EXAM 1 : Solutions

PART A : MULTIPLE CHOICE

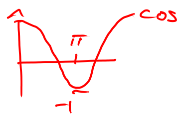
1. B , since response is typical of a second order, LTI system

2. C

3. D

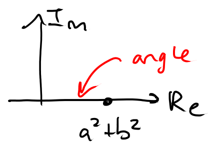
Euler's formula

$$i \cdot e^{i\pi} = i \cdot (\underbrace{\cos\pi}_{=-1} + i\underbrace{\sin\pi}_{=0})$$
$$= -i$$



4. A

$$z \cdot \bar{z} = (a+ib)(a-ib)$$
$$= a^2 + iab - iab - i^2 b$$
$$= a^2 + b^2$$



angle is zero hence $\arg(z\bar{z}) = 0$

5. A

Recall: $\mathcal{L}[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt$

Thus, if $\int_0^\infty f(t) e^{-st} dt = \frac{1}{s^2}$

then from the L.T. table (row 3)

$f(t)$ must be a ramp, $f(t) = t$

PART B : WORKOUT PROBLEMS

1) $\ddot{x} + \dot{x} - 2x = 0$, $x(0) = 4$, $\dot{x}(0) = -5$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1-4(-2)}}{2} = \frac{-1 \pm 3}{2} \Rightarrow \boxed{\begin{matrix}\lambda_1 = 1 \\ \lambda_2 = -2\end{matrix}}$$

Roots

General solution : (distinct, real)

$$x(t) = C_1 e^{t} + C_2 e^{-2t}$$

Particular solution

$$x(0) = C_1 + C_2 = 4 \Rightarrow C_1 = 4 - C_2$$

$$\dot{x}(0) = C_1 - 2C_2 = -5 \qquad 4 - C_2 - 2C_2 = -5$$

$$C_2 = (-5-4)/-3$$

$$\boxed{C_2 = 3}$$

$$C_1 = 4-3$$

$$\boxed{C_1 = 1}$$

Thus, $x(t) = e^{t} + 3e^{-2t}$

2) $$f(t) = (t+1)^2 e^{-3t}$$

$$= (t^2 + 2t + 1)e^{-3t}$$

$$F(s) = \underbrace{\mathcal{L}[t^2 e^{-3t}]}_{\text{row 9}} + 2\underbrace{\mathcal{L}[te^{-3t}]}_{\text{row 7}} + \underbrace{\mathcal{L}[e^{-3t}]}_{\text{row 6}}$$

$$= \frac{2}{(s+3)^3} + \frac{2}{(s+3)^2} + \frac{1}{(s+3)}$$

simplify

$$= \frac{2 + 2(s+3) + (s+3)^2}{(s+3)^3}$$

$$= \frac{2 + 2s + 6 + s^2 + 6s + 9}{(s+3)^3} = \frac{s^2 + 8s + 17}{(s+3)^3}$$

3)

$$F(s) = \underbrace{\mathcal{L}[u(t)]}_{\text{row 2}} + \underbrace{\mathcal{L}[\cosh(2t)]}_{\text{row 13}}$$

$$= \frac{1}{s} + \frac{s}{s^2-4} = \frac{s^2+s-4}{s(s^2-4)}$$

4)

$$\ddot{x} = 1 \qquad x(0) = a$$
$$\dot{x}(0) = b$$

Rewrite as 2nd order sys

Let $z_1 = x$
$z_2 = \dot{x}$

Then, $\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} z_2 \\ 1 \end{bmatrix}}_{F(z)\text{ vector}}$

Euler's Method, $h = 3$

$k=0 \qquad z_0 = \begin{bmatrix} a \\ b \end{bmatrix} \qquad f_0 = \begin{bmatrix} b \\ 1 \end{bmatrix}$

$k = 1 \qquad z_1 = z_0 + f_0 \cdot h$

$$= \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} b \\ 1 \end{bmatrix} 3$$

$$= \begin{bmatrix} a + 3b \\ b + 3 \end{bmatrix}$$