# Occupancy Grid Mapping

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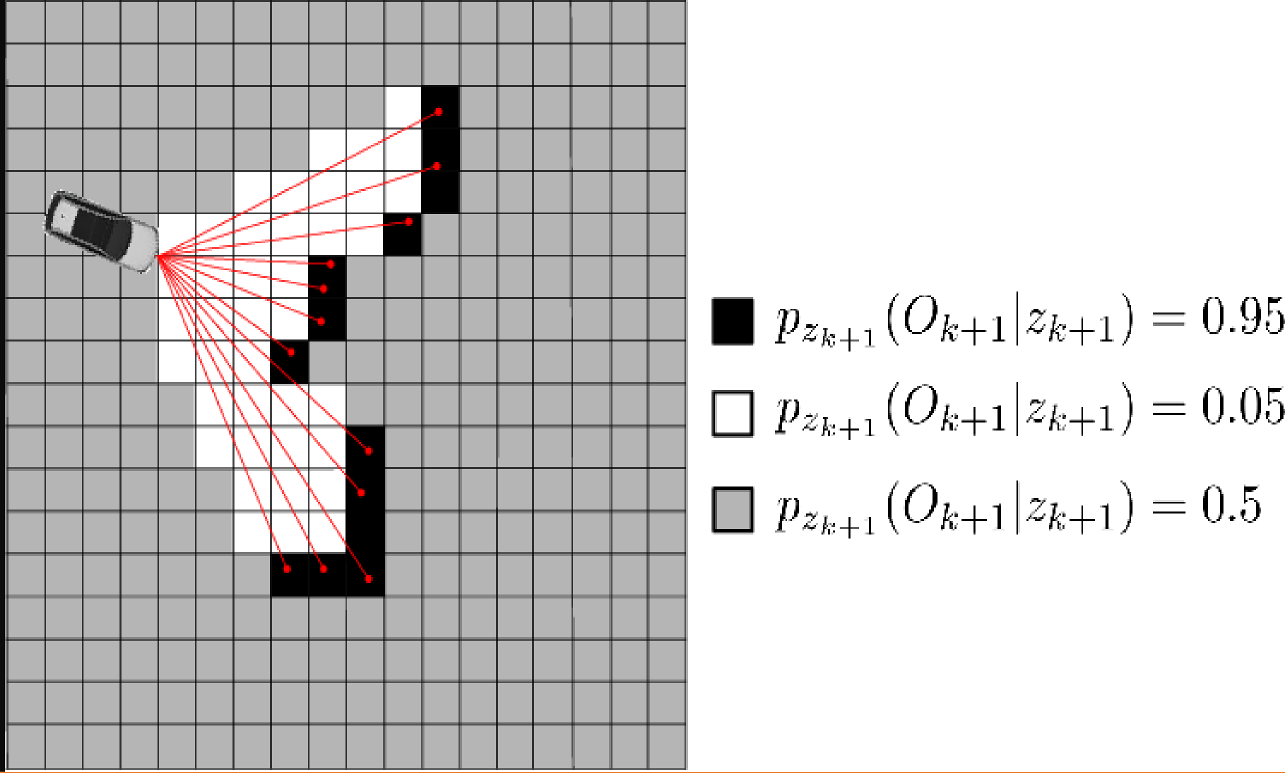
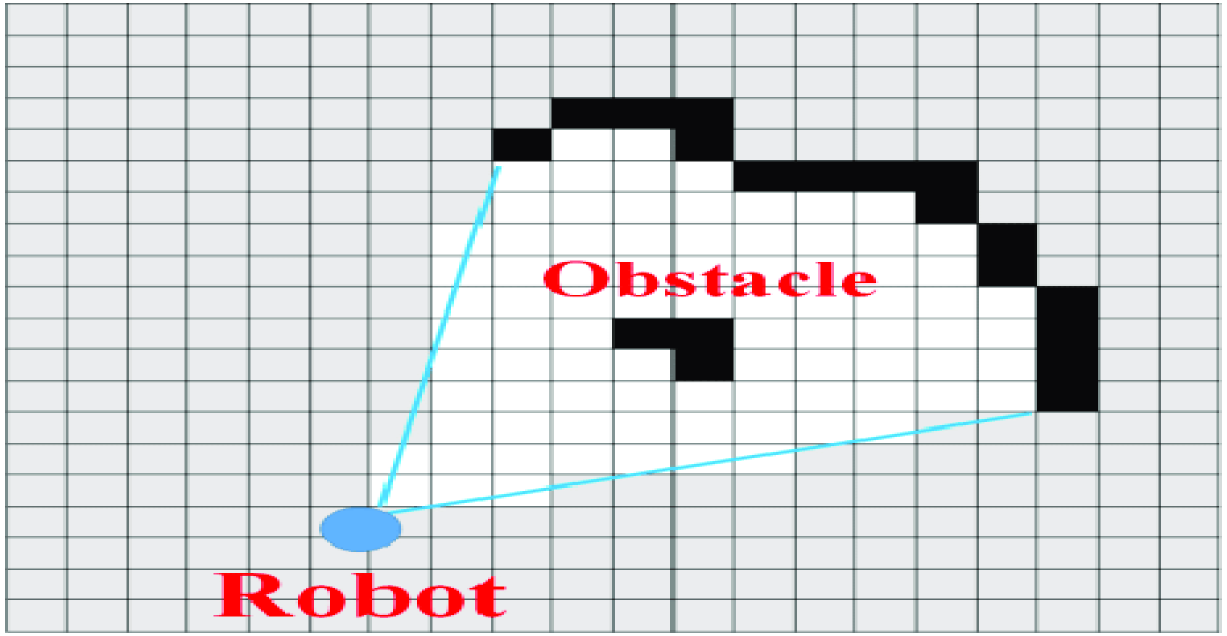


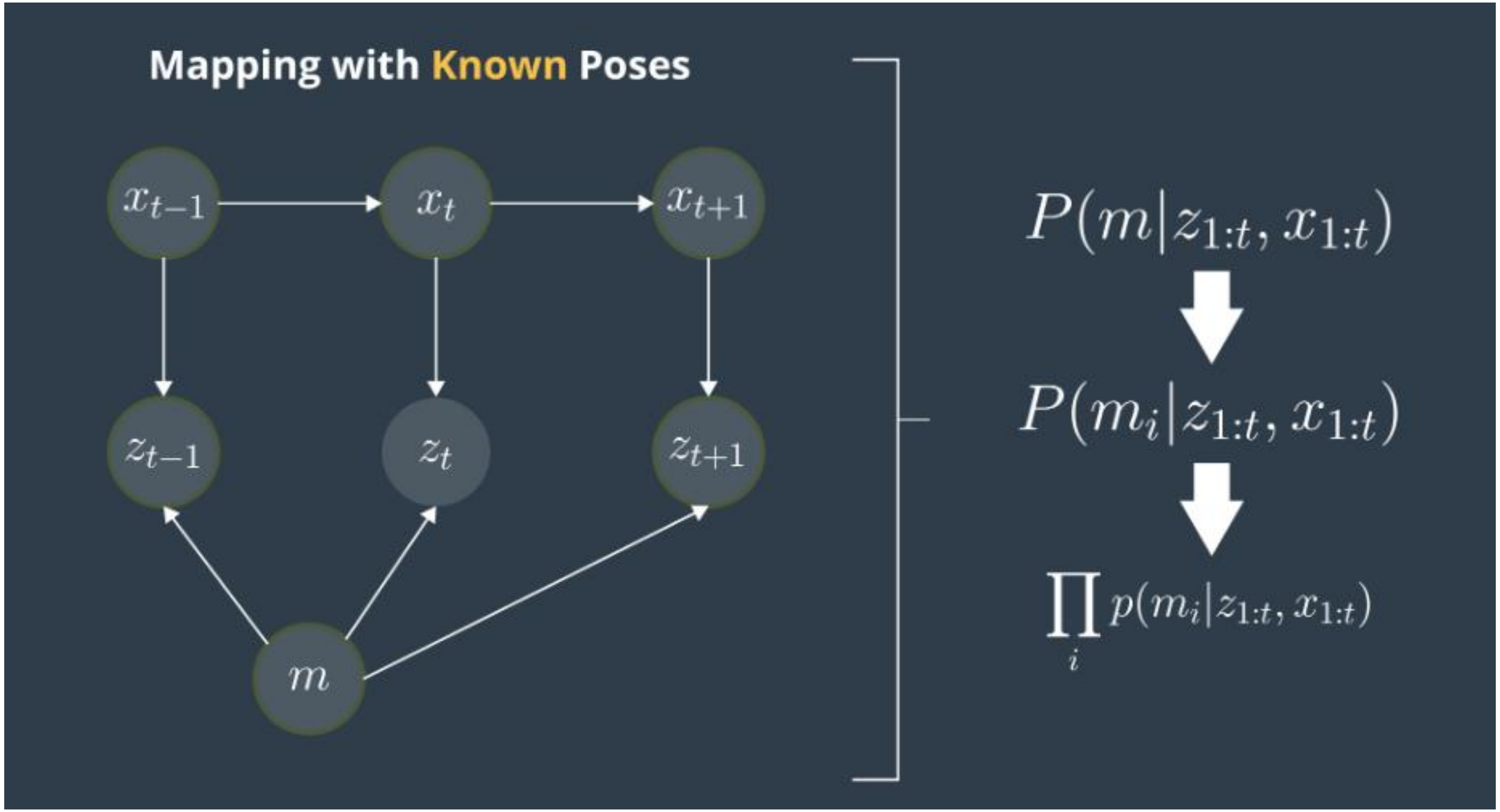
# By the end of this Presentation, you will be able to:

- Know the odds term
- Be able to know how log-odds term affect our mapping algorithm
- Apply Bayesian filter
- Know occupancy grid mapping algorithm
- Know the inverse sensor model

# Occupancy Grid Mapping | Equations

- *First Approach*: $P(m|z1{:}t,x1{:}t)$
- *Second Approach*: $P(mi|z1{:}t,x1{:}t)$
- A second or better approach to estimating the posterior map is to decompose this problem into many separate problems. In each of these problems, we will compute the posterior map $\mathbf{m}_i$ at each instant.
- *Third Approach*: $\prod iP(mi|z1{:}t,x1{:}t)$
- Now, the third approach is the best approach to computing the posterior map by relating cells and overcoming the huge computational memory, is to estimate the map with the product of marginals or factorization.

# Mapping



Mapping with Known Poses
$x_{t-1}$
$x_t$
$x_{t+1}$
$z_{t-1}$
$z_t$
$z_{t+1}$
$m$
$P(m|z_{1:t}, x_{1:t})$
$P(m_i|z_{1:t}, x_{1:t})$
$\prod_i p(m_i|z_{1:t}, x_{1:t})$

# Mapping

$$Odd := \frac{(X\ happens)}{(X\ not\ happens)} = \frac{p(X)}{p(X^c)}$$

- More convenient when we use “Odd”

$$Odd((m_{x,y} = 1)\ given\ z) = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)}$$

# Mapping | Explanation

- Odd

(Bayes' Rule)

$$p(m_{x,y} = 1|z) = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)}{p(z)}$$

$$Odd = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)/p(z)}{p(m_{x,y} = 0|z)}$$

# Mapping | Explanation

- Odd

$$Odd = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)/\cancel{p(z)}}{p(z|m_{x,y} = 0)p(m_{x,y} = 0)/\cancel{p(z)}}$$

$$p(m_{x,y} = 0|z) = \frac{p(z|m_{x,y} = 0)p(m_{x,y} = 0)}{p(z)}$$

(Bayes' Rule)

# Mapping | Explanation

- Take the log!

Odd:

$$\frac{p(m_{x,y}=1|z)}{p(m_{x,y}=0|z)}=\frac{p(z|m_{x,y}=1)p(m_{x,y}=1)}{p(z|m_{x,y}=0)p(m_{x,y}=0)}$$

**Log-Odd:**

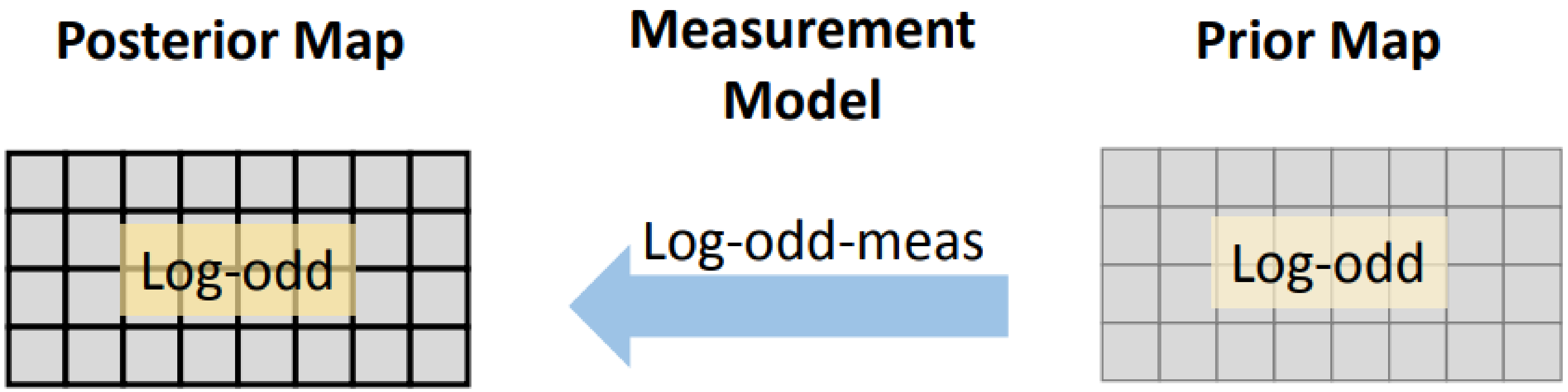
$$\log\frac{p(m_{x,y}=1|z)}{p(m_{x,y}=0|z)}=\log\frac{p(z|m_{x,y}=1)p(m_{x,y}=1)}{p(z|m_{x,y}=0)p(m_{x,y}=0)}$$

$$=\log\frac{p(z|m_{x,y}=1)}{p(z|m_{x,y}=0)}+\log\frac{p(m_{x,y}=1)}{p(m_{x,y}=0)}$$

$$\boxed{\log odd^{+}=\log odd\ meas+\log odd^{-}}$$

# Mapping | Explanation

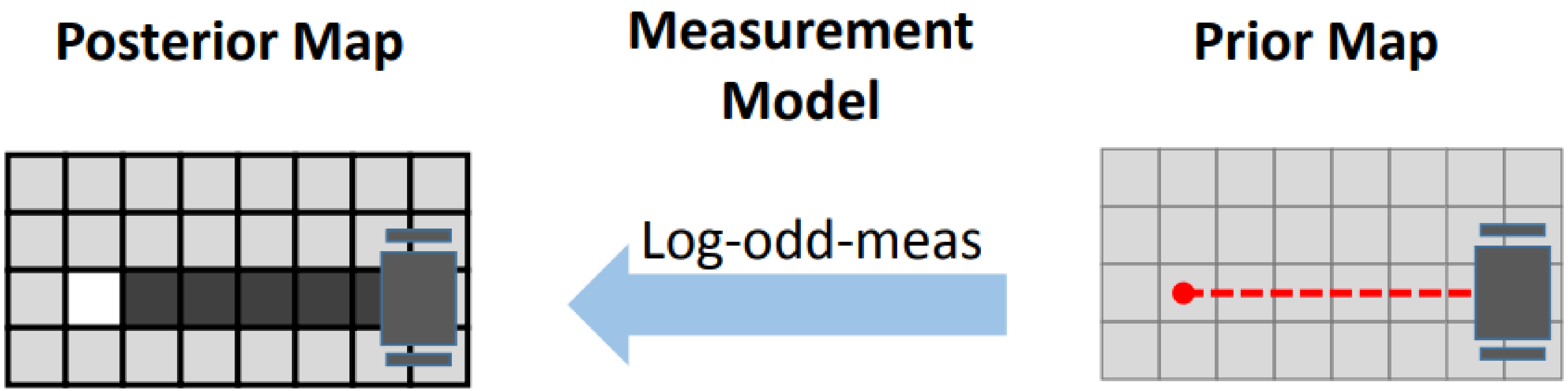
- Log-odd update




$$\log odd^{+} = \log odd\ meas + \log odd^{-}$$

# Mapping | Explanation

- Log-odd update




$$\log odd^{+} = \log odd\ meas + \log odd^{-}$$

# Mapping | Explanation

- Measurement model in log-odd form

$$\log \frac{p(z|m_{x,y}=1)}{p(z|m_{x,y}=0)}$$

- **Two possible measurement:**

<u>Case I : cells with z=1</u>

$$\log odd\_occ := \log \frac{p(z=1|m_{x,y}=1)}{p(z=1|m_{x,y}=0)}$$

<u>Case II : cells with z=0</u>

$$\log odd\_free := \log \frac{p(z=0|m_{x,y}=0)}{p(z=0|m_{x,y}=1)}$$

(Trivial Case : cells not measured)

# Mapping | Explanation

- Example

<u>Constant Measurement Model</u>

$$\log odd\_occ := 0.9$$

$$\log odd\_free := 0.7$$

<u>Initial Map:</u>

$\log odd = 0$ for all (x,y)

$$\Updownarrow$$

$$p(m_{x,y} = 1) = p(m_{x,y} = 0) = 0.5$$

**Update Rule:**

$$\boxed{\log\ odd \mathrel{+}= \log odd\_meas}$$

t0

# Mapping | Explanation

- Example

Constant Measurement Model

$$\log odd\_occc := 0.9$$

$$\log odd\_free := 0.7$$

Update

- Case I : cells with z=1

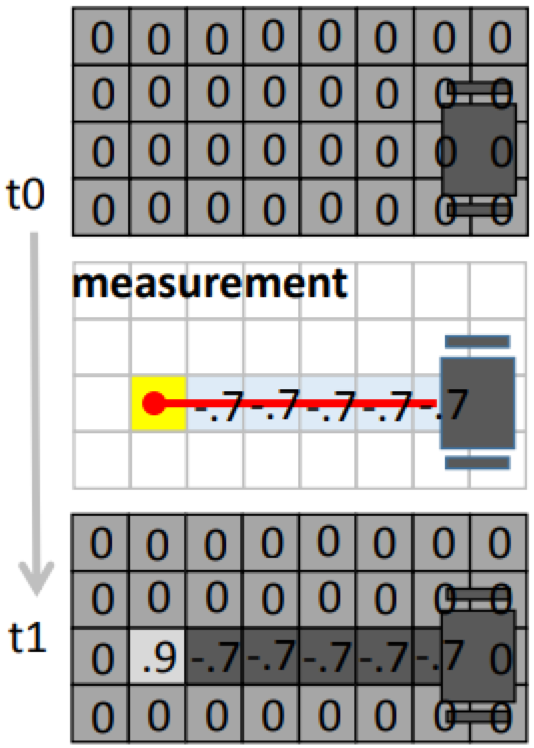
$$\log odd \leftarrow 0 + \log odd\_occ$$

- Case II : cells with z=0

$$\log odd \leftarrow 0 - \log odd\_free$$

**Update Rule:**

$$\log\ odd \mathrel{+}= \log odd\_meas$$



# Mapping | Explanation

- Example

<u>Constant Measurement Model</u>

$\log odd\_occ := 0.9$

$\log odd\_free := 0.7$

<u>Update</u>

- Case I : cells with z=1

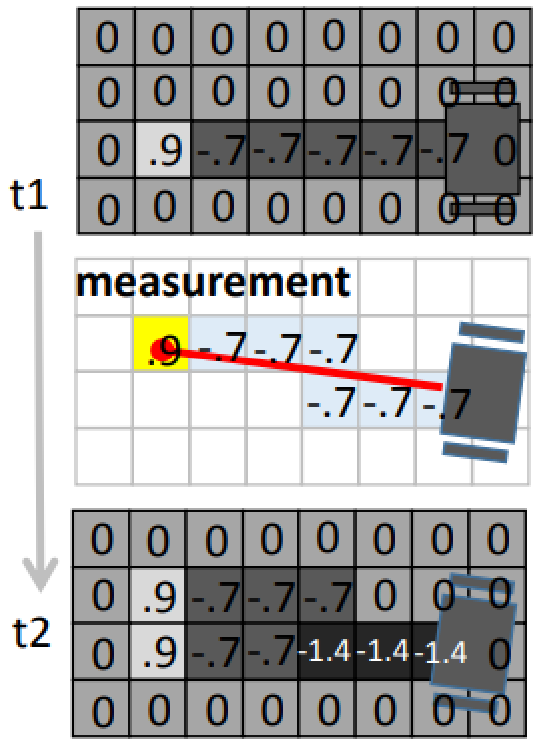
$\log odd \leftarrow 0 + \log odd\_occ$

- Case II : cells with z=0

$\log odd \leftarrow 0 - \log odd\_free$

**Update Rule:**

$$\boxed{\log odd \mathrel{+}= \log odd\_meas}$$



# Mapping | Explanation

Algorithm Binary_Bayes_Filter $(l_{t-1}, z_t)$:

$$\underbrace{l_t}_{\text{New Belief}} = \underbrace{l_{t-1}}_{\text{Previous Belief}} + \underbrace{log \frac{p(x|z_t)}{1 - p(x|z_t)}}_{\text{Log Odd of the Inverse Measurements Model}} - \underbrace{log \frac{p(x)}{1 - p(x)}}_{\text{Initial Belief}}$$

return $l_t$

# Mapping | Explanation

Algorithm Occupancy_Grid_Mapping $(\{l_{t-1,i}\}, x_t, z_t)$:

for all cells $m_i$ do

if $m_i$ in perceptual field of $z_t$ then

$$l_{t,i} = l_{t-1,i} + Inverse\_Sensor\_Model(m_i, x_t, z_t) - l_0$$

else

$$l_{t,i} = l_{t-1,i}$$

endif

endfor

# Mapping | Explanation

Algorithm Inverse_Sensor_Model($m_i$, $x_t$, $z_t$):

Let $x_i$, $y_i$ be the center of mass of $m_i$

$$r = \sqrt{(x_i - x)^2 + (y_i - y)^2}$$

$$\Phi = atan2(y_i - y, x_i - x) - \theta$$

$$k = argmin_j |\Phi - \theta_{j,sens}|$$

$if\ r > min(z_{max}, z_t^k + \alpha/2)\ or\ |\Phi - \theta_{k,sens}| > \beta/2\ then$

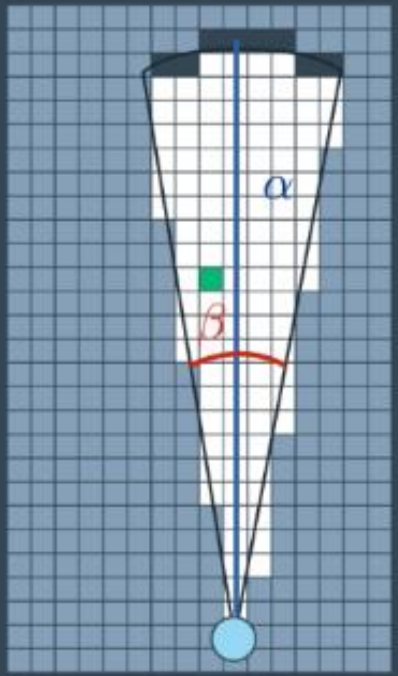
$\quad return\ l_0$

$if\ z_t^k < z_{max}\ and\ |r - z_t^k| < \alpha/2$

$\quad return\ l_{occ}$

$if\ r \leq z_t^k$

$\quad return\ l_{free}$



# Review

- Know the effect of log odds notation
- Know the occupancy grid mapping algorithm
- Show how the Bayesian filtering is used in mapping

# By the end of this session you should be able to:

- Know how continuous maps are represented using grid maps
- Probabilities represented for occupied and unoccupied region
- Types of map
- Known position of the robot