# Annotated Type Rules

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2 Apr 2015

## 1 Type system

Below you will find Figure 1, which describes the annotated type rules for the control flow analysis (CFA) for our extended lambda calculus language for the second assignment of Automatic Program Analysis at Utrecht University.

$$\frac{}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} c : \widehat{\tau}_c}\ [con]$$

$$\frac{\widehat{\Gamma}\ (x) = \widehat{\tau}}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} x : \widehat{\tau}}\ [var]$$

$$\frac{\widehat{\Gamma}\ [x \mapsto \widehat{\tau}_x] \vdash_{\mathrm{CFA}} e_1 : \widehat{\tau}_0}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} \mathbf{fn}_\pi\ x \Rightarrow e_1 : \widehat{\tau}_x \xrightarrow{\{\pi\} \cup \varphi} \widehat{\tau}_0}\ [fn]$$

$$\frac{\widehat{\Gamma}\ [f \mapsto \widehat{\tau}_x \xrightarrow{\{\pi\} \cup \varphi} \widehat{\tau}_0]\ [x \mapsto \widehat{\tau}_x] \vdash_{\mathrm{CFA}} e_1 : \widehat{\tau}_0}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} \mathbf{fun}_\pi\ f\ x \Rightarrow e_1 : \widehat{\tau}_x \xrightarrow{\{\pi\} \cup \varphi} \widehat{\tau}_0}\ [fun]$$

$$\frac{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1 : \widehat{\tau}_2 \xrightarrow{\varphi} \widehat{\tau}_0 \qquad \widehat{\Gamma} \vdash_{\mathrm{CFA}} e_2 : \widehat{\tau}_2}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1\ e_2 : \widehat{\tau}_0}\ [app]$$

$$\frac{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1 : \mathsf{Bool} \qquad \widehat{\Gamma} \vdash_{\mathrm{CFA}} e_2 : \widehat{\tau} \qquad \widehat{\Gamma} \vdash_{\mathrm{CFA}} e_3 : \widehat{\tau}}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} \mathbf{if}\ e_1\ \mathbf{then}\ e_2\ \mathbf{else}\ e_3 : \widehat{\tau}}\ [if]$$

$$\frac{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1 : \widehat{\tau}_1 \qquad \widehat{\Gamma}\ [x \mapsto \widehat{\tau}_1] \vdash_{\mathrm{CFA}} e_2 : \widehat{\tau}_2}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} \mathbf{let}\ x = e_1\ \mathbf{in}\ e_2 : \widehat{\tau}_2}\ [let]$$

$$\frac{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1 : \widehat{\tau}^1_{op} \qquad \widehat{\Gamma} \vdash_{\mathrm{CFA}} e_2 : \widehat{\tau}^2_{op}}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1\ op\ e_2 : \widehat{\tau}_{op}}\ [op]$$

$$\frac{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1 : \widehat{\tau}_1 \qquad \forall i :: \widehat{\Gamma} \vdash_{\mathrm{CFA}} e_i : \widehat{\tau}_1 \qquad \forall j :: \widehat{\Gamma} \vdash_{\mathrm{CFA}} e_j : \widehat{\tau}_0}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} \mathbf{case}\ e_1\ \mathbf{of}\ [\,e_i\ \mathbf{then}\ e_j\,]^+ : \widehat{\tau}_0}\ [case]$$

$$\frac{\widehat{\Gamma} \vdash_{\mathrm{CFA}} x : \widehat{\tau}_0 \qquad \widehat{\Gamma} \vdash_{\mathrm{CFA}} xs : \widehat{\tau}_{list(\widehat{\tau}_0)}}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} (x : xs) : \widehat{\tau}_{list(\widehat{\tau}_0)}}\ [list\text{-}cons]$$

$$\frac{}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} [\,] : \widehat{\tau}_{list(\widehat{\tau}_0)}}\ [list\text{-}nil]$$

$$\frac{\widehat{\Gamma} \vdash_{\mathrm{CFA}} e_1 : \widehat{\tau}_1 \qquad \widehat{\Gamma} \vdash_{\mathrm{CFA}} e_2 : \widehat{\tau}_2}{\widehat{\Gamma} \vdash_{\mathrm{CFA}} (e_1, e_2) : \widehat{\tau}_{pair(\widehat{\tau}_0)}}\ [pair]$$

Figure 1: Control Flow Analysis