Homework 3

1 Problem

Find the Laplace transform $F(s) = \mathcal{L}[f(t)]$ for each of the following functions. For full credit, show all of your work by expanding each Laplace transform into one or more Laplace transforms of the common functions listed in the provided Laplace Transform Table. Indicate which rows of the table you used to obtain your solution and fully simplify the result to a single term.

1. $f(t) = e^{at+b}$ where a, b are constant
2. $f(t) = \sin(\omega t - \phi)$ where ω, ϕ are constant
3. $f(t) = t^3 - 1/2$
4. $f(t) = (e^{-t}/4)[2 + t^2 + \cos(3t)]$. (Note: For this problem there is no need to simplify to a common denominator.)
5. $f(t) = (t-2)H(t-2)$, where $H(\cdot)$ is the unit step or Heaviside function

2 Problem

What is the final value of $x(t)$ as $t \to \infty$ if the Laplace transform of $x(t)$ is the following?

$$X(s) = \frac{6}{s(s+2)}$$

3 Problem

Sketch (by hand) the following function on the interval $t \in [-2\pi, 4\pi]$ and find its Laplace transform:

$$f(t) = \sin(t) - \sin(t-\alpha)H(t-\alpha)$$

where $H(t-\alpha)$ is the unit step function delayed to start at time $\alpha = 2\pi$.

Hint: Use the Laplace transform for a translated function.

4 Problem

Find the Laplace transform fraction for the following function and rearrange it such that $X(s)/F(s)$ is the only term on the left-hand-side:

$$\dddot{x}(t) + 2\zeta\omega\ddot{x}(t) + \omega^2\dot{x}(t) = f(t)$$

Assume the initial conditions are all zero, $x(t_0) = \dot{x}(t_0) = \ddot{x}(t_0) = \dddot{x}(t_0) = 0$ with initial time $t_0 = 0$. Hint: Use the differentiation theorem.

5 Problem

Compute the inverse Laplace transform $f(t) = \mathcal{L}^{-1}[F(s)]$ of the following function:

$$F(s) = \frac{1}{s+\sigma}(e^{-as} - e^{-bs})$$

where a, b, and σ are constants.

Hint: Recall that the following property holds for translated functions $\mathcal{L}[f(t-\alpha)H(t-\alpha)] = e^{-s\alpha}F(s)$, which implies that

$$f(t) = \mathcal{L}^{-1}[e^{-s\alpha}F(s)] = f(t-\alpha)H(t-\alpha)$$

The expression above can be written as a sum of two functions of this form.

6 Problem

Compute the inverse Laplace transform $f(t) = \mathcal{L}^{-1}[F(s)]$ of the following function:

$$F(s) = \frac{s+1}{s^2+6s+9}$$

7 Problem

Compute the inverse Laplace transform of

$$F(s) = \frac{s+1}{s(s+2)}$$