Homework 3

1 Problem

Find the Laplace transform fraction for the following function and rearrange it such that $X(s)/F(s)$ is the only term on the left-hand-side:

$$\dddot{x}(t) + 2\zeta\omega\ddot{x}(t) + \omega^2\dot{x}(t) = f(t)$$

Assume the initial conditions are all zero, $x(t_0) = \dot{x}(t_0) = \ddot{x}(t_0) = \dddot{x}(t_0) = 0$ with initial time $t_0 = 0$. Hint: Use the differentiation theorem.

2 Problem

Compute the inverse Laplace transform $f(t) = \mathcal{L}^{-1}[F(s)]$ of the following function:

$$F(s) = \frac{1}{s+\sigma}(e^{-as} - e^{-bs})$$

where a, b, and σ are constants.

Hint: Recall that the following property holds for translated functions $\mathcal{L}[f(t-\alpha)H(t-\alpha)] = e^{-s\alpha}F(s)$, which implies that

$$f(t) = \mathcal{L}^{-1}[e^{-s\alpha}F(s)] = f(t-\alpha)H(t-\alpha)$$

The expression above can be written as a sum of two functions of this form.

3 Problem

Compute the inverse Laplace transform $f(t) = \mathcal{L}^{-1}[F(s)]$ of the following function:

$$F(s) = \frac{s+1}{s^2+6s+9}$$

4 Problem

Compute the inverse Laplace transform of

$$F(s) = \frac{s+1}{s(s+2)}$$

5 Problem

Compute the inverse Laplace transform of

$$F(s) = \frac{s+2}{(s+1)(s+4)^2}$$

Hint: Use a combination of partial fraction expansions for real distinct and repeated poles.

6 Problem

Compute the inverse Laplace transform of

$$F(s) = \frac{3s + 9}{s^2 + 4s + 5}$$