Homework 6

1 Problem

Solve Problem 6a, 6b, 6d in the Davies book (p. 119)

2 Problem

Solve Problem 11 in the Davies book (p. 122)

3 Problem

The rolling motion of a Mariner Class cargo ship can be approximated by the following equation:

$$(I_{44} + A_{44})\ddot{\phi} + B_{44}\dot{\phi} + C_{44}\phi = M\sin(\omega_{\text{waves}}t)$$

where ϕ is the roll angle, I_{44} is the roll inertia, A_{44} is the inertia of the added mass (i.e., the surrounding water has the effect of adding inertia to the vessel), B_{44} is the roll damping due to viscous shear forces, C_{44} is the hydrostatic restoring force. The right hand side represents sinusoidal forcing caused by waves at frequency ω_{waves}, with M being the maximum applied moment to the ship. Assume $I_{44} = 1.471 \times 10^{10}$ kg-m^2, $A_{44} = 2.1 \times 10^{10}$ kg-m^2, $C_{44} = 1.1852 \times 10^{10}$ N-m/rad, $B_{44} = 6.6018 \times 10^9$ N-m/(rad/s).

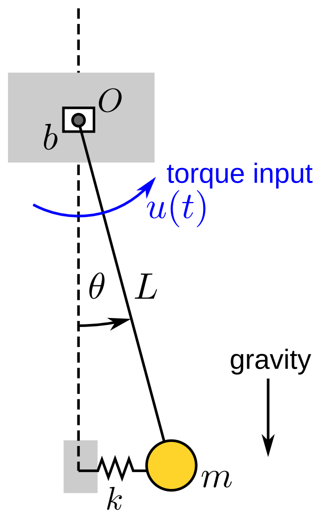
Determine the following parameters for the vessel by equation the coefficients of the above system to that of a damped harmonic oscillator:

1. undamped natural frequency (in Hz)
2. damping ratio
3. damped natural frequency (in Hz)
4. period of oscillation (in seconds, assuming the damped natural frequency)
5. If the waves suddenly stopped ($M = 0$) when the ship was at it's maximum roll angle, how long would it take the ship to settle down to 2 % of this maximum roll angle? (Hint: Use your knowledge of the time constant for the decaying oscillations.)

Side note: The system parameters above do not depend on the right hand side of the equation, which is the input or forcing term caused by the waves. However, if the frequency of the waves ω_{waves} is close to the natural frequency the ship roll angle will resonate and lead to large excursions in roll angle.

4 Problem

Consider the following mechanical system consisting of a massless rod of length L connected to a ball of mass m. The system has a rotational damper b, a spring constant k, and is subject to gravitational acceleration g. The input into the system is a *torque* $u(t)$.



Derive and *fully simplify* the transfer function:

$$G(s) = \frac{\Theta(s)}{U(s)}$$

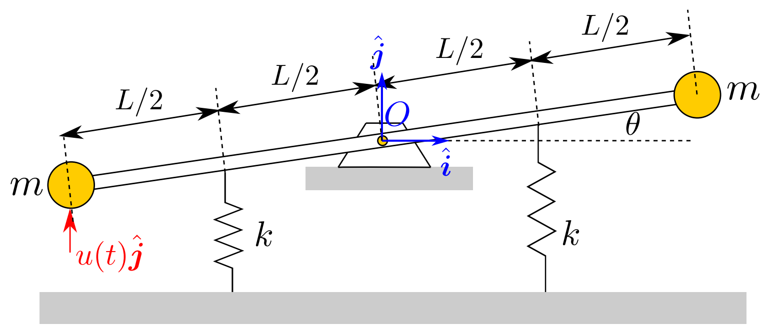
Make sure to expand/cancel terms such that the numerator and denominator are each a polynomial in terms of s (with coefficients in terms of the constants of the problem: k, m, b, L as needed). Use small angle approximations.

5 Problem

Consider the single degree-of-freedom rotational system shown below with two masses m concentrated at each end. An upward force, $u(t)$, is applied at the leftmost point, and two springs with stiffness k are attached at distances away from the center rotation point O. The springs are unstretched when the angle of the rod is $\theta = 0$. Assume small angles and account for weight of masses.

- Derive an expression for the natural frequency in terms of the parameters given.
- Show that the transfer function for this system is

$$\frac{\Theta(s)}{U(s)} = \frac{-2}{4mLs^2 + kL}$$



Hint: Recall that for N masses, the inertia around point O is:

$$I_O = \sum_{i=1}^{N} m_i ||\boldsymbol{r}_{i/O}||^2$$

where $\boldsymbol{r}_{i/O}$ is the vector that points to the ith mass from point O.