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Chapter 8

8-1

解：联立得

$$m\ddot{\theta} + b\dot{\theta}^2 + c\dot{\theta} + q\theta^2 - k\theta = m[\ddot{\theta}_d + k_v\dot{\theta} + k_p(\theta_d-\theta)] + \cos\theta + \sin(\theta_d-\theta)$$

$$\Rightarrow \ddot{\theta} = \ddot{\theta}_d + k_v\dot{\theta} + k_p(\theta_d-\theta) + \frac{[\cos\theta + \sin(\theta_d-\theta) - b\dot{\theta}^2 - c\dot{\theta} - q\theta^2 + k\theta]}{m}$$

8-3

解：设计控制律 $\tau = 2u + \dot{\theta} + 7\cos\theta - 3\sin\theta$

则代入原动力学方程有 $\ddot{\theta} = u$ ①

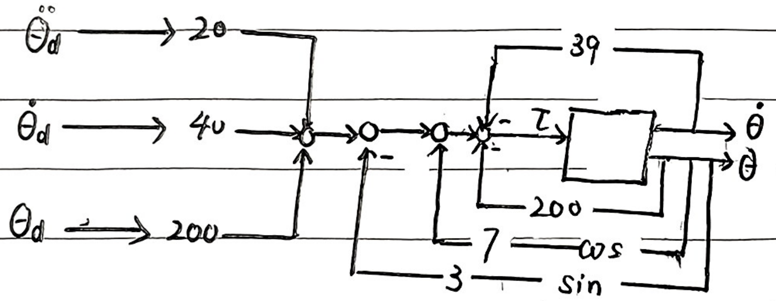
设误差 $\tilde{\theta} = \theta_d - \theta$，即设计u满足 $\ddot{\tilde{\theta}} + 20\dot{\tilde{\theta}} + 100 = 0$ ②

①代入②得

$$u = \ddot{\theta}_d + 20(\dot{\theta}_d - \dot{\theta}) + 100(\theta_d - \theta)$$

则 $\tau = 2\ddot{\theta}_d + 40(\dot{\theta}_d - \dot{\theta}) + 200(\theta_d - \theta) + \dot{\theta} + 7\cos\theta - 3\sin\theta$

如图所示：



8-8

解：

令 $\varphi_1 = m_1 l_1^2 + I_{zz1} + I_{yy2} + m_2 d_2^2$

$\varphi_2 = \cancel{4}m_2$ 通常不带数字？

$\varphi_3 = m_1 l_1 g$

$\varphi_4 = m_2 g$

则 $$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \ddot{\theta}_1 & 2d_2\dot{\theta}_1\dot{d}_2 & \cos\theta_1 & d_2\cos\theta_1 \\ 0 & \ddot{d}_2 - d_2\dot{\theta}_1^2 & 0 & \sin\theta_1 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix}$$

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8-9

由P177式8-121：内环控制律 $\tau_d = Y(\Phi,\dot{\Phi},\alpha_\Phi)\hat{\psi} = \hat{M}(\Phi)\alpha_\Phi + \hat{C}(\Phi,\dot{\Phi})\dot{\Phi} + \hat{L}\dot{\Phi} + \hat{G}(\Phi)$

由P171式8-93：外环控制律 $\alpha_\Phi = \ddot{\Phi}_d + K_D\dot{\tilde{\Phi}} + K_P\tilde{\Phi}$

被控对象模型：$M(\Phi)\ddot{\Phi} + C(\Phi,\dot{\Phi})\dot{\Phi} + L\dot{\Phi} + G(\Phi) = Y(\Phi,\dot{\Phi},\ddot{\Phi})\psi$ （P175 式8-20）

联立有：$Y(\Phi,\dot{\Phi},\ddot{\Phi})\psi = \hat{M}(\Phi)\alpha_\Phi + \underline{\hat{C}(\Phi,\dot{\Phi})\dot{\Phi} + \hat{L}\dot{\Phi} + \hat{G}(\Phi)}$

两侧同加 $\hat{M}(\Phi)\ddot{\Phi}$：$Y(\Phi,\dot{\Phi},\ddot{\Phi})\psi + \hat{M}(\Phi)\alpha_\Phi + Y(\Phi,\dot{\Phi},\ddot{\Phi})\hat{\psi}$

代入 α_Φ：$\ddot{\tilde{\Phi}} = -K_D\dot{\tilde{\Phi}} - K_P\tilde{\Phi} + \hat{M}^{-1}(\Phi)Y(\Phi,\dot{\Phi},\ddot{\Phi})\tilde{\psi}$

$$\therefore \dot{\varphi} = \begin{bmatrix}\dot{\tilde{\Phi}}\\ \ddot{\tilde{\Phi}}\end{bmatrix} = \begin{bmatrix}0 & I\\ -K_P & -K_D\end{bmatrix}\begin{bmatrix}\tilde{\Phi}\\ \dot{\tilde{\Phi}}\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix}\hat{M}^{-1}(\Phi)Y(\Phi,\dot{\Phi},\ddot{\Phi})\tilde{\psi}$$

why

$$= \bar{A}\varphi + \bar{D}\tilde{\psi}$$

8-10

由P177式8-124和8-125、8-126

$$\dot{V}_L(\varphi,\tilde{\psi}) = \dot{\varphi}^T P_L\varphi + \varphi^T P_L\dot{\varphi} + \dot{\tilde{\psi}}^T\Gamma\tilde{\psi} + \tilde{\psi}^T\Gamma\dot{\tilde{\psi}}$$

$$= (\bar{A}\varphi + \bar{D}\tilde{\psi})^T P_L\varphi + \varphi^T P_L(\bar{A}\varphi + \bar{D}\tilde{\psi}) + (-\Gamma^{-1}\bar{D}^T P_L\varphi)^T\Gamma\tilde{\psi} + \tilde{\psi}^T\Gamma(-\Gamma^{-1}\bar{D}^T P_L\varphi)$$

$$= \varphi^T\bar{A}^T P_L\varphi + \tilde{\psi}^T D^T P_L\varphi + \varphi^T P_L\bar{A}\varphi + \varphi^T P_L\bar{D}\tilde{\psi} - \varphi^T P_L^T\bar{D}(\Gamma^{-1})^T\Gamma\tilde{\psi} - \tilde{\psi}^T D^T P_L\varphi$$

$$= \varphi^T(\bar{A}^T P_L + P_L\bar{A})\varphi$$

$$= -\varphi^T Q_L\varphi$$