmitz: Section 1.1-1.5
- Ogata: Section 1.1-1.4