Homework 4

1 Problem

Compute the inverse Laplace transform of

$$F(s) = \frac{s+2}{(s+1)(s+4)^2}$$

Hint: Use a combination of partial fraction expansions for real distinct and repeated poles.

2 Problem

Compute the inverse Laplace transform of

$$F(s) = \frac{3s+9}{s^2+4s+5}$$

3 Problem

For each of the following differential equations, use Laplace transforms to find the solution to the IVP.

1. $3\ddot{x} + 12\dot{x} + 60x = \delta(t);\ x(0) = 0;\ \dot{x}(0) = 0$ where $\delta(t)$ is the impulse or dirac delta function (row 1 in the Laplace transform table).
2. $\ddot{x} + 10\dot{x} + 25x = 0;\ x(0) = 1;\ \dot{x}(0) = 0$
3. $\ddot{x} + 5\dot{x} + 6x = 2e^{-t};\ x(0) = 1;\ \dot{x}(0) = 0$
4. $\ddot{x} + 2\dot{x} = 8t;\ x(0) = 0;\ \dot{x}(0) = 0$

Show all your work/intermediate steps. Other solution methods besides Laplace transforms will not receive any credit.

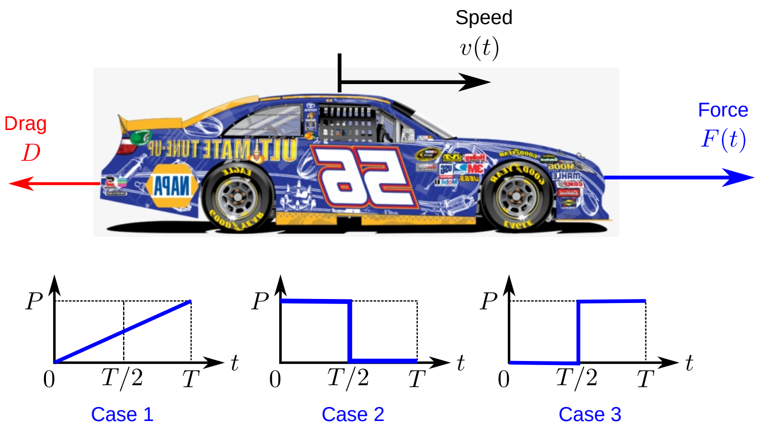
4 Problem

Use the MATLAB function `dsolve` to verify your answer for Problem 3.4. Generate a plot of the solution over the time interval $t \in [0, 3]$ seconds. Submit your code.

5 Problem

Suppose the racecar below has a mass of $m = 750$ kg and is moving down a track with an initial speed of $v(t_0) = 45$ m/s at time $t_0 = 0$ sec. The drag on the car is modeled as a linear function of velocity: $D = bv$, where $b = 20$ N/(m/s).

- Using the free-body diagram below, where $F(t)$ is an applied force, apply Newton's 2nd Law to find the equations of motion. Since $a(t) = \dot{v}(t)$ you can write this equation as a first-order ODE in speed (i.e., $\sum F = m\dot{v}$).



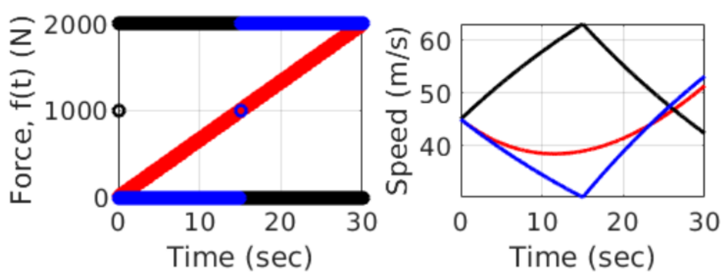
Suppose that over the next $T = 30$ seconds the driver can choose from the three possible force profiles, $F(t)$, shown above, where $P = 2000$ N is the same maximum force reached during each profile.

- Write down an expression for each of the force profiles $F_1(t)$, $F_2(t)$, $F_3(t)$ as a function of the magnitude P and time. You can construct the force profiles from a combination of Heaviside functions and ramps (straight lines) with appropriate slope. Reviewing the doublet example (Lecture 7 PDF, p.2) may be helpful.

Interestingly, each profile has the same impulse (area under the force-time curve) but results in a different final displacement and velocity. Determine the velocity profile $v(t)$ that results from each case by following these steps:

- Solve for the velocity profile in each of the three cases using MATLAB (following the methods of Lecture 10 e.g., using `dsolve`). and plot the three solutions on the same axes. Which case results in the largest final speed? Label your axes, add a legend for each line, and use a thick line type for clarity.

Note that MATLAB defines the step function as: `heaviside(t)`.Your solution should look similar to the one below:



Bonus: Which case results in the furthest distance traveled at time T? Justify your answer with a plot of distance traveled in MATLAB.