Quiz 1

Suppose that $f''(c)=0$, what are the sufficient conditions that c must furnish to be a relative minimum?

If $x=c$ is a stationary point and $f''(c)=0$, you can find a relative minimum using the limit definition of a minimum, so c must meet these conditions:

$$\begin{cases} \lim\limits_{x \to c^-} f'(x) < 0 \\ \lim\limits_{x \to c^+} f'(x) > 0 \end{cases}$$

5 pts

HW1

Prove that we have the commutativity, $x+y=y+x$, and the associativity $x+(y+z)=(x+y)+z$

say $x=[x_1, x_2]$ $y=[y_1, y_2]$

$\therefore [x_1+y_1, x_2+y_2]=[y_1+x_1, y_2+x_2]$

order will not affect the outcome in addition of vectors

ex. with numbers $x=[1,2]$ $y=[3,4]$

$[1+3, 2+4]=[3+1, 4+2]=[4,6]$ ✓

- 3 pts

$x+(y+z)=(x+y)+z$

say $x=[x_1, x_2]$ $y=[y_1, y_2]$ $z=[z_1, z_2]$

$[x_1+(y_1+z_1), x_2+(y_2+z_2)]=[(x_1+y_1)+z_1, (x_2+y_2)+z_2]$

ex with numbers $x=[1,2]$ $y=[3,4]$ $z=[5,6]$

$[1+(3+5), 2+(4+6)]=[(1+3)+5, (2+4)+6]$

$[1+8, 2+10]=[4+5, 6+6]$

$[9,12]=[9,12]$ ✓

HW2

Just as we showed the addition property of two vectors above, show the subtraction properties of two vectors x and y.

* basing answer off of (3.1.3) from class notes as I assume this is what is being referenced as "above."

$$x - y = \begin{bmatrix} x_1 - y_1 \\ x_2 - y_2 \\ \vdots \\ x_N - y_N \end{bmatrix}$$

- 3 pts

ex. say $x = [15, 10, 1]$ $y = [-3, 6, 9]$

$x - y = [15 + 3, 10 - 6, 1 - 9]$

$x - y = [18, 4, -8]$

$$x - y = [x_1 - y_1, x_2 - y_2, x_3 - y_3]$$

Hw 3

Prove that $\langle ax+by, ax+by\rangle$
$= a^2\langle x,x\rangle + 2ab\langle x,y\rangle + b^2\langle y,y\rangle$
is a non-negative quadratic form in the scalar variables a and b if x and y are real.

$$a^2\langle x,x\rangle + 2ab\langle x,y\rangle + b^2\langle y,y\rangle$$

$$Q(u,v) = a\left(u + \frac{bv}{a}\right)^2 + \left(c - \frac{b^2}{a}\right)v^2$$

$\therefore\ a = \langle x,x\rangle \quad b = \langle x,y\rangle \quad c = \langle y,y\rangle \quad u = a \quad v = b$

$$\langle x,x\rangle\left(a + \frac{\langle x,y\rangle b}{\langle x,x\rangle}\right)^2 + \left(\langle y,y\rangle - \frac{\langle x,y\rangle^2}{\langle x,x\rangle}\right)b^2$$

For $Q(u,v) > 0$ then $a > 0$ and $c - \frac{b^2}{a} > 0$

For $Q(a,b) > 0$

Prove $\langle x,x\rangle > 0$ and $\langle y,y\rangle - \frac{\langle x,y\rangle^2}{\langle x,x\rangle} > 0$

$= \sum_{i=1}^{N} x_i^2 > 0$

true as long as $\langle x,x\rangle \neq 0$

$$\sum_{i=1}^{N} y_i^2 - \frac{\left(\sum x_i y_i\right)^2}{\sum_{i=1}^{N} x_i^2}$$

all summations come out positive due to being squared so this is non-negative as long as

$$\langle x,x\rangle\langle y,y\rangle > \langle x,y\rangle^2$$

The Cauchy-Schwarz Inequality

What are you doing here?

- 3pts

Hw 4

Hence, show that for real-valued vectors x and y, that the Cauchy-Schwarz Inequality $\langle x,y\rangle^2 \le \langle x,x\rangle\langle y,y\rangle$ holds.

case 1: $x = 0$

$$\left(\sum_{i=1}^{N} \cancel{x_i y_i}\right)^2 \le \cancel{\sum_{i=1}^{N} x_i^2} \cdot \sum_{i=1}^{N} y_i^2$$

$0 \qquad\qquad 0$

$$\therefore \quad 0 = 0$$

- 3 pts

case 2: x and $y \neq 0$

so $$0 \le \left(\sum_{i=1}^{N} x_i^2\right)\left(\sum_{i=1}^{N} y_i^2\right) - \left(\sum_{i=1}^{N} x_i y_i\right)^2 \quad \checkmark$$

and

$$0 \ge \left(\sum_{i=1}^{N} x_i y_i\right)^2 - \left(\sum_{i=1}^{N} x_i^2\right)\left(\sum_{i=1}^{N} y_i^2\right) \quad \checkmark$$

HW5

Using the above result, show that for any two complex vectors x and y, $|\langle x,y\rangle|^2 \leq \langle x,\bar{x}\rangle\langle y,\bar{y}\rangle$

$\sum x_i \bar{x}_i$ $\qquad$ $\sum y_i \bar{y}_i$ $\qquad$ $\left|\sum_{i=1}^{N} x_i y_i\right|^2$

$\sum a_i^2 + b_i^2$ $\qquad$ $\sum c_i^2 + d_i^2$

$$\left|\sum_{i=1}^{N} x_i y_i\right|^2 \leq \left(\sum_{i=1}^{N} a_i^2 + b_i^2\right)\left(\sum_{i=1}^{N} c_i^2 + d_i^2\right)$$

The square is outside the summation so you can have negative values inside

values on this side are squared so you won't get a negative value so this side won't be smaller than the left side

- 4 pts

HW6

show that the triangle inequality

$$\langle x+y, x+y\rangle^{1/2} \leq \langle x, x\rangle^{1/2} + \langle y, y\rangle^{1/2}$$

holds for any two real-valued variables - 3 pts

any 2 sides of a triangle added must be greater than or equal to the 3rd side

$$\left(\sum_{i=1}^{N}(x_i+y_i)^2\right)^{1/2} \leq \left(\sum_{i=1}^{N} x_i^2\right)^{1/2} + \left(\sum_{i=1}^{N} y_i^2\right)^{1/2}$$

This is not how you prove an expression.

- If x and y are both ≥ 0 then this statement is true. Also true if x and y are both ≤ 0

- If only x or y are < 0 and the other is > 0 then this means the right hand side of the inequality will be > left.

Because the left side allows for subtraction inside the summation, while the right side does not. This allows the right side to grow larger