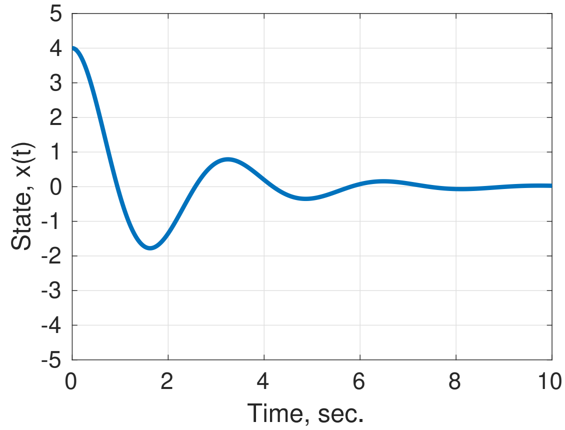
Name: ______________________________

MEGR 3122 Dynamics Systems II: Exam 2, Spring 2023

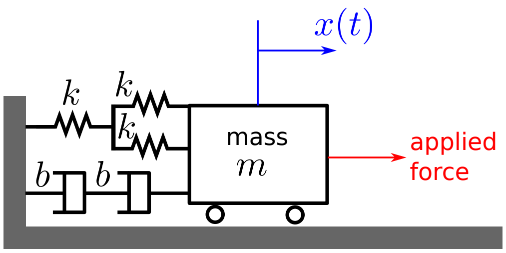
Directions: Circle the best answer. Show your work and explain your reasoning on all problems to receive full credit (unless otherwise specified).

1. Consider the response of a homogeneous (unforced) second-order system from the initial condition $x(0) = 4$ and $\dot{x}(0) = 0$.



Which ODE best matches the response shown?
A. $2\ddot{x} + 1\dot{x} - 0.5x = 0$
B. $4\ddot{x} + 4\dot{x} + 8x = 0$
C. $\ddot{x} + 2\dot{x} - 4x = 0$
D. $\ddot{x} + 8\dot{x} + 4x = 0$
E. $\ddot{x} + 2x = 0$

2. What is the damping ratio of the system shown below?



A. $\dfrac{b}{\sqrt{(2/3)km}}$

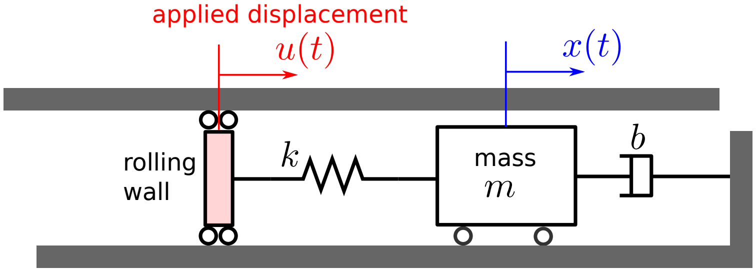
B. $\sqrt{3k/2m}$

C. $\sqrt{1-b^2}\sqrt{3k/m}$

D. $\dfrac{b}{4\sqrt{(2/3)km}}$

E. $\dfrac{3b^2}{2\sqrt{k/m}}$

3. Find the transfer function $X(s)/U(s)$ for the mechanical system shown below



A. $\frac{k}{ms^2+bs+k}$

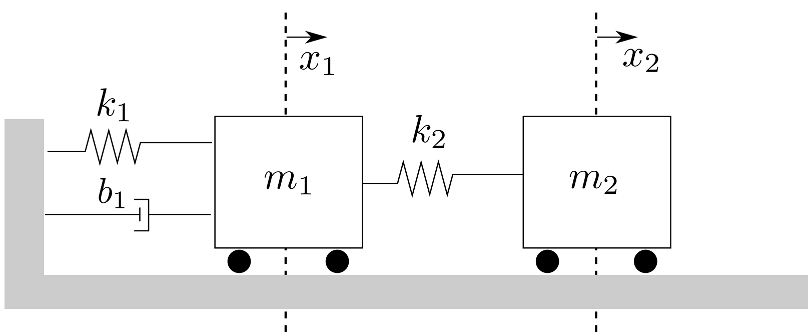
B. $\frac{1}{ms^2-ks-b}$

C. $\frac{k}{ms^2+ks+b}$

D. $\frac{b}{ms^2+bs-k}$

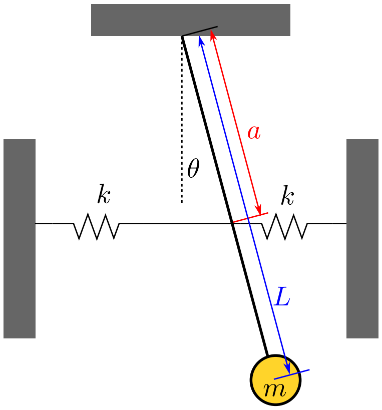
E. $\frac{b}{ms^2+bs+k}$

4. Suppose that $k_1 = 4$, $k_2 = 10$, and $b_1 = 1$. Both masses are equal $m_1 = m_2 = 1$. The first mass is at position $x_1 = 0.5$ with velocity $\dot{x}_1 = -1$ and the second mass is at position $x_2 = 0.5$ with velocity $\dot{x}_2 = 1$. What is the total force that acts on the first mass at this instant? Assume that the positive directions for x_1 and x_2 are as indicated by the arrows below. Neglect gravity/normal forces in the vertical direction (consider horizontal forces only).



A. The net force has magnitude 2 and points to the right
B. The net force has magnitude 1 and points to the right
C. The net force has magnitude 0
D. The net force has magnitude 1 and points to the left
E. The net force has magnitude 2 and points to the left

5. The system below consists of a pendulum with a masless rod and bob of mass m. Two spring forces and gravity act on the system. What is the correct equation of motion? Assume small angles.



A. $\ddot{\theta} + \left(\frac{g}{L} + \frac{2ka^2}{mL^2}\right)\theta = 0$

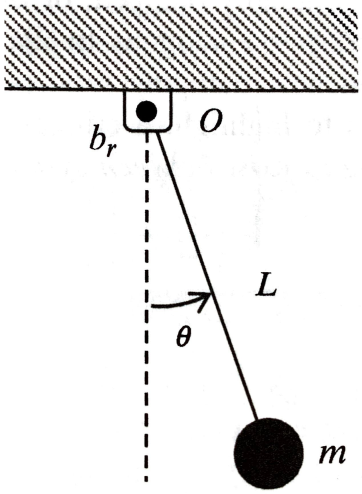
B. $\ddot{\theta} + \left(-\frac{g}{L} + \frac{2ka^2}{mL^2}\right)\theta = 0$

C. $\ddot{\theta} + \frac{g}{L}\dot{\theta} + \left(\frac{2ka}{mL^2}\right)\theta = 0$

D. $\ddot{\theta} + \frac{2ka}{mL^2}\dot{\theta} + \left(\frac{g}{L}\right)\theta = 0$

E. $\ddot{\theta} + \left(-\frac{g}{L} + \frac{2ka}{mL^2}\right)\theta = 0$

6. Suppose that the highly underdamped system below has mass $m = 3$ and length $L = 2.5$. It is released from rest and begins to oscillate (with very gradually decreasing amplitude due to the small damping $b_r = 1$). What is the period of each oscillation in seconds? Assume small angles and $g \approx 10$.



A. About two seconds
B. About half a second
C. About three seconds
D. About one second
E. About one tenth of a second

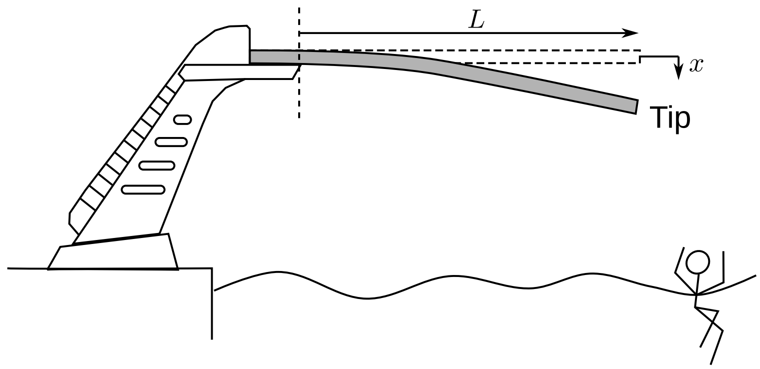
7. Consider the multi-degree-of-freedom system of coupled ODEs with zero initial conditions:

$$\ddot{x}_1 + kx_1 - kx_2 = 0$$
$$\ddot{x}_2 + kx_2 = f(t)$$

where $f(t)$ is an input into the system. What is the transfer function $G_1(s) = X_1(s)/F(s)$?

A. $\frac{2k}{(s^2+k)^2}$
B. $\frac{k^2}{(s^2+k)}$
C. $\frac{k}{(s^2+k^2)^2}$
D. $\frac{k}{(s^2+k)^2}$
E. $\frac{3k}{(s^2+2ks+k)}$

8. A diving board has Young's modulus of E, width a, thickness b, area moment of inertia I , length of L, and (volumetric) density of ρ. Use a lumped-parameter model to model the displacement of the tip of the diving board. What is the natural frequency of the tip (in rad/s)?



A. $\frac{3}{4}\sqrt{\frac{0.23EI}{\rho ab L^3}}$

B. $\frac{3.61}{L^2}\sqrt{\frac{EI}{\rho ab}}$

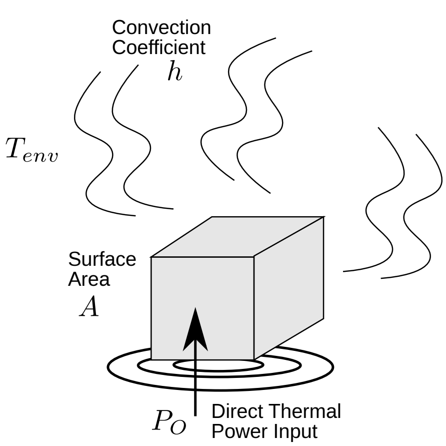
C. $\sqrt{\frac{0.23E(ab^2)}{4\rho L^3}}$

D. $\frac{0.4796}{L^2}\sqrt{\frac{EI}{\rho ab}}$

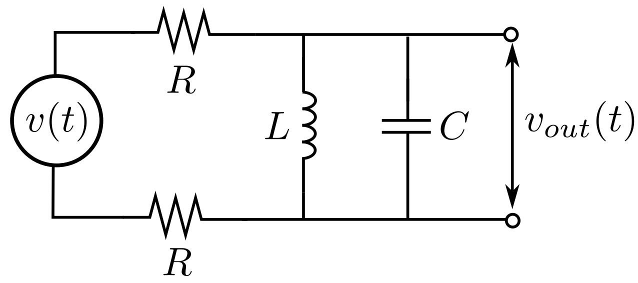
E. $\frac{L^2 b}{0.277}\sqrt{\frac{\rho a}{EI}}$

9. Suppose that an object initially at temperature $T_0 = 65$ deg C has thermal mass of $mc = 3$ J/(deg C) and is being simulatenously heated with a constant thermal power input of $P_0 = 100$ Watts and cooled via convection with the environment at $T_{\text{env}} = 15$ deg C. The exposed surface area for convection is $A = 0.8$ m^2 and the convection coefficient is $h = 2.5$ Watts/(m$^2\cdot$ deg C). What will happen to the temperature of the object after a long period of time?

A. The temperature will decay to a value $T_{\text{env}} + P_0/(mc)$
B. The temperature will approach an asymptote at $(T_{\text{env}} + P_0/(mc))/2$
C. The temperature will stay constant at T_0
D. The temperature will decay to T_{env} after approximately four time constants
E. The temperature will oscillate between T_{env} and $T_0 + P_0/(mc)$



10. For the following circuit, what is the transfer function $G(s) = V_{\text{out}}(s)/V(s)$?



A. $\frac{CLs^2+1}{CLs^2+2RLs+1}$

B. $\frac{Ls}{2RLCs^2+Ls+2R}$

C. $\frac{1}{RLCs^2+2RLs+LC}$

D. $\frac{(1/RL)}{s^2+2Cs+1}$

E. $\frac{RCs^2+Ls+R}{CLs^2+LRs+1}$