Week 4 Assignment

Submitted for RBOT250: Robot Manipulation; Planning and Control

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I. Homework 5: Verify that $R_{ij} = R_{ij}^{-1} = R_{ji}^{T}$

From the reading it is known that

$$R_{ij} = \begin{bmatrix} x_j \cdot x_i & y_j \cdot x_i & z_j \cdot x_i \\ x_j \cdot y_i & y_i \cdot y_i & z_j \cdot y_i \\ x_j \cdot z_i & y_j \cdot z_i & z_j \cdot z_i \end{bmatrix}$$

Upon visual inspection of R_{ij} it can be seen that the rows of R_{ij} are the unit vector coordinates of the frame I in the frame J. It therefore follows that:

$$R_{ij} = R_{ji}^{T}$$

From linear algebra we know that a matrix $A^{-1} = A^T$ when $A^T A = I$, that is when A is an orthonormal matrix. An orthonormal matrix has the following property = $A^T A$ = $A * A^T$ = I. Given the above definition of R_{ij}, and the operations carried out in figures 3.4.9, 3.4.10, 3.4.11 from the reading (which I will omit here due to the time it would require to write them). It follows:

$$R_{ij} = R_{ji}^{T} = R_{ij}^{-1}$$

II. Homework 6

The rotation of a matrix around the x-axis would be:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \end{bmatrix}$$

The rotation of a matrix around the y-axis would be:

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \end{bmatrix}$$

The rotation of a matrix around the z-axis would be:

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \end{bmatrix}$$

A. Verify 1:

B. Verify 2:

C. Verify 3:

III. Homework 7

A. Determine rotation matrix in Fig 3.7

By visual inspection it appears that figure 3.7 represents a rotation of 60^o about the x axis. Using the rotation matrices presented in homework 6 - the rotation matrix would be:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60 & -\sin 60 \\ 0 & \sin 60 & \cos 60 \end{bmatrix}$$

B. Explain the difference between rotationg about a current frame and rotating about a fixed frame.

The matrix corresponding to a set of rotations about moving axes, that is - the current frame, can be found by post-multiplying the rotation matrices. By post multiplying, one multiplies the matrices in the same order in which the rotations take place. When compositing rotations about fixed axes one pre-multiplies the different elementary rotation matrices.

**this probably needs more work.

IV. Homework 8: For the robot manipulator we are using in this class, suppose that you have the following point in the base frame of the robot, q_0 = [-2, 3, 1]. Furthermore, suppose that the joint angles for all six joints are respectively -90, 60, 30, 45, 90, 125, transform the point q_0 in the base frame to a coordinate frame on the sixth joint.

Let

$$
{}_0^1R = \begin{bmatrix} \cos(q_o) & -\sin(q_0) & 0 \\ \sin(q_0) & \cos(q_0) & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

and the point at base, a translation vector from 0 be

$$
{}_0^1D = [-2, 3, 1]
$$

The robotic manipulator for this class is unspecified. Assuming it is a planar open chain manipulator the

$$
\begin{aligned}
x = L_1 + \cos\theta_1 + \\
L_2 \cos(\theta_1 + \theta_2) + \\
L_3 \cos(\theta_1 + \theta_2 + \theta_3) + \\
L_4 \cos(\theta_1 + \theta_2 + \theta_3 + \theta_4) + \\
L_5 \cos(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5) + \\
L_6 \cos(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6)
\end{aligned}
$$

$$
\begin{aligned}
y = L_1 + \sin\theta_1 + \\
L_2 \sin(\theta_1 + \theta_2) + \\
L_3 \sin(\theta_1 + \theta_2 + \theta_3) + \\
L_4 \sin(\theta_1 + \theta_2 + \theta_3 + \theta_4) + \\
L_5 \sin(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5) + \\
L_6 \sin(\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6)
\end{aligned}
$$

Where Joint angles q_1 through q_6, are (-90, 60, 30 , 45, 90, 125). Ommiting the rest of the calculation in $LaTex$. I

$$
{}_0^{ee}T = {}_0^1 T_1^2 T_2^3 T_3^4 T_4^5 T_5^6 T_6^7 T =
\begin{bmatrix}
\cos(q_1)..\cos(q_n) & -\sin(q_0)..-\sin(q_n) & 0 & -\sin(q_0)(1+q_1) \\
\cos(q_1)..\cos(q_n) & -\sin(q_0)..-\sin(q_n) & 0 & -\cos(q_0)(1+q_1) \\
0 & 0 & 0 & 1 \\
..-\sin(q_n) & 0 & 0 & f(-\sin(q_n)
\end{bmatrix}
$$

V. Homework 9: Carry out the transformation above in reverse order. What do you notice?

The original transformation is:

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}
\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}
=
\begin{bmatrix} \cos\psi & -\sin\psi \\ 0 & \cos\theta\cos\psi \\ \sin\theta\sin\phi & \sin\theta\sin\phi \end{bmatrix}
$$

Carrying out the transformation in reverse order yields:

$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}
\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}
=
\begin{bmatrix} \cos\psi & -\cos\theta\sin\psi & \sin\psi\sin\theta \\ \sin\theta & \cos\theta\cos\psi & -\cos\psi\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}
$$

Trivially, the transformation is not reversible.