Homework 1

1 Problem

For each of the ODEs in the 1st column, indicate whether it is:

1. linear time-invariant (LTI), linear time-varying (LTV), or nonlinear
2. 1st order, 2nd order, or higher order
3. homogeneous or inhomogeneous (only for the case of linear systems—for nonlinear systems you can leave the last column blank)

by marking the appropriate column. The unknown function $x(t)$ represents the state of some mechanical system and t represents time. Hint: An ODE is considered nonlinear only if the nonlinearity involves the unknown function $x(t)$.

	Linearity?			Order?			Homogeneity?	
System ODE	LTI	LTV	NL	1st	2nd	Higher	Homog.	Inhomog.
$\ddot{x} + 3tx = 0$								
$(\dot{x} - x)^2 + 1 = 0$								
$t^2x + bx + c\dot{x} = 0$								
$\dddot{x} = 0$								
$\ddot{x} + \dot{x} + x - 2 = 0$								
$\ddot{x} + \sin(x) = 0$								
$e^t x + \dot{x} = \sin t$								
$\dot{x} + x = 0$								
$\dot{x}x + a + bt = 0$								
$\ddot{x} - b\dot{x}^2 = 0$								

2 Problem

Consider the following IVP:

$$\dot{x} + 2x = 0$$

with initial condition $x(t_0) = -10$ and $t_0 = 0$.

1. What is the particular solution, $x(t)$?
2. What is the value of x as time $t \to \infty$?

A. $x \to -\infty$
B. $x \to -10$
C. $x \to 0$
D. $x \to +10$
E. $x \to +\infty$

3 Problem

Consider the ODE $\dot{x} + 2x = e^{-2t}$ with initial condition $x(t_0) = 10$ and $t_0 = 0$. What is the particular solution, $x(t)$?

4 Problem

Consider the ODE $\dot{x} = 2 - 3x$ with initial condition $x(t_0) = 1$ and $t_0 = 0$. What is the value of the state x when $t = 2$? Estimate when the system will decay to a constant value using the time constant of the system (i.e., after 4 time constants)?

5 Problem

Consider the ODE $\dot{x} - 3x = 0$ with initial condition $x(1) = 40$ and $t_0 = 0$. (Note that the initial condition is not given at $t = 0$!)

1. What is the particular solution, $x(t)$?
2. What is the value of x as time $t \to \infty$?

A. $x \to -\infty$
B. $x \to -10$
C. $x \to 0$
D. $x \to +10$
E. $x \to +\infty$