Course Syllabus

### I. Course Information

# **Course Name**: Mathematical Foundations of Robotics

**Course Number :** RBOT- 101

**Course Start & End Dates:**  July 14, 2021 -

**Instructor’s Name and Contact Information**

Dr. Lekan Molu

Email: patlekno@brandeis.edu

Phone: (optional)

Office Hours/Availability – 6pm-8pm, EST Wednesdays and Fridays

## Document Overview

This syllabus contains all relevant information about the course: its objectives and outcomes, the grading criteria, the texts and other materials of instruction; and of weekly topics, outcomes, assignments, and due dates.

Consider this your roadmap for the course. Please read through the syllabus carefully and feel free to share any questions that you may have. Please print a copy of this syllabus for reference.

## Course Description

The course is intended to be a refresher for the mathematical concepts important in robotics that students should have encountered in past courses or avenues of study. The topics covered by this course include the basic linear algebra: vectors and matrices – inner products, vector-vector multiplication, matrix-matrix multiplication, noncommutativity, associativity, Hermitian matrices, Unitary matrices etc – we will learn how to solve linear homogeneous equations, Gram-Schmidt orthogonalization, the necessary and sufficient conditions for positive definiteness , Cayley-Hamilton theorem for symmetric matrices; an introduction to perturbation theory, representation of the sum of e.t.c.

In the latter third of the course, [Benjamin Gravell](https://sites.google.com/view/ben-gravell/home) will guest teach probability theory – covering the basics of sets, Boolean algebra, set algebra laws, probability measures, Baye’s theorem, random variables, distributions, probability density (mass) functions, expectations, moments, and parameter asymptotics – law of large numbers, central limit theorem – and cap the his part by touching upon information theory.

## Course Outcomes

After taking this course, each student will:

1. Develop an appreciation for linear algebra, linear systems, probability theory and information theoretical characterization of real-world systems.
2. Formulate optimal state estimation tools for solving real-time smoothing and filtering operations in robotics.
3. Possess a rich repertoire of mathematical fundamentals necessary for deconstructing complicated real-world reasoning and engineering problems..
4. Have the foundations necessary for navigating the world of a standard mechanical, electrical, process or computer science and engineering curriculum.

**Relevant Programs:** Robotics Software Engineering

**Prerequisites:** An undergraduate-level understanding of linear algebra, analytical mechanics, Python and C++ programming.

## 

## Materials of Instruction

* Introduction to Matrix Analysis by *Richard Bellman* © The RAND Corporation, 1960.
* Simon, D. (2007). Optimal state estimation: Kalman, H [infinity], and nonlinear approaches. Choice Reviews Online, Vol. 44, pp. 44-3334-44–3334. <https://doi.org/10.5860/choice.44-3334>
* John Woods and Henry Stark.Probability, Statistics, and Random Processes for Engineers.Pearson Higher Ed, 2011.
* Martin J Wainwright.High-dimensional statistics: A non-asymptotic viewpoint, volume 48.Cambridge University Press, 2019
* Victor M Panaretos and Yoav Zemel.Statistical aspects of wasserstein distances.Annual review of statistics and its application, 6:405–431, 2019.
* Mackay, D. J. C. (n.d.). Information Theory, Inference, and Learning Algorithms. Retrieved August 17, 2017, from http://www.cambridge.org/0521642981

# b. Optional Software :

# A working knowledge of \*nix Systems

# Python

# GNU C++

c. Recommended Text(s) / Journals: TBA

d. Online Course Content

This course will be conducted completely online using Brandeis’ LATTE site, available at http://moodle2.brandeis.edu. The site contains the course syllabus, assignments, our discussion forums, links/resources to course-related professional organizations and sites, and weekly checklists, objectives, outcomes, topic notes, self-tests, and discussion questions. Access information is emailed to enrolled participants before the start of the course. To begin participating in the course, review the Welcoming Message and the Week 1 Checklist.

• Students will be provided with PDF copies of additional course material and links to relevant material, if required.

**Welcome Message from the Instructor**

Hi there,

I am Dr. Lekan Molu, and I preferably go by my first name. I am the one that developed this curriculum, coordinated with the course designers, made available the course notes and will be coordinating the invited lectures portions of this class toward the end of imbibing you with much needed skills for you to enjoy your engineering journey henceforward.

Parts of the contents of this course will be run in a real-time video format over zoom, whereby the instructor will walk through course contents with you, answer your questions, provide help in your class and home works, as well as point you to helpful reading material that may enrich your mid as we journey through a concise introduction to the mathematical basics that you will need to become an excellent engineer.

I will be holding two office hours on weekdays as outlined in the earlier section of this material but if any of you have questions, please do not hesitate to shoot me an email in the middle of your week or ask for a zoom meeting to run over problems. We are all here to help you succeed in your academic and prospective professional journey in robotics and Engineering broadly.

### Course Grading Criteria

|  |  |  |  |
| --- | --- | --- | --- |
| Percent | Component | Description | Week Due |
| 20% | Discussions/Online participation) | You are encouraged to engage one another, ask questions about assignments, textbook materials, as well as contribute meaningful discussions that enlighten others on the discussion forum. | Every week |
| 40% | Assignments | There will be weekly assignments to test your knowledge and understanding of each week’s coursework. | Every other Monday |
| 20% | Mid-term take-home examination. | There will be a mid-term examination that is intended to test your knowledge of what you have learned in weeks I through V. The examination will be a set of questions from which you are to answer a subset. The grading of this will count 20% towards your final score for this course. | Week VI |
| 20% | End of term take-home examination. | There will be an end of term examination that is intended to test your knowledge of what you have learned in weeks VI through X. The examination will be a set of questions from which you are to answer a subset. The grading of this will count 20% towards your final score for this course. | Week X |

## Grading Criteria for Discussions/Online Participation

**Discussion responses to instructor posts will be graded according to the criterion outlined below:**

**An exceptional post:**

* Provides original, substantive, and thought-provoking analysis of the course material.
* Is coherent and has a central thesis.
* Contains properly cited references.
* Is grammatically correct and contains no spelling errors.

**A good post:**

* Contains most elements of an exceptional post but may lack coherency and/or have a couple minor spelling/grammatical errors.

**A fair post:**

* Provides only a surface-level analysis of the course material.
* Contains properly cited references.
* Contains a few grammatical and/or spelling errors

**A poor post:**

* Provides only a surface-level analysis of the course material.
* Does not properly cite references.
* Contains several grammatical and/or spelling errors.

Similarly, substantive responses to peer posts will be similarly graded. In addition to the grading metric outlined above, to earn full credit the responses must (1) address the author of the post directly and highlight texts/ideas from the original post and 2) provide constructive insight (i.e. not simply “I agree/disagree with you”).

**Timing**:  
Online discussions should be completed during the course week in which they are assigned. Early or late discussion posts do not earn credit. Your first post should be made by Saturday midnight of each course week; following posts should be made by Tuesday midnight. You are expected to post on at least three different days of the course week.

Unless stated otherwise, you should expect to post substantive answers to each discussion question and at least one response to a post from two other students (3 to 4 substantive posts per week). A substantive post is one that is about 150-250 words, and which makes a useful point or asks a useful question. Posts which are poorly written, which merely quote from external sources, or which merely echo agreement or disagreement

### II. Weekly Information & Assignment Outline

|  |  |
| --- | --- |
| Week 1 | Introduction to Optimization of Functions of Variables |
| **Objectives** | * Minimization, Maximization of Functions of a Variable * Minimization, Maximization of Functions of Two Variables * Analytic Approach and Algebraic Approaches * Definite and Indefinite Forms * Vectors, Vector Addition, Scalar Multiplication |
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| **Learning Materials** | * Introduction to Matrix Analysis by Richard Bellman * Chapter 1, Class Notes |
| **Discussion Topic** | * *Please see Week 1 discussion page in LATTE* |
| **Assignments / Assessments** | * Quiz 1 * Homework 1, 2, 3, 4, 5, 6 |
| Week 2 | Vectors and Matrices |
| **Objectives** | * Inner Product of Two Vectors, Orthogonality * Matrix and Multiplication – Vector by Matrix * Matrix Multiplication – Matrix by Matrix * Noncommutativity, Associativity, Invariant Vectors * Matrix Transpose, Symmetric Matrices |
|
| **Learning Materials** | * Introduction to Matrix Analysis by Richard Bellman * Chapter 2, Class Notes |
| **Discussion Topic** | * *Please see Week 2 discussion page in LATTE* |
| **Assignments / Assessments** | * Homework 7, 8, 9, 10, 11, 12, 13 |
| Week 3 | Introduction to Matrix Analysis |
| **Objectives** | * Hermitian Matrices * Invariance of Distance – Orthogonal Matrices * Unitary Matrices * The Matrix Determinant and its Properties * The Trace of a Matrix * EigenData of a Matrix * The Matrix Inversion Lemma |
|
| **Learning Materials** | * Introduction to Matrix Analysis by Richard Bellman * Chapter II/III, Class Notes |
| **Discussion Topic** | * *Please see Week 3 discussion page in LATTE* |
| **Assignments / Assessments** | * Homework 14, 15, 16, 17 * Quiz 2, 3 |
| Week 4 | Optimal State Estimation |
| **Objectives** | * Metric Spaces, Manifolds, Submanifolds * Distance between line segments, triangle sets * Distance between a point and (i) a parameterized entity, and (ii) an implicit entity * Quaternions * Kabsch Algorithm * Closed-form solutions with least squares * Point set registration with quaternions * Iterative Closest Point Algorithm |
|
| **Learning Materials** | * Besl, P. J. and N. D. McKay (1992). Method for registration of 3-d shapes. In Sensor fusion IV: Control Paradigms and Data Structures, Volume 1611, pp. 586–606. International Society for Optics and Photonics. * Kabsch, W. (1978). A discussion of the solution for the best rotation to relate two sets of vectors. Acta Crystallographica Section A 34(5), 827–828. * Horn, B. K. (1987). Closed-form solution of absolute orientation using unit quaternions. Josa a 4(4), 629–642. * Chapter III, Class Notes |
| **Discussion Topic** | * *Please see Week 4 discussion page in LATTE* |
| **Assignments / Assessments** | * Homework 18, 19, 20 * Quiz 4 |
| Week 5 | Probability Theory |
| **Objectives** | Classical/Frequentist Interpretation of Probability   * What is probability? * Sets, Boolean algebra, Set algebra * Events, Sigma algebras, Probability measure * Rules of probability: Sum rule; Product rule * Statistical independence, Total probability * Bayes’ theorem * Random variables * Probability densities, Probability mass functions * Functions of random variables |
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| **Learning Materials** | * John Woods and Henry Stark. Probability, Statistics, and Random Processes for Engineers. Pearson Higher Ed, 2011. * Martin J Wainwright. High-dimensional statistics: A non-asymptotic viewpoint, volume 48.Cambridge University Press, 2019 * Victor M Panaretos and Yoav Zemel. Statistical aspects of Wasserstein distances. Annual review of statistics and its application, 6:405–431, 2019. * Ben Gravell Slides |
| **Discussion Topic** | * *Please see Week 5 discussion page in LATTE* |
| **Assignments / Assessments** | * Homework P1-1 through P1-4, P2-1 through P2-5 |
| Week 6 | Probability Theory |
| **Objectives** | * Expectations and Moments * Probability bounds * Random vectors * Whitening and coloring transformations * Parameter estimation * Maximum likelihood estimation * Least-squares estimation * Law of large numbers * Central limit theorem |
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| **Learning Materials** | * John Woods and Henry Stark. Probability, Statistics, and Random Processes for Engineers. Pearson Higher Ed, 2011. * Martin J Wainwright. High-dimensional statistics: A non-asymptotic viewpoint, volume 48. Cambridge University Press, 2019 * Victor M Panaretos and Yoav Zemel. Statistical aspects of Wasserstein distances. Annual review of statistics and its application, 6:405–431, 2019. * Ben Gravell Slides |
| **Discussion Topic** | * *Please see Week 5 discussion page in LATTE* |
| **Assignments / Assessments** | * Homework P3-1 through P3-13, P4-1 through P4-4 |
| Week 7 | Information Theory |
| **Objectives** | * Introduction: Entropy, Wasserstein metric * Probabilities and Inference: Clustering and ML * Exact marginalization, Monte Carlo sampling, Variational methods * Sparse graph codes |
|
| **Learning Materials** | * Mackay, D. J. C. (n.d.). *Information Theory, Inference, and Learning Algorithms*. Retrieved August 17, 2017, from <http://www.cambridge.org/0521642981> * Class Slides |
| **Discussion Topic** | * *Please see Week 7 discussion page in LATTE* |
| **Assignments / Assessments** | * TBA |
| Week 8 | State Estimation and Control Systems |
| **Objectives** | * States, Linear Systems, Canonical Forms * Nonlinear Systems: Linearization * Discretization, Simulation * Trapezoidal, Rectangular, Runge-Kutta Integration * Stability, Controllability, Observability * Stabilizability and Detectability |
|
| **Learning Materials** | * Week 8 Learning Guide * Simon, D. (2007). Optimal state estimation: Kalman, H [infinity], and nonlinear approaches. Choice Reviews Online, Vol. 44, pp. 44-3334-44–3334. <https://doi.org/10.5860/choice.44-3334> |
| **Discussion Topic** | * *Please see Week 8 discussion page in LATTE* |
| **Assignments / Assessments** | * TBA |
| Week 9 | Optimal Control |
| **Objectives** | * The problem of Bolza * The problem of Mayer * The problem of Lagrange * Calculus of variations: The Hamiltonian, state, co-state equations * Necessary and sufficient optimality conditions: stationarity conditions; * Transversality conditions |
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| **Learning Materials** | * Richard Bellman, Dynamic Programming. Rand Corporation. 1957 * Differential Dynamic Programming, Jacobson and Mayne, 1970 * Optimal Control: Linear Quadratic Methods. Brian Anderson and John B. Moore |
| **Discussion Topic** | * TBA |
| **Assignments / Assessments** | * TBA |
| Week 10 | Dynamic Programming |
| **Objectives** | * The principle of optimality * Optimality via the optimality principle * Dynamic Programming * Differential Dynamic Programming * Iterative Linear Quadratic Gaussian Control * An introduction to Deep Reinforcement Learning (Tentative) |
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| **Learning Materials** | * TBA |
| **Discussion Topic** | * *TBA* |
| **Assignments / Assessments** | * TBA |

### III. Course Policies and Procedures

## Late Policies

**Late discussion posts** are strongly discouraged as the success of class discussion is dependent upon the active engagement of all participants in the course. Late policies related to discussion posts can be found in the Evaluation Criteria section above.

**For late assignments**, points will be deducted according to the following scale:

1-2 days late -5 points

3-4 days late -10 points

5-6 days late -15 points

7 or more days late - not accepted

\*Late Final Projects cannot be accepted

On rare occasions, personal or professional issues do arise that may warrant an exception to the late policy above. Please notify the instructor at least 24 hours in advance of a due date if an issue arises that will make it impossible for you to meet a stated due date. Exceptions, although rare, will be considered on a case-by-case basis.

## **Grading Standards**

* + Work expectations – Students are responsible to explore each week's materials and submit required work by their due dates.  On average, a student can expect to spend approximately 5-6 hours per week reading and approximately 3-4 hours per week completing assignments and posting to discussions.
  + How points and percentages equate to grades

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 100-94 | A |  | 76-73 | C |
| 93-90 | A- |  | 72-70 | C- |
| 89-87 | B+ |  | 69-67 | D+ |
| 86-83 | B |  | 66-63 | D |
| 82-80 | B- |  | 62-60 | D- |
| 79-77 | C+ |  | 59 or < | F |

## **Feedback**

My goal is to grade homework within a week of the due date.  I will post an announcement if I am delayed in grading for some reason.  If you submit an assignment late, I usually grade it after the following assignment is due, so that my feedback is timely for the greatest number of students.

If you have questions about assignments, the most reliable private way to reach me is via the One on One Discussion forum.  If your question will help the entire class, I may take the liberty of answering it via the Questions and Answers forum.

If you send me a message at my Brandeis email address, I normally respond within 24 hours of receiving it.  However, email may be delayed several days.

## **Confidentiality**

* + We can draw on the wealth of examples from our organizations in class discussions and in our written work. However, it is imperative that we not share information that is confidential, privileged, or proprietary in nature. We must be mindful of any contracts we have agreed to with our companies. In addition, we should respect our fellow classmates and work under the assumption that what is discussed here (as it pertains to the workings of particular organizations) stays within the confines of the classroom.
  + Finally, for your awareness, members of the University's technical staff have access to all course sites to aid in course setup and technical troubleshooting. Program Chairs and a small number of Graduate Professional Studies (GPS) staff have access to all GPS courses for oversight purposes. Students enrolled in GPS courses can expect that individuals other than their fellow classmates and the course instructor(s) may visit their course for various purposes. Their intentions are to aid in technical troubleshooting and to ensure that quality course delivery standards are met. Strict confidentiality of student information is maintained.

## **Time Management**

Students sometimes run into problems related to managing their time, especially in distance learning courses. I hope these ideas will help you to succeed in the class:

* If you are employed full-time, do not take more than two courses at a time. I have never yet met a student who could successfully manage this, especially toward the end of the term as finals and projects come due.
* Keep up with the course week-to-week. Don’t let yourself fall behind on readings, discussion posts, etc. Brandeis courses are not self-paced, and they depend on the collaboration of everyone. Participating late, or “trying to get ahead” is very disruptive.
* Take a look at course assignments early in the week and consider how you will approach the solutions. Ask questions early, so I have time to answer them. Don’t wait until Tuesday night to begin work.
* If you are planning a vacation, plan to continue participating in the class. Thanks to LATTE, you can post discussions and submit homework from virtually anywhere in the world. I will not be able to accept homework late because of a scheduled vacation.
* If you do find yourself short of time, remember that discussion posts count for 30% of your grade. Do not forgo discussions, because it’s impossible to complete the course successfully without them.
* If you experience a serious situation, such as a severe illness, contact me as soon as you can, or contact the GPS office.

## **Find out where to get help.**

For questions or problems with your LATTE course, contact the [24/7 Help Desk](https://brandeis.zendesk.com/hc/en-us). You can email, call, or text your questions using the LATTE Support Channels listed on this webpage. Please also bookmark this resource so you have easy access to these support services moving forward. To search our library materials use our [OneSearch](http://search.library.brandeis.edu/)databaseor contact Library Resources using [Find Your Librarian](http://www.brandeis.edu/library/research/help/liaison-subject-sort.html)

### IV. University and Division of Graduate Professional Studies Standards

Please review the policies and procedures of Graduate Professional Studies, found at http://www.brandeis.edu/gps/students/studentresources/policiesprocedures/index.html. We would like to highlight the following.

## Student Accessibility Support

Brandeis University is committed to providing reasonable accommodations to individuals with appropriately documented physical, learning, or psychological disabilities; short-term and long-term.

Disclosing a disability and requesting accommodations are personal decisions. Brandeis does not require you to disclose the existence of a disability, but requesting accommodations does require that you disclose your disability. Be sure to contact Student Accessibility Support (SAS) as soon as possible to ensure all your accommodations are delivered in a timely manner.

Please refer to www.brandeis.edu/academic-services/accessibility for more information. For any questions regarding the accommodation request process, please contact Jaspreet Mahal, Accessibility Specialist for Graduate Students, at 781-736-3470 or [jaspreet26@brandeis.edu](mailto:jaspreet26@brandeis.edu).

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## Academic Honesty and Student Integrity

Academic honesty and student integrity are of fundamental importance at Brandeis University and we want students to understand this clearly at the start of the term. As stated in the Brandeis Rights and Responsibilities handbook, “Every member of the University Community is expected to maintain the highest standards of academic honesty.  A student shall not receive credit for work that is not the product of the student’s own effort.  A student's name on any written exercise constitutes a statement that the work is the result of the student's own thought and study, stated in the student’s own words, and produced without the assistance of others, except in quotes, footnotes or references with appropriate acknowledgement of the source."  In particular, students must be aware that material (including ideas, phrases, sentences, etc.) taken from the Internet and other sources MUST be appropriately cited if quoted, and footnoted in any written work turned in for this, or any, Brandeis class.  Also, students will not be allowed to collaborate on work except by the specific permission of the instructor. Failure to properly cite resources may result in a referral being made to the Office of Student Development and Judicial Education.  The outcome of this action may involve academic and disciplinary sanctions, which could include (but are not limited to) such penalties as receiving no credit for the assignment in question, receiving no credit for the related course, or suspension or dismissal from the University.   
  
Further information regarding academic integrity may be found in the following publications: "In Pursuit of Excellence - A Guide to Academic Integrity for the Brandeis Community", "(Students') Rights and Responsibilities Handbook", AND " Graduate Professional Studies Student Handbook".  You should read these publications, which all can be accessed from the Graduate Professional Studies Web site.  A student that is in doubt about standards of academic honesty (regarding plagiarism, multiple submissions of written work, unacknowledged or unauthorized collaborative effort, false citation or false data) should consult either the course instructor or other staff of the Rabb School Graduate Professional Studies.

## University Caveat

The above schedule, content, and procedures in this course are subject to change in the event of extenuating circumstances. If you have questions or concerns about course content before the start of the course, please contact the instructor.