On my honor, I submit that I have neither given or received assistance on this exam or consulted prohibited materials (solutions manuals, internet, MATLAB etc.). The exam is open book and open notes, so all class material can be used for reference during this exam. Email the instructor at `awolek@uncc.edu` with any questions during the exam, and monitor your email for any mid-exam clarifications.

Name: _____ Date: March 29, 2021

MEGR 3122 Dynamic Systems II: Exam 2, Spring 2020

Multiple Choice Problems (10 Points)

Directions: Circle the best answer. Each question is worth 2 points.

Figure 1: A forcing input $f(t)$

1. The forcing input in Fig. 1 corresponds to which of the following equations (where $u(t-a)$ denotes a step input at time $t=a$):
   A. $u(t-1)(t-1) - u(t-2)(t-1)$
   B. $u(t-1)(t-1) - u(t-2)(t-2)$
   C. $u(t-1)(t-1) - u(t-2)(t-2) - 1$
   D. $u(t-1)(t-1) - u(t-2)(t-2) - u(t-2)$

2. What is the final value of $x(t)$ as $t \to \infty$ if the Laplace transform of $x(t)$ is the following?

$$X(s) = \frac{s+10}{s^2+2s}$$

   A. $x(t) \to 0$
   B. $x(t) \to 5$
   C. $x(t) \to 10$
   D. $x(t) \to 20$

3. What is the correct MATLAB code for defining the transfer function $G(s)$?

$$G(s) = \frac{5s^2+2s}{3s^5+3s^4+s^2+2s+1}$$

   A. `sys=tf([5,2],[3,3,1,2,1])`
   B. `sys=tf([5,2,0],[3,3,1,2,1])`
   C. `sys=tf([5,2,0],[3,3,0,1,2,1])`
   D. `sys=tf([5,2],[3,3,1,2,1])`

4. Which second-order system has a natural frequency of 2 and a damping ratio of 0.5?
   A. $2\ddot{x} + 1\dot{x} + 0.5x = 0$
   B. $4\ddot{x} + 4\dot{x} + x = 0$
   C. $\ddot{x} + 2\dot{x} + 4x = 0$
   D. $\ddot{x} + 0.5\dot{x} + 2x = 0$

5. The inverse Laplace Transform of

$$X(s) = \frac{1}{(s+1)^2(s^2+2)}$$

   is which of the following?

   A. $\dfrac{\mathrm{e}^{t}}{6} - \dfrac{\mathrm{e}^{-t}}{6} - \dfrac{\sqrt{2}\,\sin\left(\sqrt{2}\,t\right)}{6}$

   B. $\dfrac{\mathrm{e}^{2\,t}}{27} - \dfrac{\mathrm{e}^{-t}}{27} - \dfrac{t\,\mathrm{e}^{-t}}{9} - \dfrac{t^2\,\mathrm{e}^{-t}}{6}$

   C. $\dfrac{5\,\mathrm{e}^{-t}}{36} + \dfrac{\mathrm{e}^{2\,t}}{9} - \dfrac{\mathrm{e}^{t}}{4} + \dfrac{t\,\mathrm{e}^{-t}}{6}$

   D. $\dfrac{2\,\mathrm{e}^{-t}}{9} - \dfrac{2\,\cos\left(\sqrt{2}\,t\right)}{9} + \dfrac{t\,\mathrm{e}^{-t}}{3} - \dfrac{\sqrt{2}\,\sin\left(\sqrt{2}\,t\right)}{18}$

Workout Problem 1 (10 pts)

Obtain the solution $x(t)$ to the differential equation

$$\ddot{x} + 16x = \cos 3t$$

with initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$ using Laplace Transform methods. Write your answer as an expression of the form: $x(t) =$ _____.

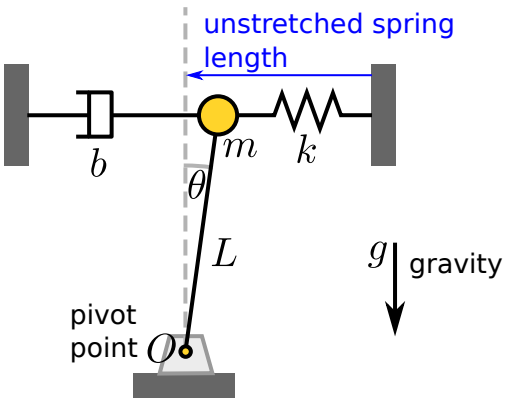
Workout Problem 2 (10 pts)

An inverted pendulum consisting of a mass m connected to a massless rigid rod of length L that rotates around point O (the mass moment of inertia is $I_0 = mL^2$). The mass is also connected to a damper and spring as shown. Gravity acts on the mass and the rod makes an angle θ with the vertical.

Find the second order differential equation describing the motion of the system using small angle approximations. Write your answer in standard form:

$$\ddot{\theta} + c_1\dot{\theta} + c_2\theta = 0$$

where c_1 and c_2 are constants. Include a free body diagram.

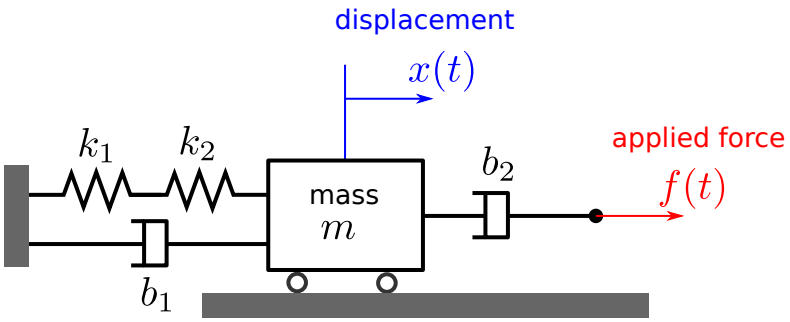


Workout Problem 3 (10 pts)

Find the transfer function $G(s) = X(s)/F(s)$ for the following system (assume zero initial conditions). Include a free body diagram.

Expand all terms and write $G(s)$ in the general transfer function format,

$$\text{e.g.,} \quad G(s) = \frac{X(s)}{F(s)} = \frac{b_m s^m + b_{m-1} s^m + \cdots + b_0}{a_n s^n + a_{n-1} s^n + \cdots + a_0} \, .$$



Workout Problem 4 (10 pts)

Determine the transfer function $G(s) = X(s)/F(s)$ for the following system (below) with two degrees of freedom, $x(t)$ and $\theta(t)$ (assume zero initial conditions). The variables: m, b, k, L are all constants and $f(t)$ is a forcing function. Hint: Take the Laplace transform of each equation first.

$$m\ddot{x}(t) + kx(t) - Lk\theta(t) = f(t)$$
$$\frac{1}{3}m\ddot{\theta}(t) - kL\dot{x}(t) = 0$$

Expand all terms and write $G(s)$ in the general transfer function format,

$$\text{e.g., } \; G(s) = \frac{X(s)}{F(s)} = \frac{b_m s^m + b_{m-1}s^m + \cdots + b_0}{a_n s^n + a_{n-1}s^n + \cdots + a_0} \; .$$