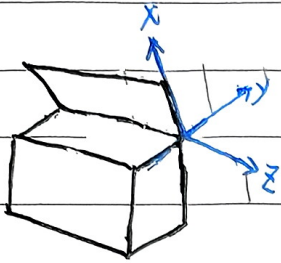


9-2



	运动学	静力学
自然约束	$V_x = V_y = V_z = 0$ $\omega_x = \omega_y = 0$	$\tau_z = 0$
人工约束	$\omega_z < 0$	$f_x = f_y = f_z = 0$ $\tau_x = 0, \tau_y = 0$

9-3

由关节空间运动学: $M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) = \tau + J_a^T(\theta)F_a$ ①笛卡尔空间: $M_x(\theta)\ddot{x} + V_x(\theta, \dot{x}) + G_x(\theta) = J_a^{-T}(\theta)\tau + F_a$ ②内环控制律: $\tau = J_a^T(\theta)[M_x(\theta)\ddot{q}_d + V_x(\theta, \dot{\theta}) + G_x(\theta) - F_a]$ ③, 代入②得闭环(双积分)系统闭环系统: $\ddot{x} = \ddot{q}_d$ ④期望阻抗关系: $M_d\ddot{\tilde{x}} + B_d\dot{\tilde{x}} + K_d\tilde{x} = F_a$ ⑤则取外环控制律: $\ddot{q}_d = \ddot{\tilde{x}} + M_d^{-1}(-B_d\dot{\tilde{x}} - K_d\tilde{x} + F_a)$ ⑥根据:
$$\begin{cases} M_x(\theta) = J_a^{-T}(\theta) M(\theta) J_a^{-T}(\theta) & ⑦ \\ V_x(\theta, \dot{\theta}) = J_a^{-T}(\theta) V(\theta, \dot{\theta}) + M_x(\theta) \dot{J}_a^{-T}(\theta) \dot{\theta} & ⑧ \\ G_x(\theta) = J_a^{-T}(\theta) G(\theta) & ⑨ \end{cases}$$
 $\therefore \dot{x} = J_a(\theta)\dot{\theta}$ $\ddot{x} = \dot{J}_a(\theta)\dot{\theta} + J_a(\theta)\ddot{\theta}$ ⑩ \therefore 联立②~⑩得

$$\tau = M(\theta) J_a^{-T}(\theta) (\ddot{\tilde{x}} - \dot{J}_a^{-T}(\theta) \dot{\theta} + M_d^{-1}(-B_d\dot{\tilde{x}} - K_d\tilde{x} + F_a)) + V(\theta, \dot{\theta}) + G(\theta) + J_a^T(\theta) [M_x(\theta) M_d^{-1} - I] F_a$$



9-5

动力学模型 $M\ddot{x} + G = \tau + F$ 对x轴方向 $(m_1 + m_2)a = \tau$ 对y轴方向 $m_2 g + m_2 a = \tau_2 + F$

$$M = \begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ m_2 g \end{bmatrix} \quad \tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad F = \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

其期望阻抗模型 $M_m(\ddot{x} - \ddot{x}_d) + D_m(\dot{x} - \dot{x}_d) + K_m(x - x_d) = F$ 其中 $x = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$

$$\text{取 } e = x - x_d, \quad M_m = \begin{bmatrix} M_{m,x} & 0 \\ 0 & M_{m,y} \end{bmatrix} > 0, \quad D_m = \begin{bmatrix} D_{m,x} & 0 \\ 0 & D_{m,y} \end{bmatrix} > 0$$

$$K_m = \begin{bmatrix} K_{m,x} & 0 \\ 0 & K_{m,y} \end{bmatrix} > 0$$

因为无扭矩传感器, 取 $M_m = M \Rightarrow \text{why?}$

$$\text{则 } \tau = M\ddot{x}_d + G + D_m\dot{e} - K_me$$

$$\text{又有 } F(s) = (Ms^2 + D_ms + K_m)e(s)$$

$$\therefore \underbrace{(I s^2 + M^{-1} D_m s + M^{-1} K_m)}_{\substack{\text{化为} \\ 1}} = \begin{bmatrix} (s+\lambda)^2 & 0 \\ 0 & (s+\lambda)^2 \end{bmatrix}$$

$$\therefore D_{m,x} = 2(m_1 + m_2)\lambda, \quad D_{m,y} = 2m_2\lambda$$

$$K_{m,x} = 0(m_1 + m_2)\lambda^2, \quad K_{m,y} = m_2\lambda^2$$

