9-1

$$^{B}f = {}^{B}_{A}R\,{}^{A}f = \begin{bmatrix} 0.500 & 0.866 & 0.000 \\ -0.866 & 0.500 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} \begin{bmatrix} 4.0 \\ 0 \\ 5.0 \end{bmatrix} = \begin{bmatrix} 2.0 \\ -3.464 \\ 5.0 \end{bmatrix}$$

$$^{B}P_{A} = -{}^{B}_{A}R\,{}^{A}P_{B} = -\begin{bmatrix} 0.500 & 0.866 & 0.000 \\ -0.866 & 0.500 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} \begin{bmatrix} 5.0 \\ 0 \\ 10.0 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 4.33 \\ -10.0 \end{bmatrix}$$

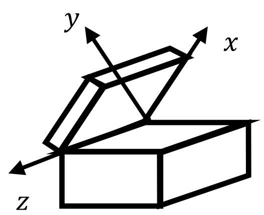
$$^{B}\tau = {}^{B}P_{A} \times {}^{B}f + {}^{B}_{A}R\,{}^{A}\tau = \begin{bmatrix} -2.5 \\ 4.33 \\ -10.0 \end{bmatrix} \times \begin{bmatrix} 2.0 \\ -3.464 \\ 5.0 \end{bmatrix} + \begin{bmatrix} 0.500 & 0.866 & 0.000 \\ -0.866 & 0.500 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{bmatrix} \begin{bmatrix} 0.0 \\ 3.0 \\ 0.0 \end{bmatrix}$$

$$= \begin{bmatrix} -10.362 \\ -6.0 \\ 0.0 \end{bmatrix}$$

六维力-力矩矢量 $^{B}F = \begin{bmatrix} 2.0 \\ -3.464 \\ 5.0 \\ -10.362 \\ -6.0 \\ 0.0 \end{bmatrix}$

9-2

坐标如下



	运动学	静力学
自然约束	$v_x = 0, v_y = 0, v_z = 0$ $\omega_x = 0, \omega_y = 0$	$\tau_z = 0$
人工约束	$\omega_z > 0$	$f_x = 0, f_y = 0, f_z = 0$ $\tau_x = 0, \tau_y = 0$

9-3

关节空间运动学：$M(\theta)\ddot{\theta}+V(\theta,\dot{\theta})+G(\theta)=\tau+J^T(\theta)F=\tau+J_a^T(\theta)F_a$

笛卡尔空间

动力学：$M_x(\theta)\ddot{x}+V_x(\theta,\dot{\theta})+G_x(\theta)=J_a^{-T}(\theta)\,\tau+F_a$

内环控制律：$\tau=J_a^T(\theta)[M_x(\theta)\ddot{x}+V_x(\theta,\dot{\theta})+G_x(\theta)-F_a]$

$\ddot{x}=a$

期望动态阻抗模型：$M_m(\ddot{x}-\ddot{x}_d)+D_m(\dot{x}-\dot{x}_d)+K_m(x-x_d)=F_a$

外环控制律：$a=\ddot{x}_d+M_m^{-1}[F_a+D_m(\dot{x}_d-\dot{x})+K_m(x_d-x)]$

根据$\begin{cases} M_x(\theta)=J_a^{-T}(\theta)M(\theta)J_a^{-1}(\theta) \\ V_x(\theta,\dot{\theta})=J_a^{-T}(\theta)V(\theta,\dot{\theta})-M_x(\theta)\,\dot{J}_a(\theta)\dot{\theta} \\ G_x(\theta)=J_a^{-T}(\theta)G(\theta) \end{cases}$

$\because \dot{x}=J_a(\theta)\theta$

$\therefore \ddot{x}=\dot{J}_a(\theta)\dot{\theta}+J_a(\theta)\ddot{\theta}$

$$\begin{aligned}\therefore \tau=&J_a^T(\theta)\big[J_a^{-T}(\theta)M(\theta)J_a^{-1}(\theta)\ddot{x}+J_a^{-T}(\theta)V(\theta,\dot{\theta})-J_a^{-T}(\theta)M(\theta)J_a^{-1}(\theta)\dot{J}_a(\theta)\dot{\theta}+J_a^{-T}(\theta)G(\theta)\\&-F_a\big]\\&=M(\theta)J_a^{-1}(\theta)\{\ddot{x}_d+M_m^{-1}[D_m(\dot{x}_d-\dot{x})+K_m(x_d-x)]-\dot{J}_a(\theta)\dot{\theta}\}+V(\theta,\dot{\theta})\\&+G(\theta)+[M(\theta)J_a^{-1}(\theta)M_m^{-1}-J_a^T(\theta)]F_a\\&=M(\theta)J_a^{-1}(\theta)\{\ddot{x}_d+M_m^{-1}[D_m(\dot{x}_d-\dot{x})+K_m(x_d-x)]-\dot{J}_a(\theta)\dot{\theta}\}+V(\theta,\dot{\theta})\\&+G(\theta)+J_a^T(\theta)[M_x(\theta)M_m^{-1}-I]F_a\end{aligned}$$

9-4

(1)

	运动学	静力学
自然约束	$v_z=0$ $\omega_x=0,\omega_y=0$	$f_x=0,f_y=0,\tau_z=0$
人工约束	$v_x=a,v_y=\sqrt{v_0^2-a^2},\omega_z=0$	$f_z=0$ $\tau_x=0,\tau_y=0$

(2)

$\omega_z=0,\tau_x=0,\tau_y=0$自动满足

9-5

动力学模型：$M\ddot{q}+G=\tau+F$

其中：$M=\begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_2\end{bmatrix}$ $G=\begin{bmatrix}0\\m_2g\end{bmatrix}$ $\tau=\begin{bmatrix}\tau_1\\\tau_2\end{bmatrix}$ $F=\begin{bmatrix}F_x\\F_y\end{bmatrix}$

期望阻抗模型：$M_m(\ddot{x}-\ddot{x}_d)+D_m(\dot{x}-\dot{x}_d)+K_m(x-x_d)=F$，其中$x=\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}d_1\\d_2\end{bmatrix}$

取$e=x-x_d, M_m=\begin{bmatrix}M_{m,x} & 0\\ 0 & M_{m,y}\end{bmatrix}>0, D_m=\begin{bmatrix}D_{m,x} & 0\\ 0 & D_{m,y}\end{bmatrix}>0, M_m=\begin{bmatrix}K_{m,x} & 0\\ 0 & K_{m,y}\end{bmatrix}>0$

因为没有力矩传感器，所以取$M_m=M$，即$\begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_2\end{bmatrix}=\begin{bmatrix}M_{m,x} & 0\\ 0 & M_{m,y}\end{bmatrix}$

那么：

$$\tau=Mx_d+G-D_m\dot{e}-K_me=\begin{bmatrix}(m_1+m_2)\ddot{x}_d\\ m_2\ddot{y}_d\end{bmatrix}+\begin{bmatrix}0\\m_2g\end{bmatrix}+\begin{bmatrix}D_{m,x}(\dot{x}_d-\dot{x})+K_{m,x}(x_d-x)\\ D_{m,y}(\dot{y}_d-\dot{y})+K_{m,y}(y_d-y)\end{bmatrix}$$

由期望阻抗模型得到：

$$(Ms^2 + D_m s + K_m)e(s) = F(s)$$

$$\therefore Is^2 + M^{-1}D_m s + M^{-1}K_m = \begin{bmatrix} (s+\lambda)^2 & 0 \\ 0 & (s+\lambda)^2 \end{bmatrix}$$

$$\therefore D_{m,x} = 2(m_1 + m_2)\lambda, D_{m,y} = 2m_2\lambda, K_{m,x} = (m_1 + m_2)\lambda^2, K_{m,y} = m_2\lambda^2$$