Homework 1

1 Problem

For each of the ODEs in the 1st column, indicate whether it is:

1. linear time-invariant (LTI), linear time-varying (LTV), or nonlinear
2. 1st order, 2nd order, or higher order
3. homogeneous or inhomogeneous

by marking the appropriate column. The unknown function $x(t)$ represents the state of some mechanical system and t represents time. Hint: An ODE is considered nonlinear only if the nonlinearity involves the unknown function $x(t)$.

	Linearity?			Order?			Homogeneity?	
System ODE	LTI	LTV	NL	1st	2nd	Higher	Homog.	Inhomog.
$\ddot{x} + 3tx = 0$								
$(\dot{x} - x)^2 + 1 = 0$								
$t^2x + bx + c\dot{x} = 0$								
$\dddot{x} = 0$								
$\ddot{x} + \dot{x} + x - 2 =$								
$\ddot{x} + \sin(x) = 0$								
$e^t x + \dot{x} = \sin t$								
$\dot{x} + x = 0$								
$\dot{x}x + a + bt = 0$								
$\ddot{x} - b\dot{x}^2 = 0$								

2 Problem

Consider the following IVP:

$$\dot{x} + 2x = 0$$

with initial condition $x(t_0) = -10$ and $t_0 = 0$.

1. What is the particular solution, $x(t)$?
2. What is the value of x as time $t \to \infty$?

A. $x \to -\infty$
B. $x \to -10$
C. $x \to 0$
D. $x \to +10$
E. $x \to +\infty$

3 Problem

Consider the ODE $\dot{x} + 2x = e^{-2t}$ with initial condition $x(t_0) = 10$ and $t_0 = 0$. What is the particular solution, $x(t)$?

4 Problem

For each of the following ODEs determine if the eigenvalues are (a) real and distinct, (b) repeated, (c) complex conjugate pairs:

1. $\ddot{x} + 2\dot{x} + 3x = 0$
2. $\ddot{x} + 4\dot{x} + x = 0$
3. $\ddot{x} + 4\dot{x} + 4x = 0$
4. $\ddot{x} + 3x = 0$

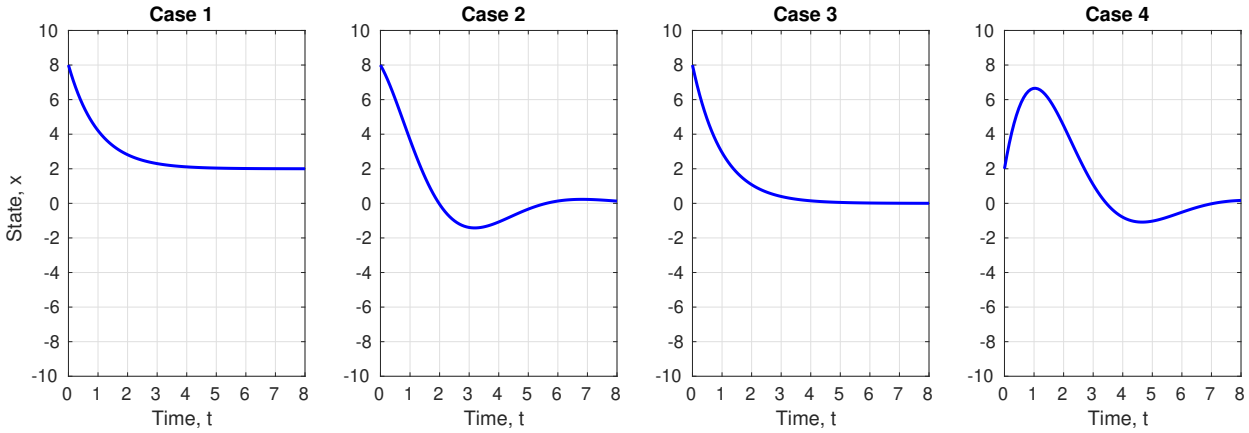
5 Problem

For each of the following linear, time-invariant, 2nd order homogeneous ODEs solve for the particular solution that satisfies the initial values given.

1. $\ddot{x} - 4\dot{x} + 4x = 0$, Initial Values: $x(0) = 12$, $\dot{x}(0) = -3$
2. $\ddot{x} + 3\dot{x} - 10x = 0$, Initial Values: $x(0) = 4$, $\dot{x}(0) = -2$
3. $\ddot{x} - 8\dot{x} + 17x = 0$, Initial Values: $x(0) = -4$, $\dot{x}(0) = -1$

6 Problem

Match each one of the responses shown below (Cases 1-4) with one of the following IVPs



(a) $\ddot{x} + \dot{x} + x = 0, x(0) = 8, \dot{x}(0) = 0$

(b) $\dot{x} - x = 2,\ x(0) = 8$

(c) $\dot{x} + x = 2,\ x(0) = 8$

(d) $\ddot{x} + \dot{x} + x = 2, x(0) = 0, \dot{x}(0) = -8$

(e) $\ddot{x} + \dot{x} + x = 2, x(0) = 2, \dot{x}(0) = 0$

(f) $\dot{x} + x = 8,\ x(0) = 2$

(g) $\dot{x} + x = 0,\ x(0) = 8$

(h) $\ddot{x} + \dot{x} + x = 0, x(0) = 8, \dot{x}(0) = -3$

(i) $\ddot{x} + \dot{x} + x = 0, x(0) = 2, \dot{x}(0) = 10$

(j) $\dot{x} + 8x = 0,\ x(0) = 0$

7 MATLAB Problem

The McGuire Nuclear Station in Huntersville, NC is testing a new isotope of radioactive material called *nineridium*. Engineers have determined that the material exhibits exponential decay according to the ODE:

$$\dot{N} = -kN$$

where $N(t)$ is the number of parent atoms, time t has units of years, and $k = \ (1/\text{year})$ is the decay rate. If the reactor starts with a chunk of nineridium that consists of $N(t_0) = 1000$ atoms at time $t_0 = 0$, then:

- Part A (5 pts). What is the expression that gives the number of atoms, $N(t)$, for any future time $t \geq t_0$?
- Part B (5 pts). What is the time constant describing the decay?
- Part C (5 pts). Using MATLAB, plot the function $N(t)$ out to 5 time constants. Label your axes and include appropriate units. Your submission should include both your code and the resulting graph. MATLAB Hints:
  - If you don't have MATLAB installed you can download it from `software.uncc.edu`. If you are feeling rusty, please review the MATLAB help files provided (you may wish to complete the "On Ramp" tutorial).

  - to plot a solid line of width 2 with circular markers in MATLAB use the the command `plot(time, x, 'ro-','linewidth',2)` where `time` is a vector of increasing time values, `x` is a vector of data points to be plotted with time. You can change the color of the line by replacing the `r` in `ro-` with other letters corresponding to colors e.g., blue `b`, magenta `m`, green `g`.

- Part D (5 pts). Based on your plot, what is the approximate half-life of nineridium (i.e., how many years does it take the material to decay to 50% of the initial amount)?