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A short treatise on robots' geometry, kinematics, and dynamics.

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Dissemination Venue: Microsoft Research RL Group,

New York City, NY 10012

June 09, 2022

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Standard Texts – Modeling and Control

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Robot Modeling and Control

Spong, Mark W., Seth Hutchinson, and Mathukumalli Vidyasagar. Robot modeling and control. Vol. 3. New York: Wiley, 2006.

Mathematical Modeling of Robots

Murray, R. M., Li, Z., & Sastry, S. S. (1994). A Mathematical Introduction to Robotic Manipulation. In Book (Vol. 29). https://doi.org/10.1.1.169.3957

Texts – Modeling, Control, and Mechanisms

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Robot Modeling and Control

Lynch, K. M., & Park, F. C. (2017). Modern Robotics Mechanics, Planning, and Control.

Mechanisms' Kinematic Geometry

Hunt, Kenneth H., and Kenneth Henderson Hunt. Kinematic geometry of mechanisms. Vol. 7. Oxford University Press, USA, 1978.

Texts – Screws and Kinematics

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Screw Theory

Ball, Robert Stawell. A Treatise on the Theory of Screws. Cambridge university press, 1998.

Mechanisms' Kinematic Geometry

Hunt, K. H. (2019). Structural Kinematics of In-Parallel-Actuated Robot-Arms. 105(December 1983), 705–712.

Outline

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Mechanism Components

Kinematic geometry. Mechanisms.

Joints: Joint closure; Pairs; Couplings.

Lower pairs and linkages; Higher and lower pairs.

Motions: Planar and spherical motions.

Synthesis: Type-, number-, and size-syntheses.

Preamble.

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Mechanics

Mechanics is an indirect study of nature via bodies – essentially the mathematical abstractions of common natural things; the mass is an *allocation* in *place* to each body; geometry, deals with the theory of places.

Geometry

Geometry, deals with the theory of places; geometry is the bedrock of robotics, control theory, and many fields of modern engineering and the physical sciences.

Mechanics Overview.

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Definition (Motion)

When a place undergoes body transformation in the course of time, we have motion.

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Preamble – Mass, Body, Rigid Body Motion.

Definition (Body – Truesdell, 1977.)

By a body, we shall mean the closure of an open set in some measure space Ω over which a non-negative measure M, called the mass, is defined, and that M can be extended to a Borel measure over the $\sigma-$ algebra of Borel sets in Ω.

Preamble – Mass, Body, Rigid Body Motion.

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Bodies – Truesdell, 1977.

That in mechanics which deals with
(i) mass points, which occupy a single point at any one time;
(ii) rigid bodies, which never deform;
(iii) strings and rods and jets, which are 1-dimensional; membranes and shells, that sweep out surfaces;
(iv) space-filling fluids and solids e.t.c. are termed bodies.

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Statics, Dynamics, Rigid Body (Motion).

Statics and Dynamics

That which studies putative equilibria is referred to as statics. That which concerns motion of all sorts is referred to as dynamics. The dynamics that are specific to particular bodies are termed constitutive.

Statics, Dynamics, Rigid Body (Motion).

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The Rigid Body

A rigid body does not stretch, buckle, contract, bend, twist, nor deform. Well, not really!

The Rigid Body

As engineers, we judge kinematic rigid hardware with the expectation that kinematic changes do not depart from rigid-body predictions.

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Statics, Dynamics, Rigid Body (Motion).

The Rigid Body

We expect that localized stresses, active noise, vibrations and heat e.t.c will not cause reasonable departures from expectations.

Rigid Body Motion

That motion that preserves distance between all points in a body is termed a rigid body motion.

Statics, Dynamics, Rigid Body (Motion).

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Rigid Body Motion

At issue are components of a rigid body's movement w.r.t to a fixed or moving frame of reference. In its most basic form, this movement is parameterized by displacement (and is sometimes time-varying e.g. for a continuum body). When solving for the movements of bodies, it is often useful to include velocities (twists) in order to characterize the motion.

Kinematics vs. Kinetics

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Dynamics

$$\dot{x} = f(t; x, u), \quad x(t_0) = t_0 \tag{1}$$

$$\dot{x} = f(t; x) + g(t; x, u), \quad x(t_0) = t_0 \tag{2}$$

Definition (Kinematics.)

Kinematics is the English version of the word *cinématique* coined by A.M. Ampère (1775-1836), who translated it from the Greek word *kίνημα*.

Kinematics vs. Kinetics.

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Definition (Truesdell)

That part of a system's dynamics that involves its motion by displacement – both linear and angular – and separated from motions owing to forces and torques, together with the successive derivatives with respect to time of all such displacements (this includes velocities, accelerations, and hyper accelerations) all form the kinematics of a rigid, continuum or laminae of bodies.

Kinematics vs. Kinetics

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Kinetics

The motion of bodies can also be conceived as resulting from the forces' action. Energy, temperature, and calory of a body are resultant effects of gains or loss of heat. Motions arising as a result of these are called kinetics.

Kinematics vs. Kinetics.

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Definition (Kinetics – Technical Definition)

That part of a system's dynamics that involves its motion by forces, energy, torque, inertia, dynamic stability, and equilibrium and similar properties all form the kinetics of a rigid, continuum or laminae of bodies.

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Kinematic Geometry.

Definition (Kinematic Geometry)

The solid geometry of relatively moving rigid bodies is termed the kinematic geometry of the rigid body. With motion, we'd have to include the successive derivatives of the displacement such as acceleration e.t.c as the 'laws of motion' stipulates in mechanics.

Joints and Links

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Links

Links may be rigid mechanical parts, elastic, (vulcanized) rubber components, diaphragms, conveyor belts, spring-damper systems e.t.c.

An Elementary Joint or Kinematic Pair.

An elementary joint or a kinematic pair consists of touching two links together at one point – then ensuring a single contact point is continuously maintained throughout relative movement.

Joint (Contact) Kinematics

Contact Kinematics

A body may slide or slip across a plane or surface, or roll over another body.

Joints

Joints are the result of the connecting points between two or more rigid bodies.

Links

Links may be rigid mechanical parts, elastic, (vulcanized) rubber components, diaphragms, conveyor belts, spring-damper systems e.t.c.

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Definition of a Mechanism

Definition (Author's Definition)

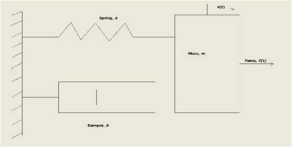
A connection of mechanical, magnetic, electrical, hydraulic, or pneumatic components forming an assemblage, meant for moving rigid, semi-rigid or non-rigid bodies via a controlled generation of (sometimes constrained) motion.

Kenneth Hunt (1978)

A means of transmitting, controlling, or constraining the relative movement between parts. Whenever we have an higher pair or more, we have a mechanism.

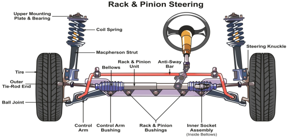
Mechanism Examples

Spring-Mass-Damper System

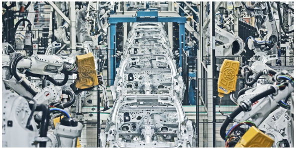


Excavator

Car suspension



Daimler Plant



Lower Pairs, Higher Pairs, Linkages

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Lower and Higher Pairs

When elements of pairs touch one another over a substantial region of a surface covering a line, curve-surface, or point of contact, we have lower pairs. When they touch along a discrete line, curve-surface, or point of contact, we have higher pairs.

Linkage (Hunt, 1978)

If all joints of a mechanism or mechanical movement belong to lower pairs, we have a linkage.

Prismatic Pairs or P-pairs

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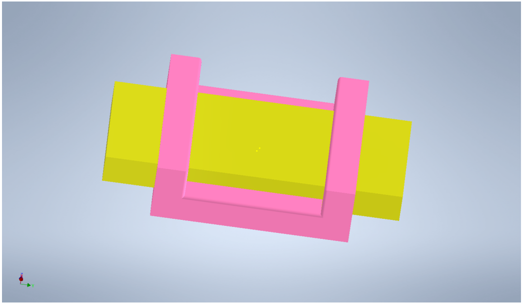
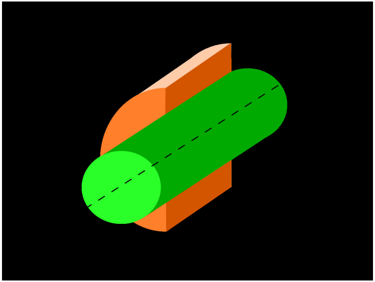
Hunt, 1978

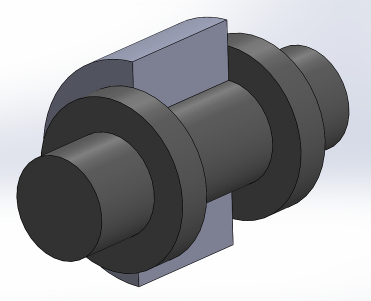
Formed by receding the axis of the revolution surface between two pairs to ∞ so that the curve that produces the surface moves parallel to itself, tracing a cylinder; or a polygonal-tracing curve generates a prism.



Revolute Pairs or R-pairs

One convex surface and one non-convex surface for a one degree of rotational freedom around the one joint the two surfaces make.





Revolute or Hinge or Turning or simply R-pairs with and without shoulder cutaway geometries. Credit: Wikimedia commons.

Helical- & U-Joints

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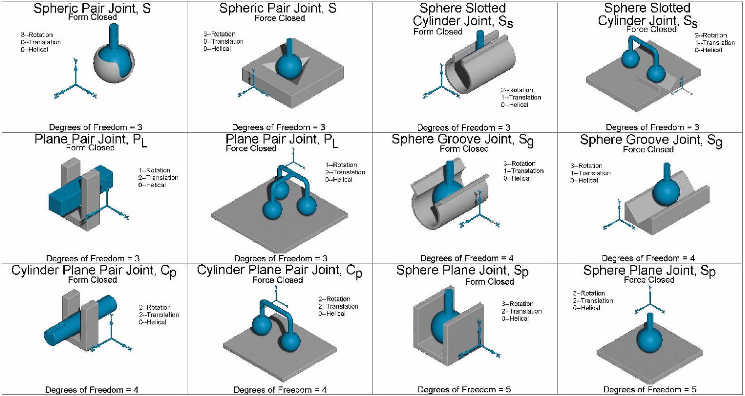
Helical Joint

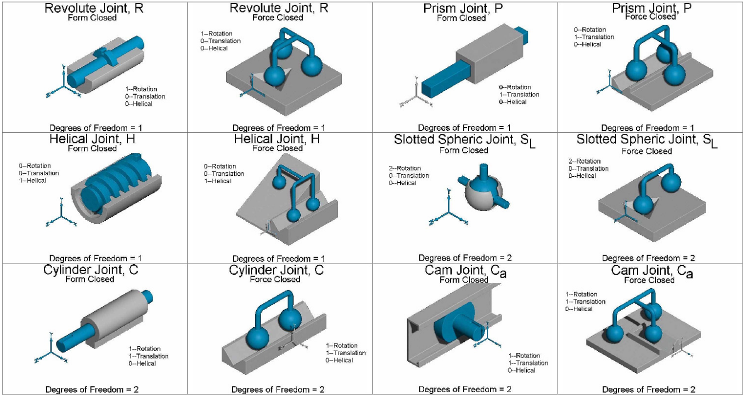
Universal Joint

Common Lower Kinematic Pairs



Credit: Wharton and Singh, 2001.

Common Lower Kinematic Pairs



Credit: Wharton and Singh, 2001.

Kinematic Geometry of Common Actuations

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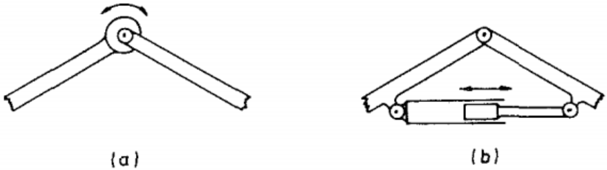
In-series vs. Parallel-actuated lower pairs



(a): In-series-actuated kinematic pair with a rotary joint that is actuated "about" the hinge. (b): Prismatic joint actuated "across" a hinge. Reprinted from Hunt, Kenneth. Structural Kinematics of In-Parallel-Actuated Robot Arms. Transactions of ASME. 1983.

Kinematic Chains

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Kinematic Chains (Reuleaux, 1975)

We can explain the structural similarity of many mechanisms by parts of kinematic chains connected by pairs.

Kinematic chains

Kinematic chains are essentially the basic building structure of mechanisms . . . and robots!

Open Kinematic Chains

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Chains

Open kinematic chains are based off the anthropomorphic construction of the human hand with cantilevered beam structures.

Chain Mechanisms and Error Amplification

Amplifies errors from waist (or base frame) all the way to the tool frame. Control difficult.

Control

Feedforward control: High power and precision hydraulic actuators for servo motors.
Sensory feedback control: Force sensing (Ernst, 1962).

Chains

Open kinematic chains are based off the anthropomorphic construction of the human hand with cantilevered beam structures.

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Amplifies errors from waist (or base frame) all the way to the tool frame. Control difficult.

Control

Feedforward control: High power and precision hydraulic actuators for servo motors.
Sensory feedback control: Force sensing (Ernst, 1962).

The PUMA arm is the world's first serial kinematic chain. Developer: Victor Scheinman, Stanford student in the '50's. Made several iterations. Patent Rights: Joe Engelberger, (Danbury Unimation, 1961). Joe – father of robotics – created world's first robotics company in '61.

Open Kinematic Robot Mechanisms

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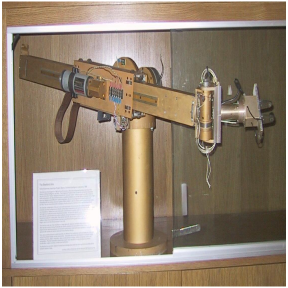
Definition (Ken Salisbury Jr., 1982)

"[Robots are] our fascination with constructing mechanical analogues of ourselves... [this fascination] has led us to place all sorts of hopes and expectations in robot capabilities."

The Stäubli PUMA Robot (1956).



The Stanford Arm (Infolab 1969).

Open Kinematic Chains

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Open kinematic chains provide unstructured environmental interaction.

Project MAC, MIT.

Tomovic and Boni's pressure sensed grasp.

Binary robot vision system (McCarthy et al, 1963).

Open Kinematic Chains

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Stanford Manipulator.

Boston arm.

The AMF (American Machines and Foundry) arm.

General electric's walking robot (1969).

Long Walk Towards Direct Drive Robot Arms

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The 50's, 60's nd 70's witnessed use of hydraulics for (feedforward) position control.

For feedback control, force sensors and pressure sensors were used in closed-loop scenarios.

Electrical actuation meant that robots had to be operated at high speeds. Needs for gear reduction for safe operations at low speeds.

With gear reduction came backlash, friction, and associated expenses.

The 50's, 60's nd 70's witnessed use of hydraulics for (feedforward) position control.

For feedback control, force sensors and pressure sensors were used in closed-loop scenarios.

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With gear reduction came backlash, friction, and associated expenses.

CMU DD I/II Arms: Workspace is donut shaped. OD: 90cm; ID: 21.7cm; $1.8m^2$ workspace area. Built by Harry Asada. Structural design similar to aircraft gimbal arm; Uses Samarium Cobalt rare earth magnet brushless DC motors on first 3 joints, and AlNiCo magnets on tip joints. No belts, transmissions making for faster transmitting of motions, less friction, low energy, low compliance. Each joint has complex AL housing which enables: (i) Control of geometrical relationships of bearing assembly; (ii) Control of servo components to bearing assembly; (iii) Controls of rotational axes to consecutive joints.

Direct Drive Robot Mechanism: CMU DD I Arm

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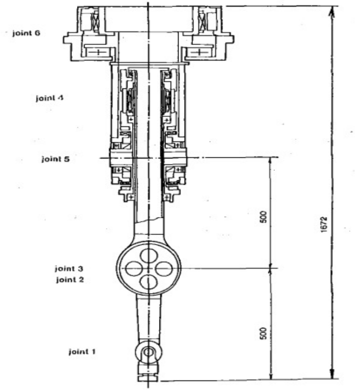
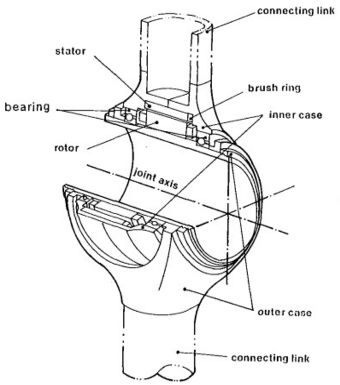
Along came Harry Asada.



Arm Schematics Transmission



Joint schematic

Direct Drive Robot Mechanism: CMU DD I Arm

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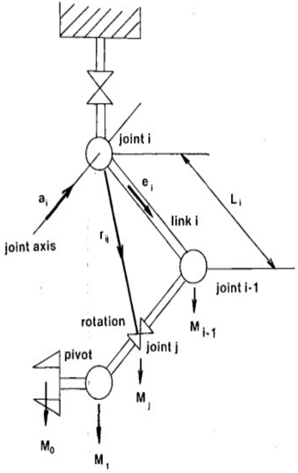
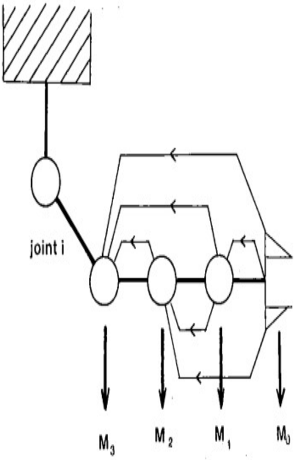
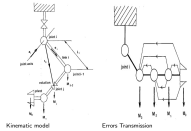


Kinematic model



Errors Transmission



First direct-drive robot without a gearbox. Selective compliance in X-Y directions given its articulated jointed arms. One-freedom motion along Z direction given its constrained arm New generations such as Cobra i600/i800 include power amplifiers, system and servo controls etc embedded in the robot's base. Kuka Scara arm: Lightweight, fast, powerful, low maintenance, energy consumption, investment costs etc.

SCARA Robot Mechanisms

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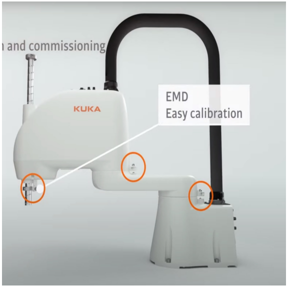
The Adept One SCARA robot (Debuted 1984).



Kuka's SCARA arm, 2022. ©Kuka Robotics

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The Stäubli anthropomorphic arm.

The Staubli 6-DOF Arm is an example of a Spherical Manipulator. Reprinted from DirectIndustry's Webpage.

Serial mechanisms research in the 80's

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Mechanisms in the 80's

With the 80's came the arrival of PCs. Lots of research went into computational algorithms for the kinematics and kinetics of (mostly) anthropomorphic robot arms.

Active control schemes

Efficient recursive Lagrangian and computational methods for the gravitational and Coriolis forces in Newton-Euler equations.

Feedback Linearization

Dynamics feedback linearization for precise bounds on manipulator performance.

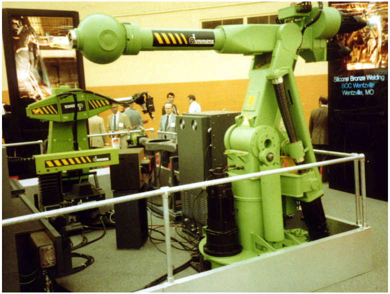
Serial mechanisms research in the 90's

Automatix

Reconfigurable robots for various assembly ops.

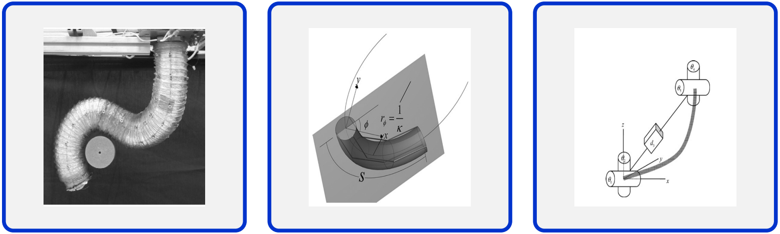
Robotworld

First industrial-scale reconfigurable robot and with machine vision components. RAIL scripting OS originally based on Motorola 68000, later on replaced by Apple Macintosh II.



©Wikipedia

Hyper-redundant Continuum Robots



The elephant trunk continuum robot. Jones & Walker, T-RO 2006.
Inspiration: Muscular hydrostats in nature.

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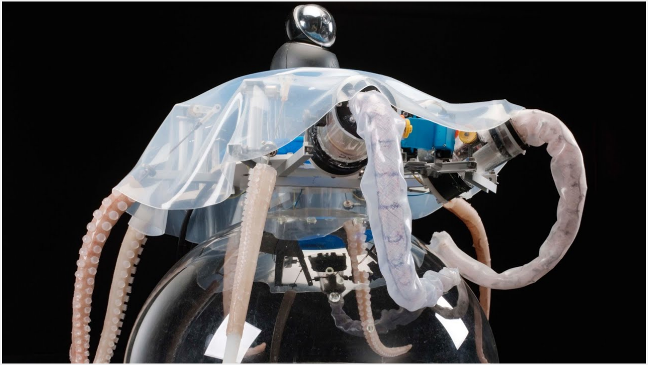
Hyper-redundant Kinematic Chains



An octopus-inspired soft robot. ©Cecilia Laschi.

Parallel Robots

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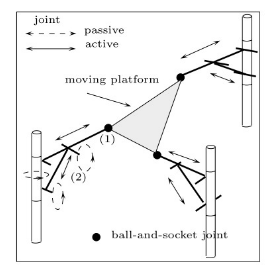
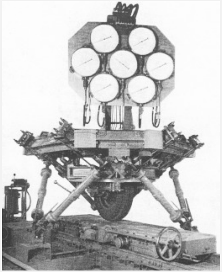
Mehlet 2015

A parallel robot is made up of an end-effector with n degrees of freedom, and of a fixed base, linked together by at least two independent kinematic chains. Actuation takes place through n simple actuators.

Parallel mechanisms: Stewart-Gough Platforms

Principles of a moving platform to test tyre wear and tear (Gough, 1947). Prototype, 1955.





Left: Stewart's 1965 mechanism. *Right:* The original 1954 octahedral hexapod proposed by Gough. Courtesy: Parallemic.org.

Truss Robots

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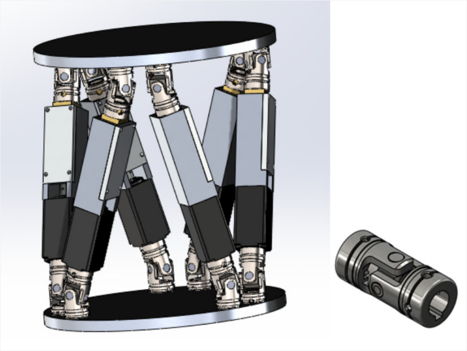
A multi-DOF Truss Robot. Courtesy of Penngineering (ICRA 2022, Philadelphia, PA).

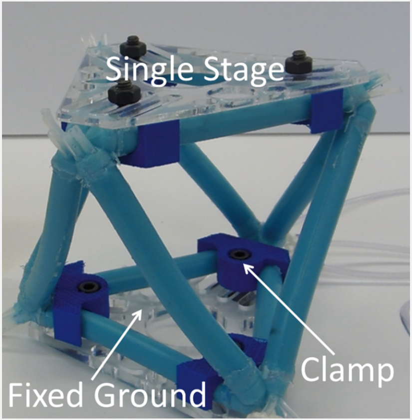
Closed kinematic chains

Connection degree ≥ 3.



A Stewart-Gough platform. SolidWorks Drawing Courtesy of Andrew Belcher. UChicago, 2018.

A Soft Stewart Platform



A soft 6-6 Stewart manipulator. Jonathan Hopkins, 2015.

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Freedom and Structure

Freedoms, Constraints, and Mobility.

Motion of linkages: Screws, and spatial motions.

Freedom and Mobility: Freedoms, unfreedoms, connectivity, mobility;

Grübler-Kutzbach's mobility criterion and examples.

Degrees of Freedom and Structure

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Definition (Connection Degree)

For any manipulator joint, we shall mean its connection degree to be the number of links attached it.

Quiz

What is the connection degree of the u-joints of a Stewart-Gough platform.

Members and Dual Graphs

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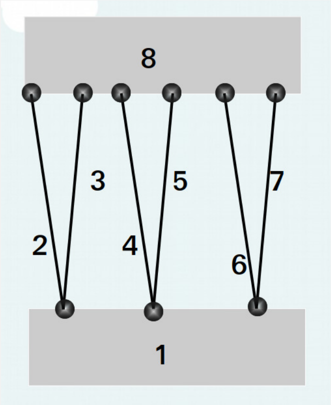
Dual graph of a Stewart platform



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Degrees of Freedom and Structure

Members and Freedoms

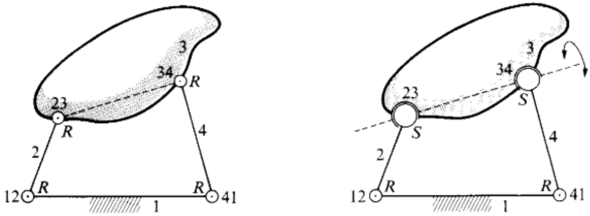
Degrees of freedoms (or freedoms) concerns the relative motion of members of a pair that do not touch one another directly.

Connectivity

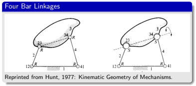
By the dual graph of the Stewart platform as seen on Frame 52, the total number of freedoms that connect the two members (1 and 8) that do not connect to one another directly is six.

Planar Linkages

Four Bar Linkages



Reprinted from Hunt, 1977: Kinematic Geometry of Mechanisms.



The planar $RRRR$ linkage, (*left*) is modified in (*right*) to an $RSSR$ linkage to allow spatial spin-movement of the coupler 3; the connectivity $\mathscr{C}_{13} = 2$.

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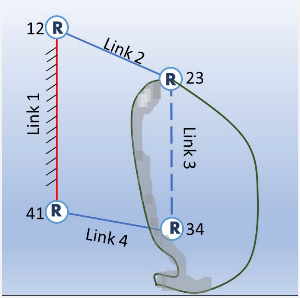
A (Hacked) Four-Bar Linkage



The Four Bar Linkage

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Couplings and Freedom

Links $2\&4$ complete a coupling or connection between links $1\&3$.

Connectivity

The R-pairs are said to have a connectivity of $\mathscr{C}_{ij} = 1$ for all $i, j = 1, 2, 3, 4$. Thus, total degree of freedom is 1.

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Mobility of Mechanisms

The Mobility and Relative Mobility, $\mathfrak{M}$

Simply put, the number of a mechanism's freedoms is its mobility, or relative mobility, $\mathfrak{M}$.

The Mobility, $\mathfrak{M}$

It specifies the independent variables needed to determine every relative location of a mechanism's members with respect to one another.

A Note on Serial and Parallel Mobility

A little tricky to determine for parallel mechanisms but straightforward for serial mechanisms.

Mobility of Mechanisms

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Quiz

What is the mobility $\mathfrak{M}$ of the $RSSR$ four bar linkage of Frame 54? Why?

Quiz

What is the mobility $\mathfrak{M}$ of the $RRRR$ four bar linkage of Frame 55? Why?

Definition (The mobility criterion (well, not yet))

Let's not get ahead of ourselves. A little introduction to screws are in order for us to grasp the Grübler-Kutzbach mobility criterion.

Unique Location of a Rigid Body in 3D Space

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Inhomogeneity of Displacements and Angles

Quiz: Three translations and three rotations are ill-posed for uniquely determining the freedoms of a body. Why?

They are not homogeneous.

For true kinematic wholeness and generality, displacement that is purely translatory and purely rotary is needed.

Screws for Kinematic Generality

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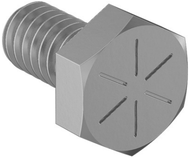
Need for Screws

From a kinematic standpoint, six homogeneous screw coordinates – each having an independent screw freedom – are needed to uniquely determine a rigid body's location.

Definition (What is a screw anyway?)

A screw is a straight line in space, called the axis, with an associated direction, called pitch, p.



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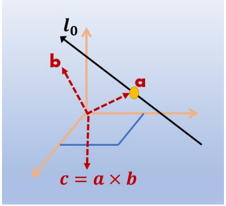
Definition (Screw Coordinates)

Six-vector, $\boldsymbol{s}$, related to the Plücker coordinates (see right inset) parameterize a screw i.e. $\boldsymbol{s} = (s_1, s_2, s_3, s_4, s_5, s_6)$.



Plücker Coordinates

Let $\boldsymbol{a}$ be a point on line ℓ_0. Let $\boldsymbol{a}$'s direction cosine vector (to be introduced shortly) be $\boldsymbol{b}$. Then, its binormal (moment) vector is $\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$. We say the pair $(\boldsymbol{b}, \boldsymbol{c})$ is the Plücker Coordinates of the point $\boldsymbol{a}$ on axis ℓ_0.

Screws and Plücker Coordinates Relationship

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Screw axis and Plücker Coordinates Relationship

$$b_1 = s_1, \quad b_2 = s_2, \quad b_3 = s_3 \tag{3}$$

$$c_1 = s_4 - p s_1, \quad c_2 = s_5 - p s_2, \quad c_3 = s_6 - p s_3. \tag{4}$$

p: pitch! How to find it?

Screw and Plücker Coordinates Relationship

Suppose that

$$h = \sqrt{b_1^2 + b_2^2 + b_3^2}. \tag{5}$$

Then $(\boldsymbol{b}/h, \boldsymbol{c}/h)$ are respectively the direction cosines of the line, l_0 and its moment.

Pitch and Magnitude of the screw

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Pitch of a screw

$$p = \frac{s_1\, s_4 + s_2\, s_5 + s_3\, s_6}{\sqrt{s_1^2 + s_2^2 + s_3^2}}, \tag{6}$$

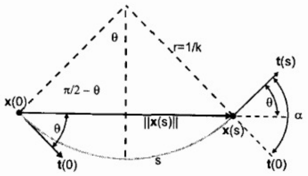
$$|\, s \,| = \sqrt{s_1^2 + s_2^2 + s_3^2} \quad \text{if } p \neq \infty \tag{7}$$

$$|\, s \,| = \sqrt{s_4^2 + s_5^2 + s_6^2} \quad \text{if } p = \infty \tag{8}$$

Plücker Coordinates Example

Chasles' Theorem Applied to The Serret-Frenet Frame

Consider a spatial curve $\boldsymbol{C}$ on the elephant continuum trunk shown earlier. Suppose $\boldsymbol{C}$ is parameterized by its arc length $\boldsymbol{s} \in [0, 1]$. For a point $\boldsymbol{x} = [x, y, z]^T$ on $\boldsymbol{C}$, the unit tangent vector to $\boldsymbol{C}$ is $\boldsymbol{t} = \boldsymbol{dx}/\boldsymbol{ds}$. Denote by $\boldsymbol{n}$ the principal normal to $\boldsymbol{C}$ at $\boldsymbol{n}$; then we must have $\boldsymbol{b} = \boldsymbol{t} \times \boldsymbol{n}$ as the binormal. We say $(\boldsymbol{b}, \boldsymbol{n})$ together form the Plücker coordinates of the tangent $\boldsymbol{t}$.



Plücker Coordinates Example

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Poinsot's Theorem Quiz on a Force and its Moment

Suppose that a force $\boldsymbol{F}$ acts at the point $\boldsymbol{a}$ in the image of Frame 61. What are the Plücker coordinates of the line of force?

Homogeneous Coordinates!

Plücker Coordinates give six unit parameters of a point on a line. Plücker Coordinates are in homogeneous coordinates!

Poinsot's Theorem Quiz on a Force and its Moment

Suppose that a force $\boldsymbol{F}$ acts at the point $\boldsymbol{a}$ in the image of Frame 61. What are the Plücker coordinates of the line of force?

Homogeneous Coordinates!

Plücker Coordinates give six unit parameters of a point on a line. Plücker Coordinates are in homogeneous coordinates!

Poinsot's Theorem Quiz on a Force and its Moment

Imagine that a force $\boldsymbol{F}$ is acting at the point $\boldsymbol{a}$ in the image of Frame 61. Suppose that $\boldsymbol{\tau}$ is torque acting along the normal to point $\boldsymbol{a}$. Then $(\boldsymbol{f}, \boldsymbol{\tau})$ are the Plücker coordinates of the line of force.

Arithmetics on Screws

Scalar and vector arithmetic operations are valid on infinitesimal screws e.g.

$$c_1\boldsymbol{s}_1 + c_2\boldsymbol{s}_2 = 0 \text{ for } c_1,\ c_2 \neq 0 \text{ on screws } \boldsymbol{s}_1, \boldsymbol{s}_2. \tag{9}$$

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Freedom and Constraints

Suppose a screw $\boldsymbol{f} = (f_1, \cdots, f_6)$ "fixes" a body in 3D space.

- Each constraint $u_i \neq f_j$ for $(i, j) \in \{1, \cdots, 6\}$.
- Rather each u_i has influence on every $\{f_i\}_{i=1}^6$.
- Each u_i from the six independent equations, $g(s_1, s_2, s_3, s_4, s_5, s_6) = 0$, suppresses a freedom, f_i.
- Progressively relaxing each u_i, or unfreedom, adds an extra body f_i.

Freedoms, Unfreedoms, and Mobility

Freedom and Unfreedoms

Suppose the total freedoms is $\boldsymbol{f}$ and the total unfreedoms is $\boldsymbol{u}$, then

$$\boldsymbol{u} + \boldsymbol{f} = 6.$$

Note: A rigid body's freedoms is also referred to the dimension of its configuration space.

Relative Freedoms

Suppose there are a total of n unconstrained bodies. Suppose further that we choose one out of the bodies as a reference body. Then the total number of relative freedoms is $6(n-1)$.

Freedoms, Unfreedoms, and Mobility

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Constraints and Joints

Now, consider k independent constraints[a] such as joints along points, lines, curves or surfaces.

[a] NB: The total *allowable* constraints is 5 for a body in relative motion. I
6 for a fully rigid body.

The Mobility Criterion

Let the constraint of joint, i (e.g. a joint along points, lines, curves or surfaces) be u_i. Then the mobility criterion $\mathfrak{M}$ is

$$\mathfrak{M} = 6(n-1) - \sum_{i=1}^{k} u_i. \tag{10}$$

General Grübler-Kutzbach Mobility Criterion

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General Grübler-Kutzbach Mobility Criterion

Recall that $\sum_i u_i + f_i = 6$ from Frame (68) so that

$$\mathfrak{M} = 6(n - k - 1) - \sum_{i=1}^{f} f_i. \tag{11}$$

Exceptions: Relative Planar and Spherical Motions

For bodies restricted to relative planar or spherical motions, the total freedoms + constraints is 3 (not 6)!

$$\mathfrak{M} = 3(n - k - 1) - \sum_{i=1}^{f} f_i. \tag{12}$$

General Grübler-Kutzbach Criterion References

The Grübler-Kutzbach Mobility Criterion References

Attributed to Grübler:

Schoenflies, Arthur, and M. Grübler. "Kinematik." In Encyklopädie der Mathematischen Wissenschaften mit Einschluss ihrer Anwendungen, pp. 190-278. Vieweg+Teubner Verlag, Wiesbaden, 1908;

Grübler, Martin Fürchtegott. Getriebelehre: eine Theorie des Zwanglaufes und der ebenen Mechanismen. Springer, 1917.

and Kutzbach:

Kutzbach, Karl. "Mechanische leitungsverzweigung, ihre gesetze und anwendungen." Maschinenbau 8, no. 21 (1929): 710-716.

Loops

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Loops

A kinematic chain often comprises members called loops.

Binary Link

Members in a binary link constitute a single loop. Example: The four-bar linkage.

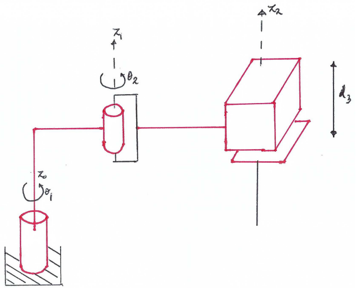
Single loops

For single loops, $k = n$ so that $\mathfrak{M} = \sum_{i=1}^{f} f_i - 6$.

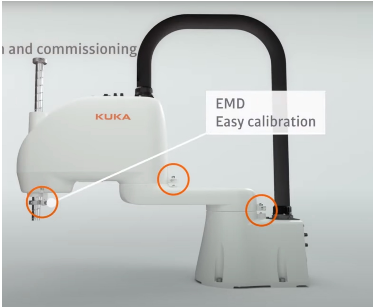
Mobility of Mechanisms

$\mathfrak{M} \leq 1$ for at least one actuator-pair to produce mobility at a successor joint which depends on that actuator-pair's input.

Mobility of Common Robot Configurations



Configuration of the SCARA Arm.



Courtesy of Fanuc America Inc.

Mobility of The SCARA Robot

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Mobility Analysis

Two rotary joints. One prismatic joint acting along the z axis, and constrained along the xy plane.

Mobility Parameters

Four rigid bodies (links). Three constraints. Four freedoms. Therefore, $\mathfrak{M} = 6(4-3-1)+4 = 4$

Mobility Analysis of The Universal Robot

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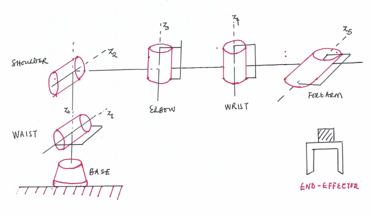
The Revolute Arm

Falls under so-called RRR kinematic arrangements. Also called a revolute, elbow, or anthorpomorphic manipulator.



$n = 6$; $k = 6$; $f = 5 \times 3 := 18$

$\therefore \mathfrak{M} = 6(n - k - 1) + \sum f_i$

$\implies 6(6 - 1 - 5) + 18$ or $\mathfrak{M} = 6$.

Mobility of The Stewart-Gough Platform

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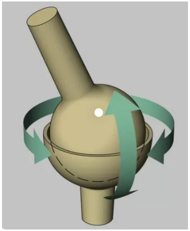

Mobility of The Stewart-Gough Platform

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Unconstrained bodies, n

There are six universal joints that connect the base platform to the prismatic linear actuators. There are six spherical joints that connect the top platform to the top of the prismatic actuators. Altogether, there are $n = 6 + 6 + 2$ or 14 unconstrained rigid links.

Constraints, k

Six u-joints. Six spherical joints. Six prismatic joints. Altogether, there are $f = 6 + 6 + 6 := 18$ constraints.

Freedoms, f

Each u-joints has two freedoms. Each spherical joint has three (rotary) freedoms. Each prismatic joint has one freedom. Altogether, there are $f = 6 \times 2 + 6 \times 3 + 6 \times 1 := 36$ freedoms.