Further Results in Complex Backward Reach-Avoid Tubes

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***Abstract*— In a recently Algorithm Foundations of Robotics Workshop article submission, Ogunmolu proposed a verification scheme using Hamilton-Jacobi-Isaacs continuous-time model of complex nonlinear systems inspired by murmuratuions' emergent collective behavior. A many-bodied system of agents all moving in a plane with the same speed but with different headings were investigated and he gave a compelling analysis that large value functions can be decomposed into separated cells within the state space and then separately computed by leveraging local nearest neighbor rules utilized by European Starlings in murmurations. TO-DO: In this paper, we provide (i) a learning-based scheme for the dynamical system even in the presence of stochastic noise; (ii) an H_∞ norm bound on the worst-possible disturbance that keeps the system maximally robust; (iii) numerically robust scheme for assemblying the respective local value functions in a way that respects gaps, boundaries, and inter-flocking motion; and (iv) a demonstration of the verification scheme on a system of n autonomous flocks.**

I. INTRODUCTION

II. NOTATIONS AND DEFINITIONS.

Let us now introduce the notations that are commonly used in this article. Time variables e.g. t, t_0, τ, T will always be real numbers. We let $t_0 \le t \le t_f$ denote fixed, ordered values of t. Vectors shall be column-wise stacked and be denoted by small bold-face letters i.e. $\mathbf{e}, \mathbf{u}, \mathbf{v}$ e.t.c. Matrices will be denoted by bold-math Latin upper case fonts e.g. $\mathbf{T}, \mathbf{S}$. Exceptions: the unit matrix is $\boldsymbol{I}$; and i, j, k, p are indices. Positive, negative, increasing, decreasing e.t.c. shall refer to strict corresponding property.

The set S of all $\boldsymbol{x}$ such that $\boldsymbol{x}$ belongs to the real numbers $\mathbb{R}$, and that $\boldsymbol{x}$ is positive shall be written as $S = \{\boldsymbol{x} \,|\, \boldsymbol{x} \in \mathbb{R}, \boldsymbol{x} > 0\}$. The cardinality of S shall be written as $[S]$. We define Ω as the open set in $\mathbb{R}^n$. To avoid the cumbersome phrase "the state $\boldsymbol{x}$ at time t", we will associate the pair $(\boldsymbol{x}, t)$ with the *phase* of the system for a state $\boldsymbol{x}$ at time t. Furthermore, we associate the Cartesian product of Ω and the space $T = \mathbb{R}^1$ of all time values as the *phase space* of $\Omega \times T$. The interior of Ω is denoted by int Ω; whilst the closure of Ω is denoted $\bar{\Omega}$. We denote by $\delta\Omega\, (:= \bar{\Omega} \backslash \text{int } \Omega)$ the boundary of the set Ω.

Unless otherwise stated, vectors $\boldsymbol{u}(t)$ and $\boldsymbol{v}(t)$ are reserved for admissible control (resp. disturbance) at time t. We say $\boldsymbol{u}(t)$ (resp. $\boldsymbol{v}(t)$) is piecewise continuous in t, if for each t, $\boldsymbol{u} \in \mathscr{U}$ (resp. $\boldsymbol{v} \in \mathscr{V}$), $\mathscr{U}$ (resp. $\mathscr{V}$) is a Lebesgue measurable and compact set. At all times, any of $\boldsymbol{u}$ or $\boldsymbol{v}$ will be under the influence of a *player* such that the motion of a state $\boldsymbol{x}$ will be influenced by the will of that player. Our operational domain involves conflicting objectives between various agents e.g. with a heading convergence goal under an external disturbance' influence. For agents that are members of a local coordination group, collision avoidance shall apply so that agents within a local neighborhood cooperate to avoid entropy and predatory pursuer(s). Thus, the problem at hand assumes that of a pursuit *game*. And by a game, we do not necessarily refer to a single game, but rather a *collection of games*. Such a game will terminate when *capture* occurs, that is the distance between players falls below a predetermined threshold.

Each player in a game shall constitute either a pursuer ($\boldsymbol{P}$) or an evader ($\boldsymbol{E}$). The cursory reader should not interpret $\boldsymbol{P}$ or $\boldsymbol{E}$ as controlling a single agent. In complex settings, we may have several pursuers (enemies) or evaders (peaceful citizens). However, when $\boldsymbol{P}$ or $\boldsymbol{E}$ governs the behavior of but one agent, these symbols will denote the agents themselves. The first component of an agent labeled say 5 within a flock labeled 2 is written $\boldsymbol{x}_1^{(5)_2}$. Each evading agent, identified by its label i as a state superscript, is parameterized by three state components: its linear velocities $(\boldsymbol{x}_1^{(i)}, \boldsymbol{x}_2^{(i)})$, and its heading $w^{(i)}$. When we need to distinguish an agent within a flock say F_j from another flock F_k, we shall use the index of the flock e.g. j as a further subscript for a particular agent's state i.e. $\boldsymbol{x}_1^{(i)_j} \neq \boldsymbol{x}_1^{(i)_k}$.

Given the various possibilities of outcomes, the question of what is "best" will be resolved by a *payoff*, Φ, whose extremal over a time interval will constitute a *value*, $\boldsymbol{V}$[1]. We adopt Isaac's [?] language so that if the payoff for a game is finite, we shall have a *game of kind*; and for a game with a continuum of payoffs, we shall have *a game of degree*. The *strategy* executed by $\boldsymbol{P}$ or $\boldsymbol{E}$ during a game shall be denoted by $\alpha \in \mathscr{A}$ (resp. $\beta \in \mathscr{B}$). With this definition, a control law e.g. $\boldsymbol{u}^{(i)}$ played by a player e.g. $\boldsymbol{P}$ will affect *agent i*; and a collection of agents under $\boldsymbol{P}$'s *willpower* be referred to as a *flock*. We shall refer to an aggregation of flocks on a state space as a *murmuration* [2].

III. CONCLUSIONS

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate

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[1]The functional $\boldsymbol{\Phi}$ may be considered a functional mapping from an infinite-dimensional space to the space of real numbers.

[2]The definition of murmurations we use here has a semblance to the murmurations of possibly thousands of starlings observed in nature.

the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

APPENDIX

ACKNOWLEDGMENT