RBOT 250: Homework Solutions

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Homework 1. Find a closed-form expression for the solution to

$$d(\boldsymbol{p}, l) = \min_{x+y=1} \|x\boldsymbol{r}_1 + y\boldsymbol{r}_2 - \boldsymbol{p}\| \tag{1}$$

where $x, y \in [0, 1]$.

Solution 1. Since p is 3-D, we have

$$\boldsymbol{p} = \begin{pmatrix} p_{1x} \\ p_{1y} \\ p_{1z} \end{pmatrix}, \quad \begin{pmatrix} p_{2x} \\ p_{2y} \\ p_{2z} \end{pmatrix}, \ldots, \begin{pmatrix} p_{nx} \\ p_{ny} \\ p_{nz} \end{pmatrix}, \tag{2}$$

and since $\boldsymbol{l}$ is a point set, we can write

$$\boldsymbol{l} = \{l_1, l_2, \ldots, l_n\}. \tag{3}$$

Now, rewrite $\boldsymbol{r}_1 = \begin{pmatrix} \boldsymbol{r}_{1x} \\ \boldsymbol{r}_{1y} \\ \boldsymbol{r}_{1z} \end{pmatrix}$, and $\boldsymbol{r}_2 = \begin{pmatrix} \boldsymbol{r}_{2x} \\ \boldsymbol{r}_{2y} \\ \boldsymbol{r}_{2z} \end{pmatrix}$ so that

$$u\boldsymbol{r} + v\boldsymbol{r} - \boldsymbol{p} = \begin{pmatrix} u\boldsymbol{r}_{1x} \\ u\boldsymbol{r}_{1y} \\ u\boldsymbol{r}_{1z} \end{pmatrix} + \begin{pmatrix} v\boldsymbol{r}_{2x} \\ v\boldsymbol{r}_{2y} \\ v\boldsymbol{r}_{2z} \end{pmatrix} - \begin{pmatrix} p_{ix} \\ p_{iy} \\ p_{iz} \end{pmatrix} \tag{4}$$

where $i = 1, \ldots, n$

This informs,

$$d(\boldsymbol{p}, \boldsymbol{l}) = \left\| \begin{matrix} u\boldsymbol{r}_{1x} + v\boldsymbol{r}_{2x} - p_{ix} \\ u\boldsymbol{r}_{1y} + v\boldsymbol{r}_{2y} - p_{iy} \\ u\boldsymbol{r}_{1z} + v\boldsymbol{r}_{2z} - p_{iz} \end{matrix} \right\|. \tag{5}$$

Write $D(\boldsymbol{p}, \boldsymbol{l}) = d(\boldsymbol{p}, \boldsymbol{l})^2 = \begin{bmatrix} u\boldsymbol{r}_{1x} + v\boldsymbol{r}_{2x} - p_{ix} \\ u\boldsymbol{r}_{1y} + v\boldsymbol{r}_{2y} - p_{iy} \\ u\boldsymbol{r}_{1z} + v\boldsymbol{r}_{2z} - p_{iz} \end{bmatrix}$.

From which,

$$D'(\boldsymbol{p}, \boldsymbol{l}) = \tag{6}$$

References

[1] G. Gogu, "Maximally regular t3-type translational parallel robots," *Structural Synthesis of Parallel Robots: Part 2: Translational Topologies with Two and Three Degrees of Freedom*, pp. 687–748, 2009.