Name: ______________________________

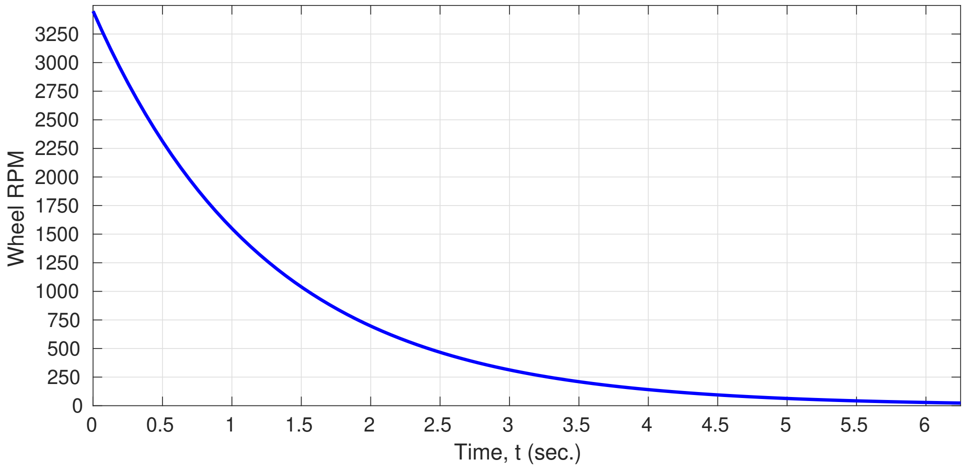
MEGR 3122 Dynamics Systems II: Exam 1, Spring 2023

Directions: Circle the best answer. Show your work and explain your reasoning on all problems to receive full credit (unless otherwise specified).

1. (2 points) A bench grinder is spinning at a rate of $\omega_0 = 3{,}450$ RPM when it is turned off and the wheel coasts to a stop. The wheel velocity is modeled according to:

$$\dot{\omega} + b\omega = 0, \ \omega(t_0) = \omega_0, \ t_0 = 0$$

where b is a rotational damping coefficient. A plot of the angular velocity measured with a tachometer is shown below. What is a reasonable estimate for the value of b?



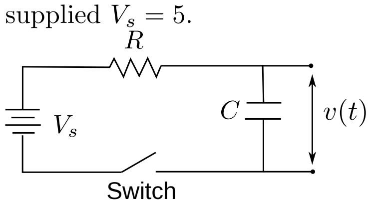
A. 1/3
B. 0.8
C. 1.4
D. 3.1
E. 6.3

2. (4 points) What is the imaginary part of the quantity below?

$$z = \frac{i-4}{2i-3} \cdot e^{i\pi/2}$$

A. 14/13
B. 5/13
C. 5/14
D. $5/(2\pi)$
E. $-5/13$

3. (4 points) The RC circuit shown below has a resistor $R = 0.5$ and a capacitor $C = 2$ and voltage supplied $V_s = 5$.



The equation modeling the system is

$$RC\frac{dv(t)}{dt} + v(t) = V_s$$

where $v(t)$ is the voltage measured at the output across the capacitor and V_s is a constant voltage supplied by a battery. What is the value of $v(t)$ at one second after the switch is closed? Assume the initial output voltage is $v(t_0) = 0$. (Hint: re-write the above equation in more familair notation.)
A. 0.47 V
B. 1.66 V
C. 2.30 V
D. 3.16 V
E. 5.00 V

4. (4 points) The general solution of a second order ODE with initial conditions $x(0) = 1$ and $\dot{x}(0) = 3$ is found to be $x(t) = e^{-t}(c_1 \cos 2t + c_2 \sin 2t)$. What is the particular solution?

A. $x(t) = e^{-t}(6\cos 2t + 4\sin 2t)$
B. $x(t) = e^{-t}(\cos 2t + 4\sin 2t)$
C. $x(t) = e^{-t}(\cos 2t + 2\sin 2t)$
D. $x(t) = 2e^{-t}(\cos 2t + \sin 2t)$
E. $x(t) = 6e^{-t}\sin 2t$

5. (4 points) What is the partial fraction expansion of the Laplace transform of $\ddot{x} + 4\dot{x} + 5x = 0$ with $x(0) = 1$ and $\dot{x}(0) = -1$?

   A. $X(s) = \frac{(s+2)}{(s+2)^2+1}$

   B. $X(s) = \frac{s+3}{s^2+4s+6}$

   C. $X(s) = \frac{1}{(s+2)^2+1} - \frac{2(s+1)}{(s+1)^2+2}$

   D. $X(s) = \frac{1}{(s+2)^2+1} - \frac{(s+2)}{(s+2)^2+1}$

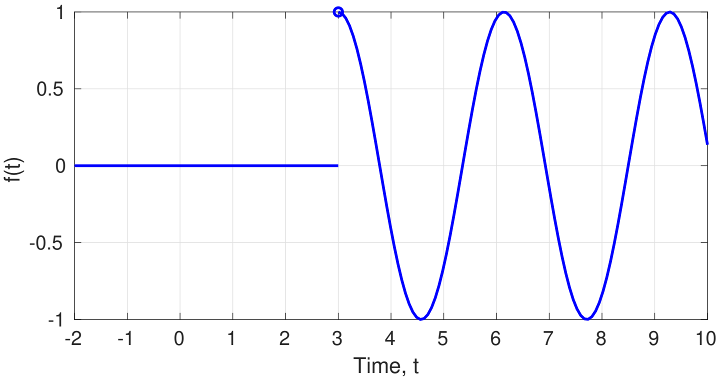
   E. $X(s) = \frac{1}{(s+2)^2+1} + \frac{(s+2)}{(s+2)^2+1}$

6. (4 points) What is the initial value of $x(t)$ if the Laplace transform of $x(t)$ is the following?

$$X(s) = \frac{5s^2 + 2s + 7}{4s^3 + 3s^2 + 2s}$$

   A. $x(0) = 7/3$
   B. $x(0) = 5/4$
   C. $x(0) = 2/3$
   D. $x(0) = 7/9$
   E. $x(0) = 4/5$

7. (4 points) The input signal $f(t)$ below is applied to a dynamic system starting at $t = 3$ seconds. The signal has a frequency $\omega = 2$ rad/s. What is the Laplace transform $F(s)$?



A. $e^{-s}\ \frac{(s+3)}{(s+3)^2+4}$

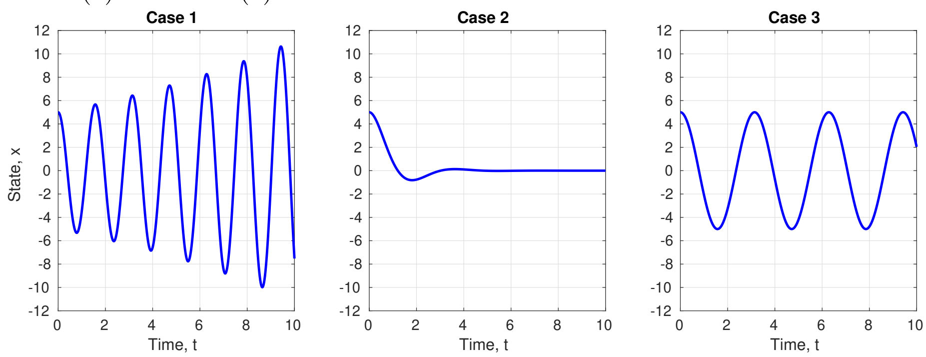
B. $e^{-(s-3)}\ \frac{(s-3)}{(s-3)^2+4}$

C. $e^{-3s}\ \frac{s}{s^2+4}$

D. $\frac{1}{s^2+4}$

E. $e^{-3s}\ \frac{(s-3)}{(s-3)^2+4}$

8. (2 points) Which case below could plausibly represent the response of a system $\ddot{x} - 0.16\dot{x} + 16x = 0$ with $x(0) = 5$ and $\dot{x}(0) = 0$?



A. Case 1
B. Case 2
C. Case 3
D. None of the above
E. All of the above

9. (4 points) The inverse Laplace transform of

$$X(s) = \frac{7}{(s+3)(s+5)}$$

is which of the following?

A. $x(t) = \frac{5}{2}e^{3t} - \frac{5}{2}e^{5t}$

B. $x(t) = \frac{7}{2}e^{5t} - \frac{5}{2}e^{3t}$

C. $x(t) = \frac{7}{2}e^{-3t} - \frac{7}{2}te^{-5t}$

D. $x(t) = \frac{7}{2}e^{-3t} - \frac{7}{2}e^{-5t}$

E. $x(t) = \frac{5}{3}\cos 3t - \frac{5}{3}\sin 5t$

10. (6 points) What is the solution to the initial value problem below?

$$\ddot{x} + 16x = \cos 3t,\ \ x(0) = 0,\ \ \dot{x}(0) = 0$$