1. There are 52 states in the United States of America. Suppose each state in the union is described by a variable xi, where i is the index of a state in the union, how would you "column-vectorize" the United States as a vector X of all states?

95%

To column vectorize the states in the United States of America you can create the vector

You don’t need this exponentiation in your LHS

Depending on the data about each state I wish to capture, it’s possible that each state may have its own states, such at latitude and longitude. These can also be column vectorized and then have X also increase in size by a factor of the number of sub-states to match.

1. Suppose you wanted to travel from Boston to Southampton, UK, by a sailboat. For simplicity purposes, let us assume that the earth's surface is approximated by a Euclidean space. You generate a vector A consisting of waypoints that mark locations on the earth's surface that you must travel through in order to reach Southampton. On your way over to Southampton, though, due to the accumulation of integration errors on your GPS device and your use of the geographic north pole instead of the magnetic north pole as a calibration yardstick, you miss course from your planned waypoint trajectory by a few hundred nautical miles. You arrived in Greenland instead. Call B the vector that includes the actual waypoints with respect to the magnetic north pole (and in the absence of noise on your GPS device). How would you calculate the actual amount of distance missed between your planned waypoint and your actual waypoint as a function of vectors A and B?

If Vector A is the waypoints that I must follow and Vector B is the path I followed due to my waypoint followingerrors, I would calculate the actual amount of distance missed between my planned waypoint and actual waypoint as , assuming A and B are of equal size.

I’ve assumed that due to the use of magnetic north in vector B and geometric north in A we have our vectors A and B in 2 different coordinate systems. Because of this difference we have one vector in what is known as the Earth Centered Inertial frame (vector A) and another in the Earth Centered Earth Fixed (vector B). As a result of this we can write both A and B in terms of either x,y, and z coordinates in their relative systems or . Because we used geometric north in A, and magnetic north in B, to directly correlate these we need to use a rotation matrix. After we have both our vectors in the same coordinate frame we can finally calculate the distance using the Euclidean distance formula with:

1. Suppose I give you an identity matrix  . Suppose further that I give you the representation of an image as a matrix of pixels . If you multiplied P by I, what is the effect that you would observe on the image P?

Multiplying any matrix by the identity matrix I results in the original matrix. Therefore, I would expect no change in the image defined by matrix P. Correct!

1. In (3) above, suppose I was replaced by a scalar c=0.5, if you multiplied c by P, what is the effect on the resolution of P? What happens to the image's resolution when c=2?

Again, assuming that an image is described by the matrix P256x256and multiplying it by a scalar c would change all values equally. Generally, resolution is determined by the size of the matrix, so this 256x256 image wouldn’t change resolution.

If we imagine P as a black and white image, where the value of each ‘cell’ in the matrix is its brightness, with 0 being black, and for instance 255 being white, multiplying the value by c = 0.5 would darken the image by half and reduce contrast, while using c=2 would lighten the image and increase contrast – up until cell value begin to hit the upper limit. If we set no maximum value then this risk is mostly mitigated. Excellent!

1. Now, assuming P were orthonormal, suppose I multiply P by its transpose, what would you expect the values in the resulting matrix to contain?

Assuming P is an orthonormal matrix, multiplying P by its transpose would result in values of 0 or 1 in the orientation of the identity matrix I. No, you should have an Identity. For an orthonormal matrix,

1. Suppose I give you a cube of volume V. If I stacked ten cubes, each of volume V together, how would you use the determinant to explain a physical interpretation of the resulting volume based on what we learned in the class this week?

Given a cube of side length l, and defined by matrix M, the absolute value of M’s determinant would be equal to the cube’s volume. But because we are effectively multiplying one row of our matrix by 10, we get 10\*det M. so the determinant would remain the same but the volume would be multiplied by 10.

This is incorrect, Ryan. If you stacked 10 cubes, each of volume. V, on top of one another, this is equivalent to stretching a side (or all sides) of the cube by a constant amount (in this case, 10). The volume of the overall stack of cubes would be multiplied by 10 and so would the determinant, since the determinant is basically the volume of the cube.

I’ll add my own references:

1. Intuitive way to think about the matrix determinant: <https://math.stackexchange.com/questions/668/whats-an-intuitive-way-to-think-about-the-determinant>
2. Geometric Interpretation of Trace (an aside, but a goodie): <https://mathoverflow.net/questions/13526/geometric-interpretation-of-trace>
3. https://www.askamathematician.com/2013/05/q-why-are-determinants-defined-the-weird-way-they-are/

<https://www.khanacademy.org/math/linear-algebra/matrix-transformations/determinant-depth/v/linear-algebra-determinant-when-row-multiplied-by-scalar>

For the first matrix, if l = 2, det(A) = 8, and det(B) = 80.

So I would use the greatest vertex in each

1. If you flip two rows of a matrix that defines a unit cube, what happens to its orientation?

Defining a unit cube matrix as the following:

Using its axes as x, y, and z and the coordinate system as i, j, and k

Flipping any 2 rows of the coordinate system would cause the unit cube to rotate around the axis of the untouched row. For example, swapping the first and 3rd rows would cause the cube to rotate about the j axis

If you flip two rows of a matrix, you flip the sign of the determinant. Essentially, you are exchanging two directions, or turning the cube inside out.