On my honor, I submit that I have neither given or received assistance on this exam or consulted any prohibited materials (beyond the one page crib sheet allowed for the exam).

Name: ______________________________ Date: March 24, 2022

MEGR 3122 Dynamics Systems II: Exam 2, Spring 2022

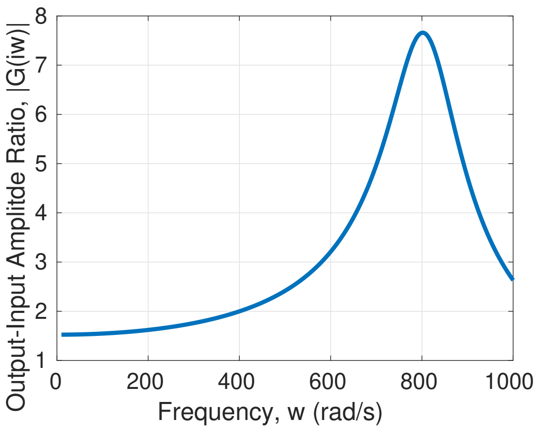
Multiple Choice Problems (Total 20 Points)

Directions: Circle the best answer. Each question is worth 2 points.

1. Which second-order system has a natural frequency of 2 rad/s and a damping ratio of 0.5?
   A. $2\ddot{x} + 1\dot{x} + 0.5x = 0$
   B. $4\ddot{x} + 4\dot{x} + x = 0$
   C. $\ddot{x} + 2\dot{x} + 4x = 0$
   D. $\ddot{x} + 0.5\dot{x} + 2x = 0$

2. A mass-spring-damper system has a spring of stiffness $1,000$ N/m, a mass of 10 kg, and a damping coefficient of 10 N/(m/s). What is the resonant frequency of the system?
   A. $10/\sqrt{2}$ rad/s
   B. $10\sqrt{0.75}$ rad/s
   C. 10 rad/s
   D. 100 rad/s

3. Consider the frequency response diagram below for $G(s) = X(s)/U(s)$. Suppose the input $u(t) = A\sin\omega t$ is a sinusoid of amplitude $A = 10$ units and, at steady-state, the output $x(t)$ is a sinusoid that has an amplitude of 20 units. What is the input frequency ω?



   A. $\approx$ 1000 rad/s
   B. $\approx$ 800 rad/s
   C. $\approx$ 660 rad/s
   D. $\approx$ 400 rad/s
   E. $\approx$ 200 rad/s
   F. $\approx$ 1 rad/s

4. For a undamped system, what is the magnitude $|G(i\omega)|$ at resonance?
   A. $|G(i\omega)| = 0$
   B. $|G(i\omega)| = \sqrt{2}$
   C. $|G(i\omega)| = \infty$
   D. Not enough information given

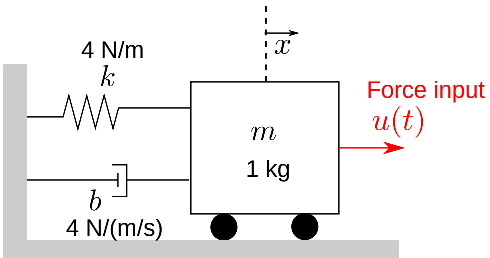
5. Suppose that the transfer function of a system is

$$G(s) = \frac{-1}{s^2} \ .$$

What is the phase of the system at an input frequency of $\omega = 10$ rad/s?

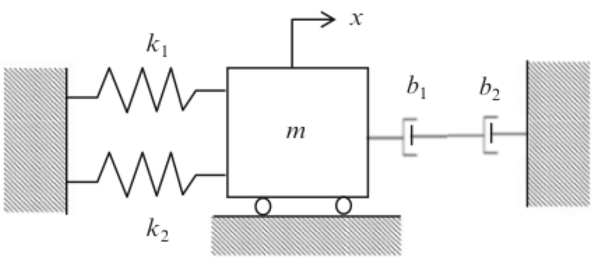
A. $\phi = 0$
B. $\phi = \pi/2$
C. $\phi = \pi$
D. $\phi = \text{atan}(-1/\omega^2)$

6. Suppose the system shown below starts from rest and is driven by a sinusoidal force input $u(t)$. How long does it take (approximately) for the transients to decay and the system response $x(t)$ to reach a steady state sinusoidal output?



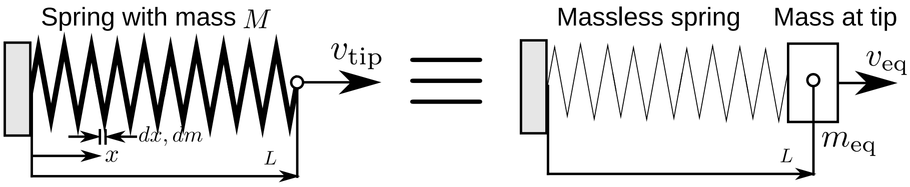
A. $\approx$ 0.5 sec.
B. $\approx$ 1 sec.
C. $\approx$ 2 sec.
D. $\approx$ 4 sec.
E. $\approx$ 16 sec.

7. What is the equation of motion for the following system with $k_1 = k_2 = b_1 = b_2 = 1$?



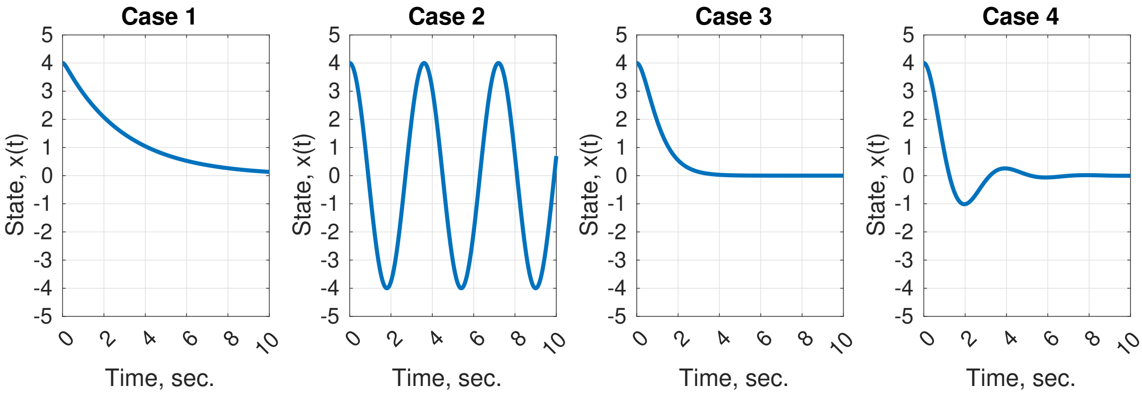
A. $\ddot{x} + 2\dot{x} + 2x = 0$
B. $\ddot{x} + 0.5\dot{x} + 2x = 0$
C. $\ddot{x} + 2\dot{x} + 0.5x = 0$
D. $\dot{x} + 0.5x = 0$

8. In class, we derived the lumped parameter model of the spring with distributed mass M (left image below) as a massless spring with a mass m_{eq} at the tip (right image below). What was an assumption used in this derivation?



A. The thickness of the spring wire is small compared to the length
B. The spring is suspended in a viscous medium that preserves momentum
C. The kinetic energy in both systems is equal
D. The elastic energy is dissipates in both systems

9. Consider the responses below of a homogeneous second-order system from the initial condition $x(0) = 4$ and $\dot{x}(0) = 0$.



Which of the above cases corresponds to the response of a second-order system with a damping ratio of 3 and natural frequency of 2 rad/s?
A. Case 1
B. Case 2
C. Case 3
D. Case 4

10. What is the correct MATLAB code for defining the transfer function $G(s)$?

$$G(s) = \frac{5s^2 + 2s}{3s^5 + 3s^4 + s^2 + 2s + 1}$$

A. `sys=tf([5,2],[3,3,1,2,1])`
B. `sys=tf([5,2,0],[3,3,0,1,2,1])`
C. `sys=tf([5,2,0],[3,3,1,2,1])`
D. `sys=tf([5,2],[3,3,1,2,1])`
E. `sys=tf([(5,2),(2,1)],[(3,5),(3,4),(1,2),(2,1),1])`

Workout Problem Instructions

To receive full credit on the workout problems show all of your work.

Workout Problem 1 (10 pts)

Consider the following system

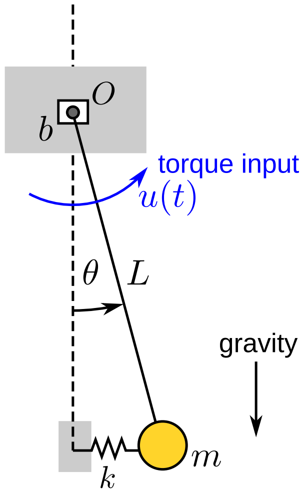
$$\ddot{x} + 6\dot{x} + 34x = 0$$

with initial conditions $x(0) = 3$ and $\dot{x}(0) = -11$.

- Take the Laplace transform and solve for the coefficients of the partial fraction expansion of $X(s)$
- State the particular solution $x(t)$ to the initial value problem

Workout Problem 2 (10 points)

Consider the following mechanical system consisting of a massless rod of length L connected to a ball of mass m. The system has a rotational damper b, a spring constant k, and is subject to gravitational acceleration g. The input into the system is a *torque* $u(t)$.



- (5 points) Derive the transfer function:

$$G(s) = \frac{\Theta(s)}{U(s)}$$

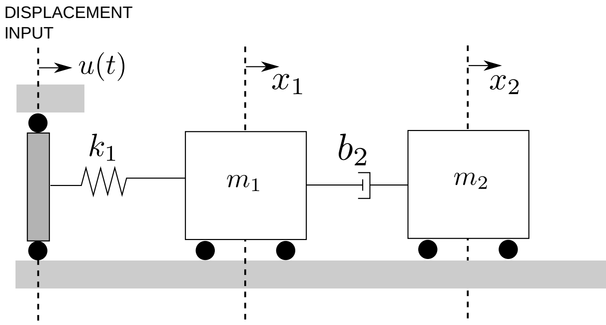
  For full credit *fully simplify* the transfer function. In other words, expand/cancel terms such that the numerator and denominator are each a simply polynomial in terms of s (with coefficients in terms of the constants of the problem: k, m, b, L as needed). Use small angle approximations.

- (5 points) Compute the sinusoidal transfer function and find an expression for the magnitude $|G(i\omega)|$ and phase lag ϕ as a function if input frequency ω.

Workout Problem 2 continued (extra page)

Workout Problem 3 (10 pts)

Consider the following mechanical system:



where $u(t)$ is a *displacement* input describing the position of the moveable wall supported by rollers on the left-hand side. Derive the transfer function:

$$G(s) = \frac{X_1(s)}{U(s)}$$

For full credit *fully simplify* the transfer function. In other words, expand/cancel terms such that the numerator and denominator are each a simply polynomial in terms of s (with coefficients in terms of the constants of the problem: $m_1, m_2, b_1, b_2, k_1, k_2$ as needed).

Workout Problem 3 continued (extra page)