

Q. A) SOLVE THE IVP USING PFE GIVEN THE LAPLACE TRANSFORM:  $X(s) = \frac{s-3}{s^2+6s+5}$  B) WHAT WAS THE IC?

C) AFTER HOW MANY SECONDS HAS  $x(t)$  DECAYED TO WITHIN 2% OF  $x(0)$  FOR ALL FUTURE TIME?  
SOLN FACTOR DENOMINATOR

$$X(s) = \frac{s-3}{(s+1)(s+5)}$$

WRITE AS PFE

$$X(s) = \frac{(s-3)}{(s+1)(s+5)} = \frac{a_1}{(s+1)} + \frac{a_2}{(s+5)}$$

EVALUATE COEFFS

$$a_1 = \left. \frac{(s-3)}{(s+5)} \right|_{s=-1} = \frac{-4}{+4} = -1$$

$$a_2 = \left. \frac{(s-3)}{(s+1)} \right|_{s=-5} = \frac{-8}{-4} = +2$$

INVERSE LAPLACE

$$x(t) = -1 \mathcal{L}^{-1} \left[ \frac{1}{s+1} \right] + 2 \mathcal{L}^{-1} \left[ \frac{1}{s+5} \right]$$

$$\boxed{x(t) = -e^{-t} + 2e^{-5t}}$$

IC  $x(0) = -1 + 2 = \boxed{1 = x(0)}$

b) or (using IVT)

$$\lim_{t \rightarrow 0} x(t) = \lim_{s \rightarrow \infty} s X(s)$$

$$= \lim_{s \rightarrow \infty} \frac{s^2 - 3s}{s^2 + 6s + 5} \cdot \frac{1/s^2}{1/s^2}$$

$$= \lim_{s \rightarrow \infty} \frac{1 - 3/s}{1 + 6/s + 5/s^2}$$

$$\boxed{x(0) = 1}$$

c) There are two time constants

$$x(t) = \underbrace{-e^{-t}}_{\downarrow} + 2 \underbrace{e^{-5t}}_{\downarrow}$$

$$\tau = \frac{1}{a}$$

$$= 1 \text{ sec.}$$

slow

$$\tau = \frac{1}{5}$$

$$= 0.2 \text{ sec}$$

fast

decays in  $4\tau = 4 \text{ sec.}$

decays in  $4\tau = 0.8 \text{ sec}$

After about 4 seconds

See p. 39-40 Davies