Homework 2

1 Problem

For each of the following ODEs determine if the eigenvalues are (a) real and distinct, (b) repeated, (c) complex conjugate pairs:

1. $\ddot{x} + 2\dot{x} + 3x = 0$
2. $\ddot{x} + 4\dot{x} + x = 0$
3. $\ddot{x} + 4\dot{x} + 4x = 0$
4. $\ddot{x} + 3x = 0$

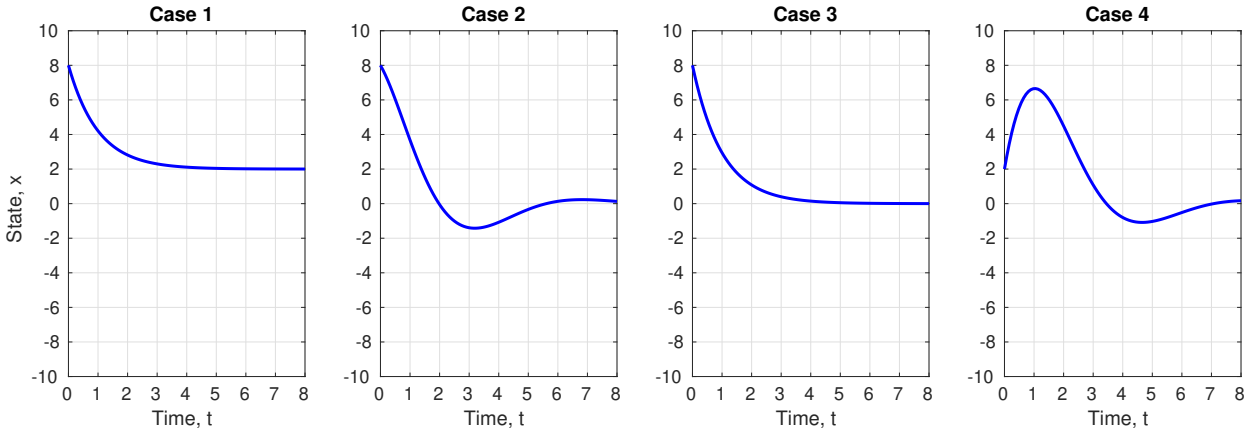
2 Problem

For each of the following linear, time-invariant, 2nd order homogeneous ODEs solve for the particular solution that satisfies the initial values given.

1. $\ddot{x} - 4\dot{x} + 4x = 0$, Initial Values: $x(0) = 12$, $\dot{x}(0) = -3$
2. $\ddot{x} + 3\dot{x} - 10x = 0$, Initial Values: $x(0) = 4$, $\dot{x}(0) = -2$
3. $\ddot{x} - 8\dot{x} + 17x = 0$, Initial Values: $x(0) = -4$, $\dot{x}(0) = -1$

3 Problem

Match each one of the responses shown below (Cases 1-4) with one of the following IVPs



(a) $\ddot{x} + \dot{x} + x = 0, x(0) = 8, \dot{x}(0) = 0$

(b) $\dot{x} - x = 2,\ x(0) = 8$

(c) $\dot{x} + x = 2,\ x(0) = 8$

(d) $\ddot{x} + \dot{x} + x = 2, x(0) = 0, \dot{x}(0) = -8$

(e) $\ddot{x} + \dot{x} + x = 2, x(0) = 2, \dot{x}(0) = 0$

(f) $\dot{x} + x = 8,\ x(0) = 2$

(g) $\dot{x} + x = 0,\ x(0) = 8$

(h) $\ddot{x} + \dot{x} + x = 0, x(0) = 8, \dot{x}(0) = -3$

(i) $\ddot{x} + \dot{x} + x = 0, x(0) = 2, \dot{x}(0) = 10$

(j) $\dot{x} + 8x = 0,\ x(0) = 0$

4 Problem

Let $z = x + iy$. Determine the value of x and y in each of the following cases below. Show all of your work for full credit. You can check your answer in MATLAB by typing in the expression with `1i` representing the imaginary number (e.g., `(3+1i)*(1+3i)`).

1. $z = (3 + i)(1 + 3i)$
2. $z = i^4 - 1$
3. $z = \frac{3+i}{1+3i}$
4. If $w = 1 + 2i$, then what is $|w|$ and $\theta = \arg(w)$?
5. If $z_1 = -i$ and $z_2 = e^{i\pi/2}$, then what is the sum $z_1 + z_2$?

5 MATLAB Problem

The McGuire Nuclear Station in Huntersville, NC is testing a new isotope of radioactive material called *nineridium*. Engineers have determined that the material exhibits exponential decay according to the ODE:

$$\dot{N} = -kN$$

where $N(t)$ is the number of parent atoms, time t has units of years, and $k = 1$ $(1/\text{year})$ is the decay rate. If the reactor starts with a chunk of nineridium that consists of $N(t_0) = 1000$ atoms at time $t_0 = 0$, then:

- Part A (5 pts). What is the expression that gives the number of atoms, $N(t)$, for any future time $t \geq t_0$?
- Part B (5 pts). What is the time constant describing the decay?
- Part C (5 pts). Using MATLAB, plot the function $N(t)$ out to 5 time constants. Label your axes and include appropriate units. Your submission should include both your code and the resulting graph. MATLAB Hints:
  - If you don't have MATLAB installed you can download it from `software.uncc.edu`. If you are feeling rusty, please review the MATLAB help files provided (you may wish to complete the "On Ramp" tutorial).
  - to plot a solid line of width 2 with circular markers in MATLAB use the the command `plot(time, x, 'ro-','linewidth',2)` where `time` is a vector of increasing time values, `x` is a vector of data points to be plotted with time. You can change the color of the line by replacing the `r` in `ro-` with other letters corresponding to colors e.g., blue `b`, magenta `m`, green `g`.
- Part D (5 pts). Based on your plot, what is the approximate half-life of nineridium (i.e., how many years does it take the material to decay to 50% of the initial amount)?