Homework 7

1 Problem

Solve Problem 7 in the Davies book (p. 74). Note that the area moment of inertia of circular cross-section is $J = \pi R^4/4$. Assume aluminimum has an elastic modulus of $E = 70 \times 10^9$ N/m^2

2 Problem

Solve Problem 8 in the Davies book (p. 75). Note that the polar moment of inertia of circular cross-section is $J = \pi R^4/2$. Assume a shear modulus of $G = 25.5 \times 10^9$ N/m^2

3 Problem

An 80 kg boy jumps from rest on the ground onto the middle of slackline that is tied across two trees that are about 9 meters apart with an initial tension of $T = 1,000$ N.

- Using the effective spring table from Lecture 15, develop an equivalent mass-spring model of this system. That is, draw a free body diagram of the equivalent system and write down a differential equation that describes the dynamics of $y(t)$. Assume that $u(t)$ is a step force input equal to the weight of the boy and ignore any damping effects.
- Write down the solution the solution to the differential equation $y(t)$ assuming that $y(0) = \dot{y}(0) = 0$.
- After jumping on the slackline the boy bounces up and down. How many seconds does each up-and-down cycle last?

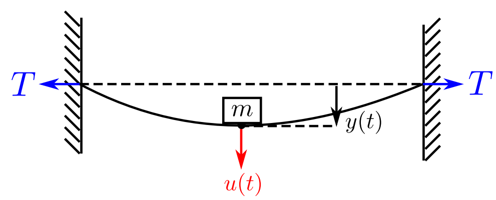
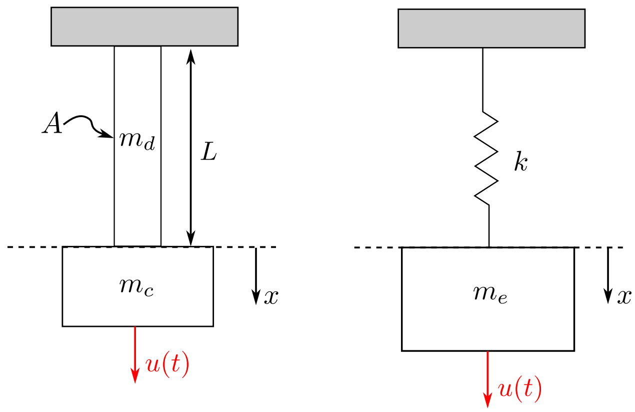


Figure 1: Left: Image Source: [Link], Right: Model

4 Problem

Consider an aluminum rod of length $L = 2$ m and cross-sectional area $A = 0.01$ m^2 (shown on the left). Attached to the rod tip is a concentrated mass $m_c = 10$ kg, and a force $u(t)$ is applied which causes a small amount of extension/compression of the rod. An equivalent lumped parameter



model of the system is shown on the right. Assume that the density of aluminum is $\rho = 2710$ kg/m^3 and Young's modulus is $E = 68 \times 10^9$ Pa.
Using the lumped parameter tables from Lecture 15 to find:

- the equivalent lumped mass m_e
- the equivalent stiffness k
- using the above expressions determine the transfer function $G(s) = X(s)/U(s)$.

5 Problem

A vibrating machine of mass m_1 is mounted on a flexure with stiffness k_1 to a simply supported beam with elastic modulus E, inertia I and length L and modeled by the original system below. The beam has a distributed mass of m_d. The gravitational force with constant g acts on the element of this system. Using the lumped parameter tables from Lecture 15 to find:

- The equivalent lumped mass m_e of the beam (note: the machine with mass m_1 is connected by a spring to the beam and cannot be lumped together with m_d as a concentrated mass)
- The equivalent stiffness k_e
- Then, derive two ODEs (one for $\ddot{x}_1$ and one for $\ddot{x}_2$) that model the motion of the system. The equations should *only* be in terms of the original system variables: $m_1, k_1, x_1, m_d, x_2, g, E, I, L$.