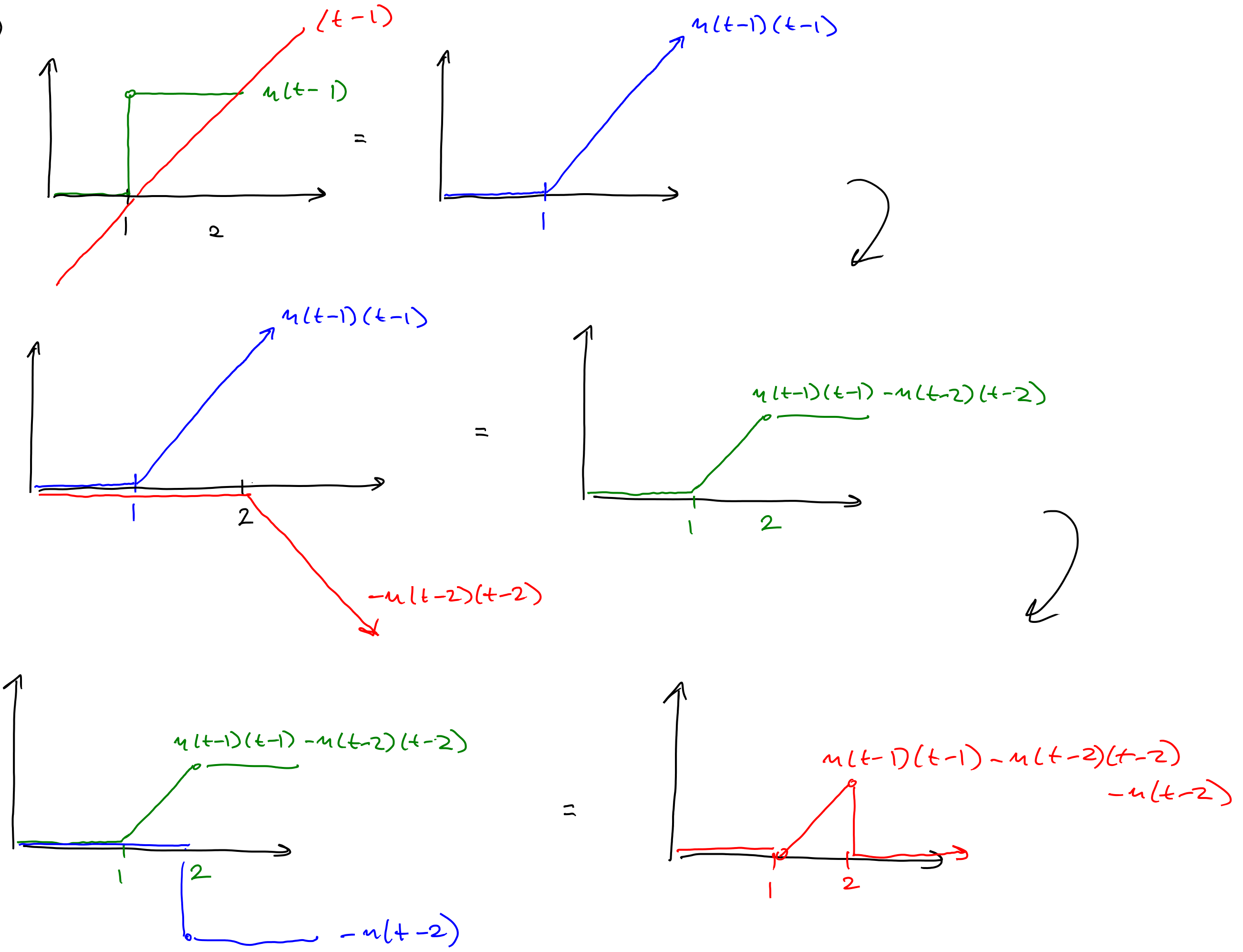
Exam 2 solutions

Multiple choice

1) The solution consists of these four signals below. Solve by inspection.

(D)



2. Final value theorem

(B)

As $t \to \infty$ $\quad x(\infty) = \lim_{s \to 0} s \cdot F(s)$

$$= \lim_{s \to 0} \frac{\cancel{s}(s+10)}{\cancel{s}(s+2)}$$

$$= \frac{10}{2} = 5$$

3. $G(s) = \dfrac{5s^2 + 2s}{3s^5 + 3s^4 + s^2 + 2s + 1} = \dfrac{5s^2 + 2s + 0}{3s^5 + 3s^4 + 0s^3 + s^2 + 2s + 1}$

(C)

use coefficients of numerator and denom.

sys = tf([5 2 0],[3 3 0 1 2 1])

Problem 4

If $\omega_n = 2$ and $\zeta = 0.5$ then the second order DHO system is

(C)

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 0$$

$$\ddot{x} + 2\left(\frac{1}{2}\right)(2)\dot{x} + 4x = 0$$

$$\ddot{x} + 2\dot{x} + 4x = 0$$

5. (D)

$$X(s) = \frac{1}{\underbrace{(s+2)^2}_{\substack{\text{repeated poles} \\ p_{1,2} = -2}}\underbrace{(s^2+2)}_{\substack{\text{imaginary poles} \\ p_{3,4} = \pm\sqrt{2}}}}$$

Since the poles are both repeated and complex, the solution will have the form

$$x(t) = c_1 e^{p_1 t} + c_2 t e^{p_1 t} + c_3 \cos(\sqrt{2}t) + c_4 \sin(\sqrt{2}t)$$

Problem 1

$$\ddot{x} + 16x = \cos 3t \qquad x(0) = 0 \quad \dot{x}(0) = 0$$

row 11

$$X(s)(s^2+16) = \frac{s}{s^2+9}$$

$$X(s) = \frac{s}{(s^2+9)(s^2+16)}$$

Two ways to solve :

Approach 1 : LT table

From row 30 with $\omega_1 = 3$, $\omega_2 = 4$

$$\boxed{x(t) = \underbrace{\frac{1}{16-9}}_{=\frac{1}{7}}\left[\cos(3t) - \cos(4t)\right]}$$

Approach 2 : PFE

$$X(s) = \frac{s}{(s^2+9)(s^2+16)}$$

$p_{1,2} = \pm 3i$ $\quad$ $p_{3,4} = \pm 4i$

all complex poles, so expand as

Recognizing that poles are purely imaginary we can expand as sine/cos terms

$$X(s) = \frac{s}{(s^2+9)(s^2+16)} = \frac{C_1}{(s^2+9)} + \frac{C_2 s}{(s^2+9)} + \frac{C_3}{(s^2+16)} + \frac{C_4 s}{(s^2+16)}$$

Multiply to eliminate denominators

$$s = C_1(s^2+16) + C_2 s(s^2+16) + C_3(s^2+9) + C_4 s(s^2+9)$$

$$= C_1 s^2 + 16C_1 + C_2 s^3 + 16C_2 s + C_3 s^2 + 9C_3 + C_4 s^3 + 9C_4 s$$

$$= (C_2 + C_4)s^3 + (C_1 + C_3)s^2 + (16C_2 + 9C_4)s + 16C_1 + 9C_3$$

equating coefficients

s^3: $\quad C_2 + C_4 = 0 \Rightarrow C_2 = -C_4$

s^2: $\quad C_1 + C_3 = 0 \qquad C_1 = -C_3$

s: $\quad 16C_2 + 9C_4 = 1 \Rightarrow -16C_4 + 9C_4 = 1$

0: $\quad 16C_1 + 9C_3 = 0$

$$-7C_4 = 1$$

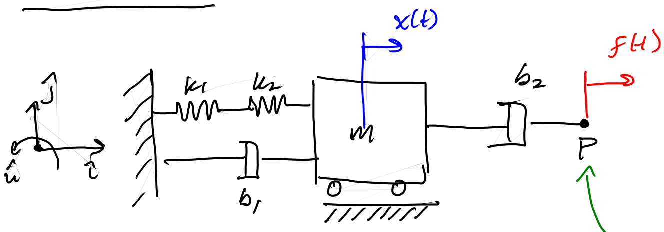
$$\boxed{C_4 = -1/7} \Rightarrow \boxed{C_2 = +1/7}$$

$$-16C_3 + 9C_3 = 0 \Rightarrow \boxed{\begin{matrix} C_3 = 0 \\ C_1 = 0 \end{matrix}}$$

Thus, $\displaystyle X(s) = \frac{1}{7}\frac{s}{(s^2+9)} - \frac{1}{7}\frac{s}{s^2+16}$

ILT, $\displaystyle \boxed{x(t) = \frac{1}{7}\cos 3t - \frac{1}{7}\cos 4t}$

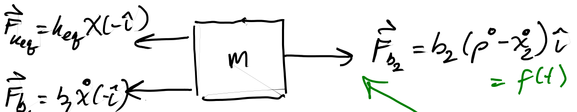
Problem 2

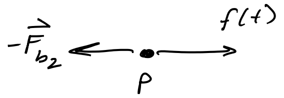


(springs in series) $\qquad$ Assume zero ICs

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

since a force not a displacement is applied, it follows that pt. P is another DOF and will move with velocity that causes balancing damper force.





$f(t) = \vec{F}_{b_2}$

here we can think of f(t) as being applied directly to mass m (i.e, as a single DOF system)

N2L ($\hat{\imath}$-dir) $\quad m\ddot{x} = -k_{eq}x - b_1\dot{x} + f(t)$

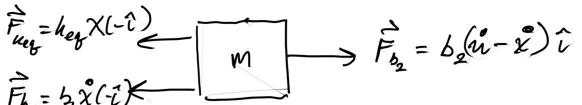
Rearrange

$$m\ddot{x} + b_1\dot{x} + k_{eq}x = f(t)$$

$$X(s)\left(ms^2 + sb_1 + k_{eq}\right) = F(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{ms^2 + sb_1 + \left(\frac{k_1 k_2}{k_1 + k_2}\right)}}$$

For those who misread applied force f(t) and considered this a applied displacement u(t), partial credit was given for this solution:



N2L ($\hat{\imath}$-dir) $\quad m\ddot{x} = -k_{eq}x - b_1\dot{x} + b_2\dot{u} - b_2\dot{x}$

Rearrange

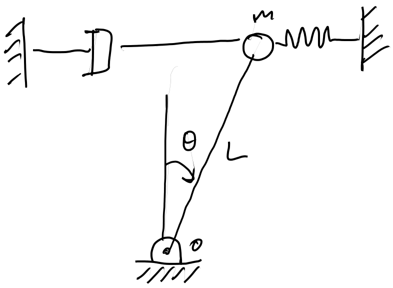
$$m\ddot{x} + (b_1 + b_2)\dot{x} + k_{eq}x = b_2 \dot{u}$$

Laplace Transform

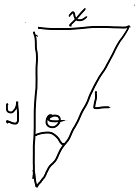
$$X(s)\left(ms^2 + (b_1 + b_2)s + k_{eq}\right) = b_2 U(s)s$$

$$\frac{X(s)}{U(s)} = \frac{b_2 s}{ms^2 + (b_1 + b_2)s + \left(\frac{k_1 k_2}{k_1 + k_2}\right)}$$

Problem 3

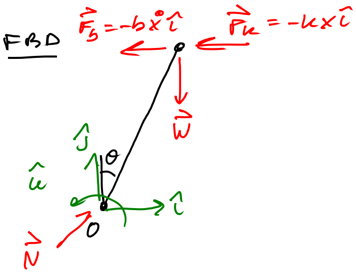


Geometry



$x = \sin\theta\, L$
$y = \cos\theta\, L$

FBD



Sum of Moments about pt. O $\quad \Sigma\vec{M} = -x(mg) + y\, b\dot{x} + ykx$

N2L $\quad \underbrace{I_o}_{=mL^2}\ddot{\theta} = -\sin\theta L mg + \cos\theta L b \dot{x} + \cos\theta L k \sin\theta L$

$\left(\dot{x} = \frac{d}{dt}\sin\theta L = \cos\theta L\dot{\theta}\right)$

Assume small angles $\quad \sin\theta \approx \theta$, $\cos\theta \approx 1$

$$mL^2\ddot{\theta} = -Lmg\theta + L^2 b\dot{\theta} + L^2 k\theta$$

$$mL^2\ddot{\theta} - L^2 b\dot{\theta} + (Lmg - L^2 k)\theta = 0$$

$$\ddot{\theta} - \frac{b}{m}\dot{\theta} + \left(\frac{g}{L} - \frac{k}{m}\right)\theta$$

Problem 4 :

$$① \quad m\ddot{x} + kx - Lk\theta = f(t)$$

$$② \quad \frac{1}{3}m\ddot{\theta} - kL\dot{x} = 0$$

LT of ① : $X(s)(ms^2 + k) - Lk\,\Theta(s) = F(s)$ ③

LT of ② : $\frac{1}{3}m s^2 \Theta(s) - kLX(s)s = 0$ ④

Rearrange ④ $\Theta(s) = \frac{3kL\cancel{s}}{ms^{\cancel{2}}}X(s) = \frac{3kLX(s)}{ms}$ ⑤

Plug ⑤ → ③ $X(s)\left(ms^2 + k - \frac{3k^2L^2}{ms}\right) = F(s)$

$$\boxed{G(s) = \frac{X(s)}{F(s)} = \frac{ms}{(m^2s^3 + kms - 3k^2L^2)}}$$