Homework 8

1 Problem

Solve Problem 1a and 1b in the Davies book (p. 312)

Solution

Calculate the volume of the coil as

$$V = \pi(R_o^2 - R_i^2)$$

where R_o and R_i are the outer and inner radii, and L is the length. Multiplying by density gives the mass, and multiplying by c gives the thermal mass.

The rate of heating is $P_0/mc = 2.558$ deg C/s

The time required to heat from 30 C to 250 C is the temperature difference divided by the rate of heating

$$t_f = \frac{(T_f - T_i)}{P_0/(mc)} = 93.1560 sec$$

It requires only 93.1560 sec for the insulation to begin to burn. This is why motor stall is damaging to motor coils – the insulation burns and then the wires in the coils short out.

See MATLAB on following pages.

```
% Chapter 9, Problem 1
clear all
clc
close all
% Parameters
P0 = 350; % Power input, W
Do = 50e-03; % Outer Diameter, m
Di = 40e-03; % Outer Diameter, m
L = 60e-03; % Length, m
c = 390; % Specific heat, J/kg-C
rho = 8960; % Density, kg/m^3
T0 = 30; % Initial temperature, C
Tf=250; % Final Temperature

% Calculated parameters
Ro = Do/2;
Ri = Di/2;
V = pi*(Ro^2-Ri^2)*L;
m = rho*V;
% time at Tf
tf = m*c/P0*(Tf - T0)
% Time vector and temperature versus time
t = [0:tf/100:tf]; % s
T = P0/(m*c)*t + T0; % C
% time at Tf (using find, alternatve soln)
k_ind = find(T >= Tf);% using find
tf = t(k_ind(1)) % use the first index
% Display results
fprintf('The time required for copper cylinder to reach %0.1f degrees \n
 Celsius is %5.2f minutes. \n', Tf, tf/60);

% Plot the temperature versus time
figure(1)
plot(t, T)
grid
xlabel('t (s)')
ylabel('T(t) (\circC)')
axis([0 max(t) 0 max(T)])
```

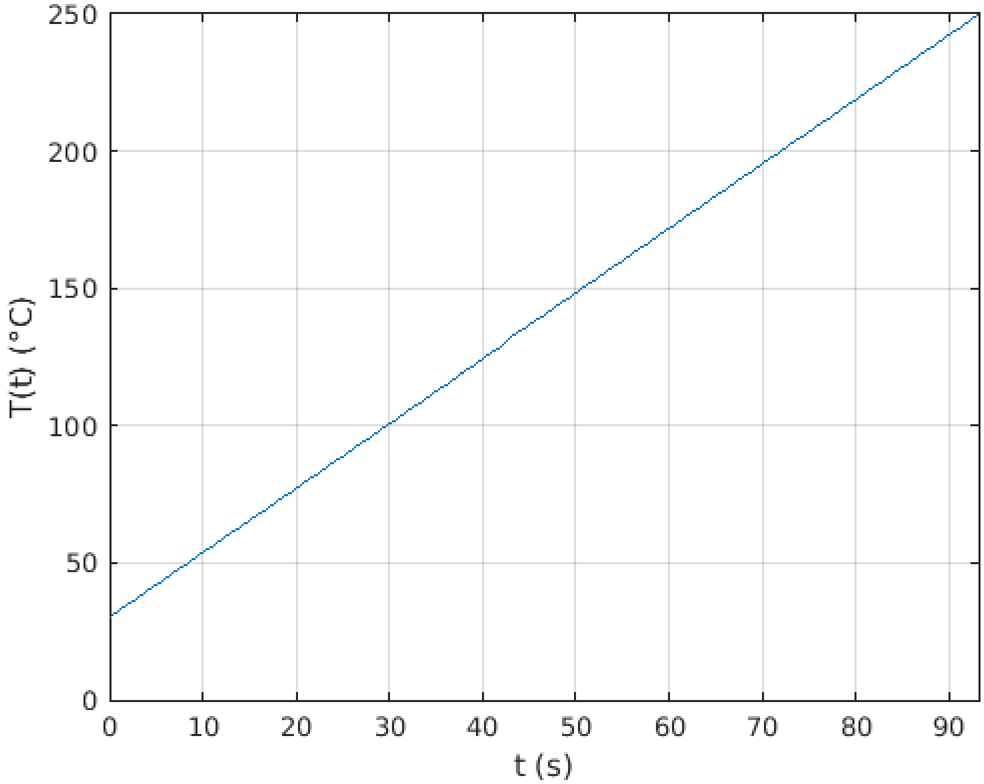
```
tf =

   93.1560


tf =

   93.1560

The time required for copper cylinder to reach 250.0 degrees
 Celsius is  1.55 minutes.
```



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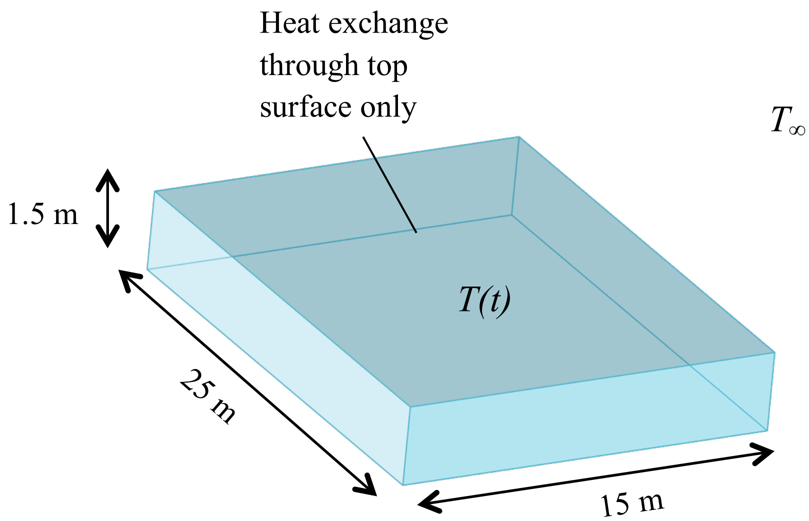
2 Problem

Solve Problem 2a and 2b in the Davies book (p. 313)

Solution

2. A rectangular swimming pool is 1.5 meters deep, 15 meters wide and 25 meters long. Assume the bottom and sides of the swimming pool are insulated and so pool exchanges heat with its environment through the *top surface only*. (The density of water is 1000 kg/m^3, the specific heat is 4180 J/kg-°C, and the heat transfer coefficient is 10 W/m^2-°C.)



Complete the following

(a) Determine the thermal time constant of the pool and express your answer in days.

(b) The swimming pool is being held at T_0=30°C when the power fails and external heating stops. The air temperature around the pool (T_∞) drops quickly and can be assumed to be 10°C at time t=0. Write a script file in MATLAB® that plots the pool temperature for four thermal time constants.

Solution

Step 1: Time constant.

$$\tau = \frac{mC_p}{hA}$$

$$V = LWH = (25m)(15m)(1.5m) = 562.5m^3$$

$$A = LW = (25m)(15m) = 375m$$

$$m = \rho V = \left(1000kg/m^3\right)\left(562.5m^3\right) = 562500kg$$

$$\tau = \frac{mC_p}{hA} = \frac{\left(526500kg\right)\left(4180J/kg-^oC\right)}{\left(10W/m^2-^oC\right)\left(375m^2\right)} = 627000s$$

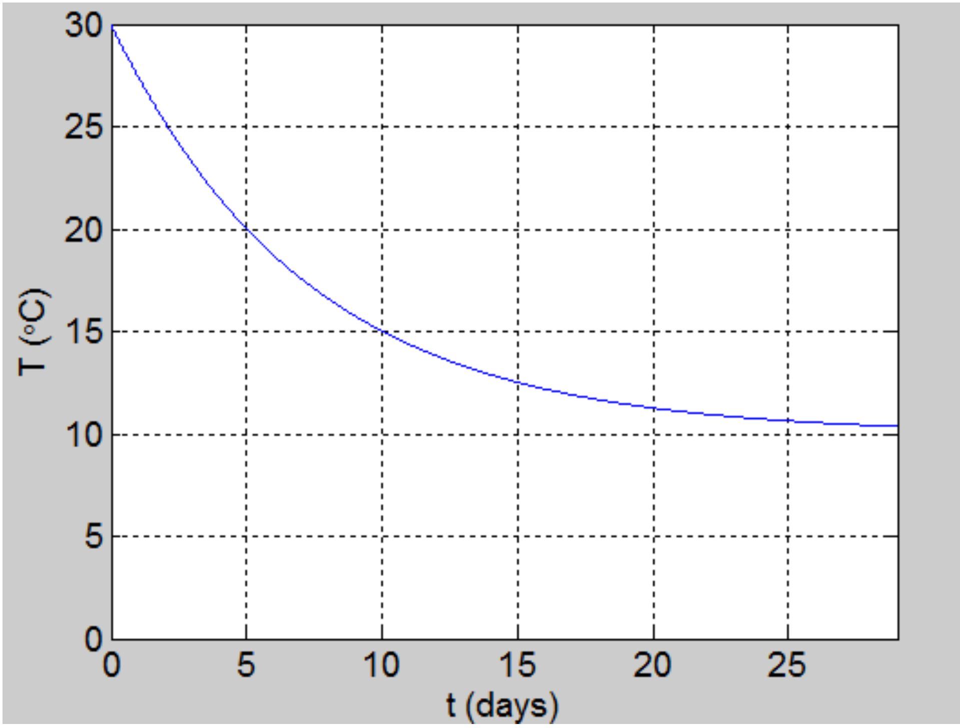
$$\tau = 7.26days$$

$$4\tau = 29days$$

Step 2: The MATLAB® code is given.

```
% Chapter 9, Problem 2
clear
clc
clf
close all

% Parameters
L=25;
W=15;
H=1.5;
A=L*W;
V=L*W*H;
rho=1000;
m=rho*V;
Cp=4180;
h=10;
T_env=10;
T0=30;

% Calculated parameters
tau=m*Cp/(h*A);
a=1/tau

% Find the temperature as a function of time
t=[0:tau/1000:4*tau];
T=T_env*(1-exp(-a*t))+T0*exp(-a*t);

% Plot the results
figure(1)
set(gca,'fontsize',14)
plot(t/3600/24, T);
grid;
xlabel('t (days)')
ylabel('T (\circC)')
axis([0 max(t/3600/24) 0 max(T)]);
```

And the plot is given.


30
25
20
15
10
5
0
T (°C)
0
5
10
15
20
25
t (days)

3 Problem

Solve Problem 3a and 3b in the Davies book (p. 313–314)

Solution

3. Consider again the swimming pool from Problem 2. Suppose the pool has chilled to T_0 =10°C when the heat comes back on. The air temperature quickly warms up so that $T_\infty = T_{env} \cdot 1(t)$ where T_{env}=22°C. The water heater comes back on at the same time providing a step input in power to the pool $P_0 \cdot 1(t)$.

Complete the following.

(a) Using the final value theorem on the Laplace domain solution for T(s), determine the value of P_0 required to bring the pool back to and equilibrium temperature of 30°C and hold it at that temperature.
(b) Using the value of P_0 determined in part (a), write a script file in MATLAB® that plots the pool temperature for four thermal time constants.

Solution

Step 1: The pool temperature as a function of time is given by the following expression in the Laplace domain.

$$T(s) = P_0 \frac{a}{hA}\left(\frac{1}{s(s+a)}\right) + T_{env} a\left(\frac{1}{s(s+a)}\right) + T_0\left(\frac{1}{(s+a)}\right)$$

Step 2: Applying the final value theorem, gives the equilibrium temperature.

$$T_{eq} = \lim_{s\to 0}\left(sT(s)\right) = \lim_{s\to 0}\left(P_0 \frac{a}{hA}\left(\frac{1}{(s+a)}\right) + T_{env} a\left(\frac{1}{(s+a)}\right) + T_0 s\left(\frac{1}{(s+a)}\right)\right)$$

$$T_{eq} = \frac{P_0}{hA} + T_{env}$$

$$P_0 = hA\left(T_{eq} - T_{env}\right)$$

Step 3: Solve for P_0.

$$P_0 = hA\left(T_{eq} - T_{env}\right) = 3750 W/°C\left(30°C - 22°C\right)$$

$$= 30,000\,\text{W} = 30\,\text{kW}$$

Step 4: The temperature as a function of time for the pool is given.

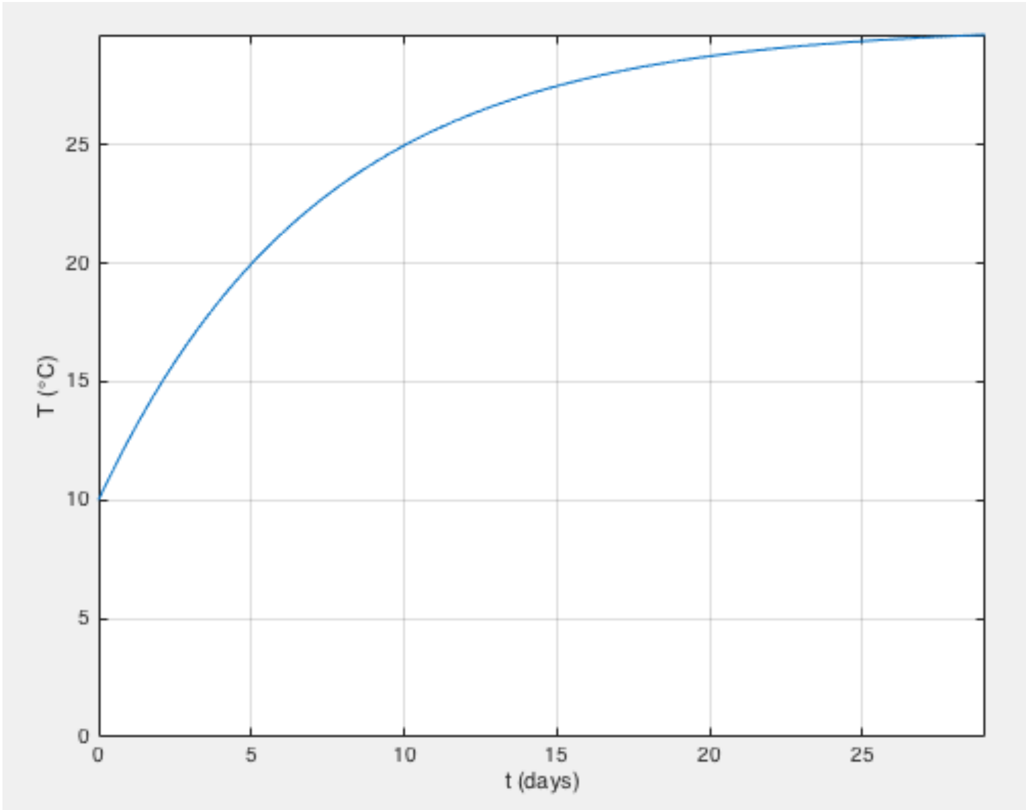
$$T(t) = \frac{P_0}{hA}\left(1 - e^{-at}\right) + T_{env}\left(1 - e^{-at}\right) + T_0 e^{-at}$$

$$\tau = \frac{1}{a} = \frac{mC_p}{hA}$$

Step 5: The MATLAB® code is given.

```
% Chapter 9, Problem 2
clear
clc
clf
close all

% Parameters
L=25;
W=15;
H=1.5;
A=L*W;
V=L*W*H;
rho=1000;
m=rho*V;
Cp=4180;
h=10;
T_env=22;
T0=10;
T_des=30;

% Calculated parameters
tau=m*Cp/(h*A);
a=1/tau

% Calcuate the necessary P0
P0=h*A*(T_des-T_env);

% Find the temperature as a function of time
t=[0:tau/1000:4*tau];
T=P0/(h*A)*(1-exp(-a*t))+T_env*(1-exp(-a*t))+T0*exp(-a*t);

% Plot the results
figure(1)
set(gca,'fontsize',14)
plot(t/3600/24, T);
grid;
xlabel('t (days)')
ylabel('T (\circC)')
axis([0 max(t/3600/24) 0 max(T)]);
```

And the resulting plot is shown.

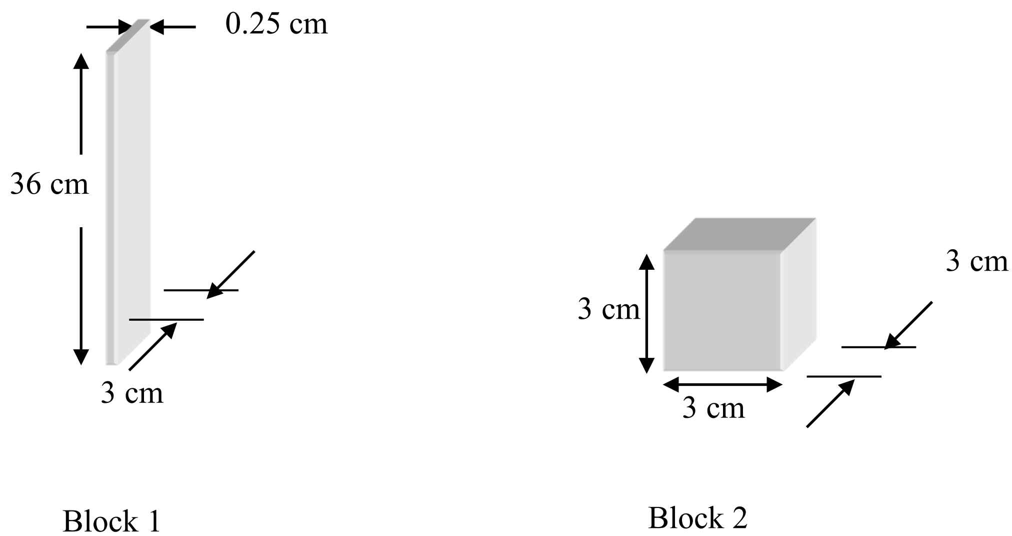

25
20
15
10
5
0
T (°C)
0
5
10
15
20
25
t (days)

4 Problem

Solve Problem 4a and 3b in the Davies book (p. 314)

Solution

4. Two pieces of aluminum of different dimensions but the same total mass are shown schematically below. The first is 0.25 cm in thickness, 3 cm in width and 36 cm in length. The second is a cube that is 3 cm on a side. Aluminum has a density of 2700 kg/m^3 and a specific heat capacity of 900 J/kg-ºC. The value of the heat transfer coefficient to the surrounding air is 10 W/m^2-ºC.



Complete the following.

(a) Calculate thermal time constant for each piece of aluminum in minutes.

(b) Each block is heated to an initial temperature 120ºC and then allowed to cool in the surrounding air, which is at 20ºC. Write a script file in MATLAB® that plots the temperature as a function of time *T(t)* for both blocks on the same graph. Plot temperature of block 1 using a solid black line and the temperature of block 2 using a dotted black line. Plot for a total time duration equal to four of the longer time constant between blocks 1 and 2. The time axis should be in units of minutes.

(c) Investigate cooling fins such as those used in an automobile radiator or an air conditioner and comment on cooling fin design based on the results of this problem.

Solution

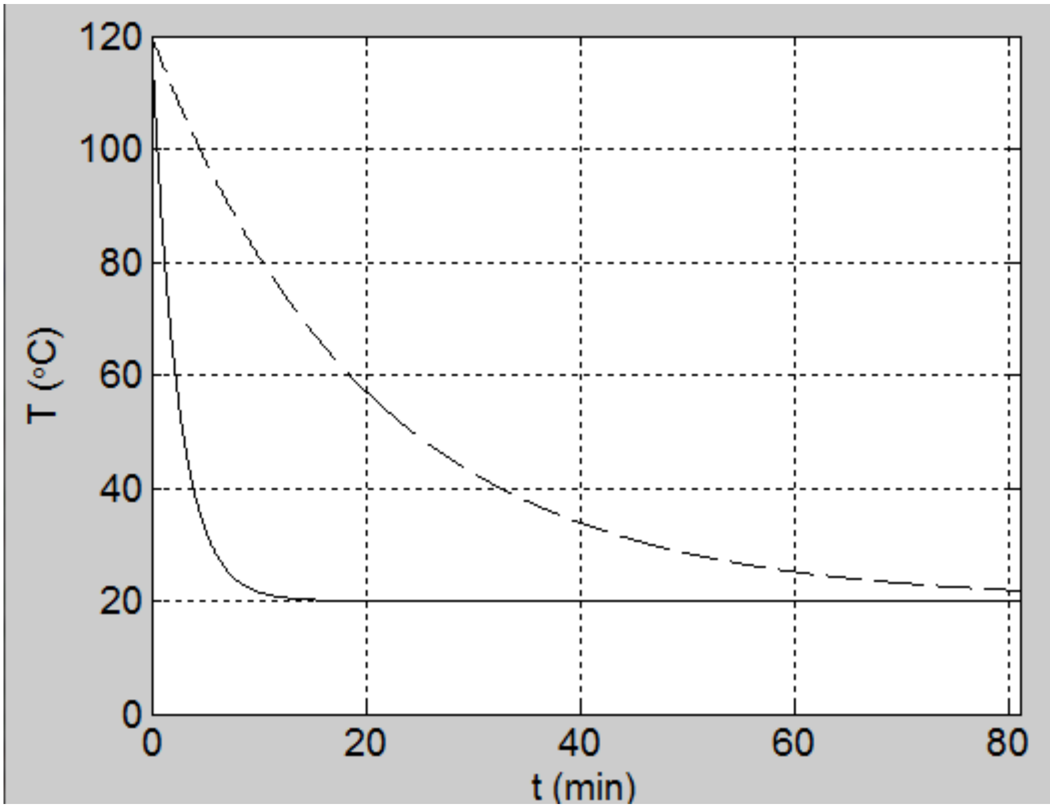
The following MATLAB® code is used for parts (a) and (b).

```
% Chapter 9, Problem 2
clear
clc
clf
close all

% General Parameters
rho=2700;
c=900;
```

```
h=10;
T_env=20;
T0=120;

% Block 1 Parameters
L1=36e-02;
W1=3e-02;
T1=0.25e-02;
V1=L1*W1*T1;
m1=rho*V1
A1=2*L1*W1+2*L1*T1+2*L1*W1;
tau1=m1*c/(h*A1)
a1=1/tau1;

% Block 1 Parameters
L2=3e-02;
W2=3e-02;
T2=3e-02;
V2=L2*W2*T2;
m2=rho*V2
A2=2*L2*W2+2*L2*T2+2*L2*W2;
tau2=m2*c/(h*A2);
a2=1/tau2;

% Find the temperature as a function of time
if tau1>tau2
   t=[0:tau1/1000:4*tau1];
else
   t=[0:tau2/1000:4*tau2];
end

% Temperature of block 1
T1=T_env*(1-exp(-a1*t))+T0*exp(-a1*t);

% Temperature of block 1
T2=T_env*(1-exp(-a2*t))+T0*exp(-a2*t);

% Plot the results
figure(1)
set(gca,'fontsize',14)
plot(t/60, T1,'k-',t/60, T2,'k--');
grid;
xlabel('t (min)')
ylabel('T (\circC)')
axis([0 max(t/60) 0 max(T1)]);
```

```
clc
fprintf('The time constant of block 1 is %5.2f minutes.\n',tau1/60);
fprintf('The time constant of block 2 is %5.2f minutes.\n',tau2/60);
```

And the resulting plot.



And the time constants.

The time constant of block 1 is 2.43 minutes.

The time constant of block 2 is 20.25 minutes.

5 Problem

How long should it take to boil an egg? Model the egg as a sphere with radius of 2.3 cm that has properties similar to water with a density of $\rho = 1000$ kg/m^3 and thermal conductivity of $k = 0.606$ Watts/(m·°C) and specific heat of c = 4182 J/(kg·° C). Suppose that an egg is fully cooked when the temperature at the center reaches 70° C. Initially the egg is taken out of the fridge at 4° C and placed in the boiling water at 100° C. Since the egg shell is very thin assume that it quickly reaches a temperature of 100° C. The protein in the egg effectively immobilizes the water so the heat conduction is purely conduction (no convection). Plot the temperature of the egg over time and use the data tooltip in MATLAB to make your conclusion on the time it takes to cook the egg in minutes.

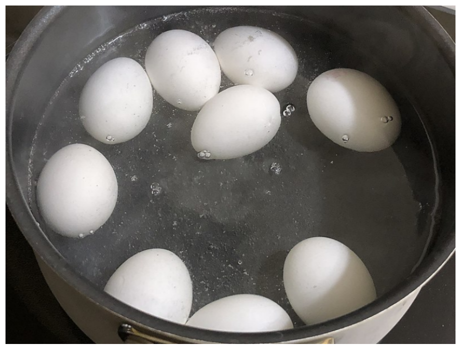


Figure 1: Image source: [Link]

Solution

This example involves heat conduction (refer to Lecture 16 notes) where the temperature profile (in general) for a plate with two exposed areas to temperatuers T_1 and T_2 has a time-varying center temperature given by

$$T(t) = \frac{1}{2}(T_1 + T_2) + (T_0 - \frac{1}{2}(T_1 + T_2))e^{-at}$$

where $a = 4kA/(mcL)$.

Approach 1: Although our egg boiling problem is 3D, it can be approximated by a plate model with thickness $L = 2R$ (twice the radius) and area equal to the surface area of a sphere $A = 4\pi R^2$. The mass can be determined from the volume of a sphere $m = \rho V = \rho(4/3)\pi R^3$. Setting $T_1 = T_2 = T_{\text{env}}$ we have:

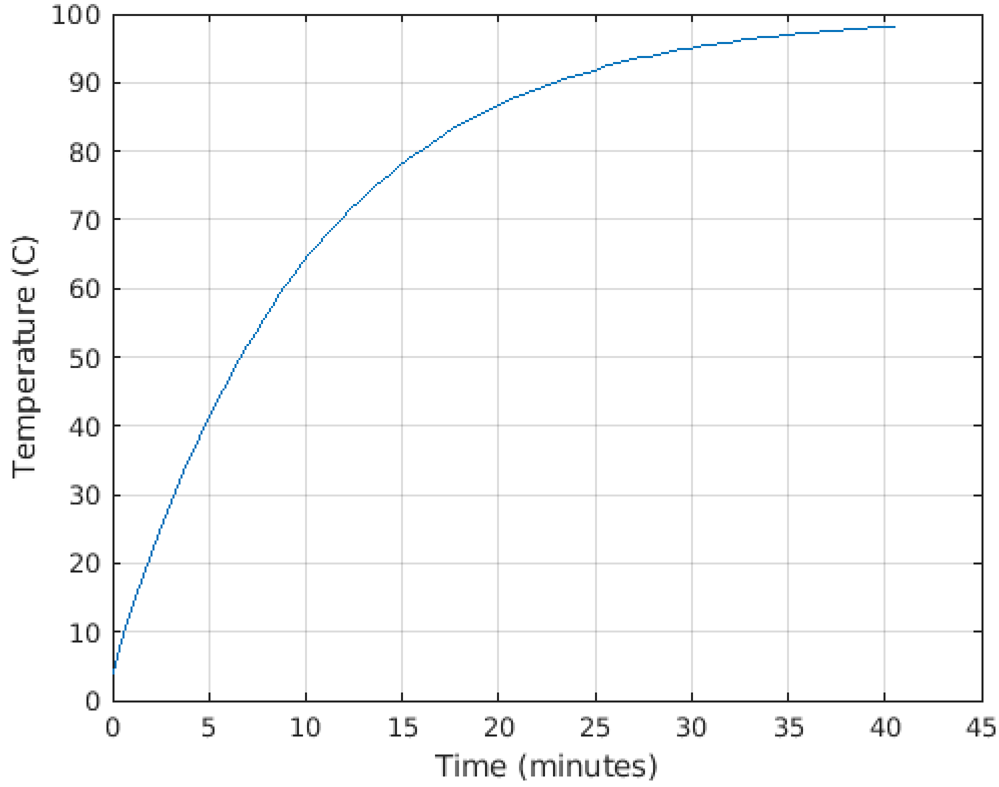
$$T(t) = T_{\text{env}} + (T_0 - T_{\text{env}})e^{-at}$$

See MATLAB on following pages. Using the data tooltip the time is approximately 11.8 minutes when using $L = 2R$ in the time constant.

Approach 2: An alternative view uses the spherical geometry and consider just one thermal resistor so that the thickness is $L = R$. Both answers are graded correctly in this assignment.

```
clear; close all; clc;

k = 0.606;
c = 4182;
R = 2.3/100; % m
A = 4*pi*R^2;
V = 4/3*pi*R^3;
rho = 1000; % kg/m^3
m = rho*V
L = 2*R;
a = 4*k*A/(m*c*L)
tau = 1/a


T0 = 4;
Tenv = 100;
t = linspace(0,4*tau);
T = Tenv + (T0 - Tenv)*exp(-a*t);
plot(t./60, T)
hold on;
xlabel('Time (minutes)')
ylabel('Temperature (C)')
grid on;
```

```
m =

    0.0510


a =

    0.0016


tau =

  608.4373
```



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