RBOT 101 - Foundations Mathematics Pre-test

December 13, 2020

The following test is meant to test your knowledge in key areas of mathematics useful in robotics. If you are unfamiliar with or don't know how to solve a particular problem, just write "unknown" for the answer. Some of this material may be familiar to some students and completely unknown to others: the intent is to provide the instructor with sufficient understanding of what is unfamiliar to ensure adequate coverage on those topics.

1 Linear Algebra

1.1 Perform the following multiplication. If you cannot, then explain why.

$$\begin{bmatrix} 1 & 2 & 0 \\ 10 & -1 & -10 \\ -1 & 3 & 11 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

1.2 Perform the following multiplication. If you cannot, then explain why.

$$\begin{bmatrix} 1 & 2 & 9 \\ 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

1.3 Given $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}$, compute $det(A)$

1.4 Given A from the previous problem, compute $tr(A)$.

1.5 Given A as before but with all off-diagonal elements equal to 0, compute A^{-1}.

1.6 Solve the given system of equations. If no solution is possible, explain why. If more than one solution is available, provide all solutions.

$$
\begin{aligned}
x_1 &+ x_2 &+ 8x_3 &= 0 & (1) \\
2x_1 &+ 3x_2 &+ 19x_3 &= 0 & (2) \\
x_1 &+ 2x_2 &+ 11x_3 &= 0 & (3)
\end{aligned}
$$

1.7 Solve the given system of equations, if possible. If more than one solution is available, provide all solutions.

$$
\begin{array}{rcrcll}
5x_1 & + & 14x_2 & = & -35 & \quad (4) \\
x_1 & + & 3x_2 & = & 28 & \quad (5)
\end{array}
$$

2 Calculus

2.1 Find the derivative of

$$f(x) = 25x^{22} + 12x^8 + 17 + \frac{9}{x^5}$$

2.2 For $f(x, y, t) = x^2 t^3 + 4xyt^2 + 2y^2 t + y^3$

   (a) Find $\frac{\partial f}{\partial x}$

   (b) Find $\frac{\partial f}{\partial y}$

   (c) Find $\frac{\partial f}{\partial t}$

   (d) Find $\frac{\partial^2 f}{\partial x \partial t}$

2.3 Differentiate

$$f(x) = xe^{-x^3}$$

2.4 Differentiate

$$f(\theta) = cos^2(\theta)sin(2\theta)$$

2.5 Evaluate $\int x^2 \, dx$

2.6 Evaluate

$$\int_0^{2\pi} 2\cos(\theta)\,\sin(\theta)\,d\theta$$

3 Differential Equations

3.1 Solve the following

$$\frac{du}{dx} = cu + x^2$$

3.2 Solve the following equation and identify it's use

$$\frac{d^2u}{dx^2} + \omega^2 u = 0$$

3.3 Solve the following equation and identify it's use. What can you say about the constant k in its effect on the solution?

$$\frac{dA}{dt} = -kA$$

3.4 Solve the following
$$m\frac{dv}{dt} = F$$

3.5 Solve the following
$$\frac{dy}{dx} = x^2(1+y), \quad y(0) = 3$$

3.6 Find the general solution to
$$\frac{du}{dt} = \alpha(1-u) - \beta u$$

3.7 Solve the following
$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

4 Probability and Statistics

4.1 A cloth bag has 3 green marbles, 2 blue marbles, 4 yellow marbles, 6 red marbles, and 5 purple marbles.

   (a) If you draw 1 marble 10 times (replacing the marble each time), how many times would you expect to get a marble that is not green or blue?

   (b) If you draw 1 marble 10 times (without replacing the marble each time), how many times would you expect to get a marble that is not red?

4.2 Describe the difference between a deterministic and a probabilistic statistical model. Under what conditions would you choose a deterministic model over a probabilistic model?

4.3 Following is a statement of Bayes' Theorem

$$P(H\,|\,E) = \frac{P(E\,|\,H) \cdot P(H)}{P(E)}$$

   (a) Give a description of each of the terms, including E and H.

   (b) What is the value of using a Bayesian model versus using other statistical models?

4.4 What is the Markov property?