Delft University of Technology

AE4866 Propagation and Optimization in Astrodynamics

Assignment 1

Author:
Rob Rikken - 4206738

`https://github.com/robrikken4/PropagationOptimizationAssignments`
Time spend: 41 hours

March 1, 2019

1 Introduction

This report will cover the analysis for selecting the best integrator and propagator for a high thrust transfer from Earth, via Venus, Mars, Venus, to Jupiter. This is done in preparation of the optimisation of this transfer. Before being able to optimise a proper model, integration and propagation settings need to be selected. In this assignment an investigation into the correct integration and propagation settings will be performed. For this analysis the TUDAT will be used to model this high thrust transfer.

2 Set requirements

To be able to determine the best integrator and propagator settings we first need to define requirements with which we can test the results from the analysis. The first requirement that is set will be requirement **REQ-HT-Position-1**: The maximum position error with respect to the benchmark shall not exceed 1 km. This is a very rough estimate and based on the following reasoning. For a one meter order of magnitude accuracy the environment models would be a the dominant error source since only point mass accelerations are used. This can be quantified in the second assignment but for now I will assume the error source due to this model is larger than the one meter order of magnitude. Also a 1000 km accuracy is the same order of magnitude as the radius of the Earth and therefore I find this to large. This concludes me to have a 1 km requirement for the maximum position error.
The second requirement is as follows; **REQ-HT-Time-1**: The computation time for a single full transfer shall not exceed 0.144 second. This is set to be able to run roughly 25000 runs in an hour during the optimisation.

- **REQ-HT-Position-1**: The maximum position error with respect to the benchmark shall not exceed 1 km.
- **REQ-HT-Time-1**: The computation time for a single full transfer shall not exceed 0.144 second.

3 Model and Benchmark

The model used for this case is the point mass gravity of the sun, the departure planet and the destination planet for each leg. For each leg first a semi-analytical solution is found via a patch conic trajectory. From this solution the half way time is determined and used as initial state for the numerical propagation. The numerical propagation propagates forwards and backwards. The termination condition is when the vehicle is within the sphere of influence of the departure and destination planet for the forward and backward propagation.
To be able to determine the error during the analysis a benchmark model needs to be made. The goal of this benchmark is to prove that the error for this run is some orders of magnitude smaller than the requirement for the position error. For the benchmark I selected the RungeKuttaFehlberg4(5) variable step size integrator with a relative and absolute tolerance ranging from 1E-2 till 1E-15. This is the lowest tolerance possible since the precision of the double variable used is a 16 digit precision variable. This integrator is used since it behaves stable for both low and high eccentricity for varying tolerances, as can be seen in the lecture slide Numerical integration slide 72-75. The Encke propagator is used ant it uses an analytical unperturbed orbit as refference and only propagate the deviation from unperturbed orbit. This is perfect since each transfer leg is very close to an unperturbed orbit since the only perturbing forces are the point mass gravity effects from the departure and destination planet for each leg. The full propagation is performed for each of these tolerances and than compared to the reference tolerance, which is the smallest tolerance, 1E-15.

To be able to compare each run with the reference run at the same epoch, the reference run will be interpolated to the same time steps for each run respectively. For this interpolation a Lagrange interpolator with a 8th degree polynomial. During the course Numerical astrodynamics there is proven that the error introduced by such an interpolator is negligible compared to the other error sources. Due to the interpolator the first 4 and last 4 points of the reference run can not be compared since the interpolator can not properly estimate the values due the fact that it uses a 8th degree polynomial. Taking this in to account the final state of the forward and backward integration for each tolerance is compared to the 10E-15 tolerance. The results can be seen in figure 1 and 2.

Here three important things can be noted, the order of magnitude error for both the forward and backward integration is the same therefore I assume this is the case during the analysis and will only look at the forwamr propagation. The second observation is that the the integrator is operating in the truncation error regime due to the linear behaviour. Lastly and most important observation is that the position difference between the 10E-14 and 10E-15 tolerance at final epoch has a order of magnitude of 1E-3 meters. With this it is possible to assume that this error source is multiple orders of magnitude smaller than requirement REQ-HT-Position-1 and therefore it is a valid benchmark with a negligible position error.

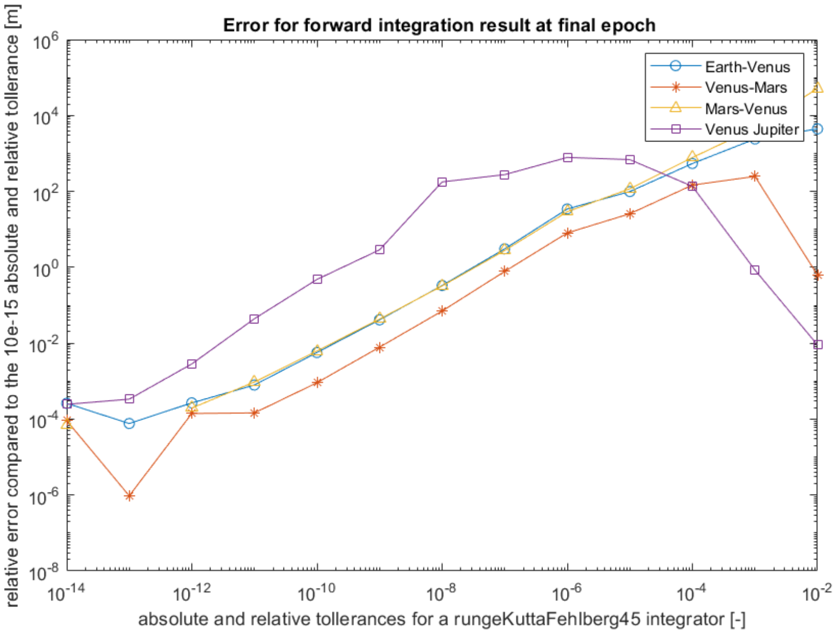

Figure 1: Benchmark run RKF4(5), Cowell, Results for forward integration at final Epoch, comparing 1E-2 till 1E-14 tolerances compared to 1E-15 tolerance.

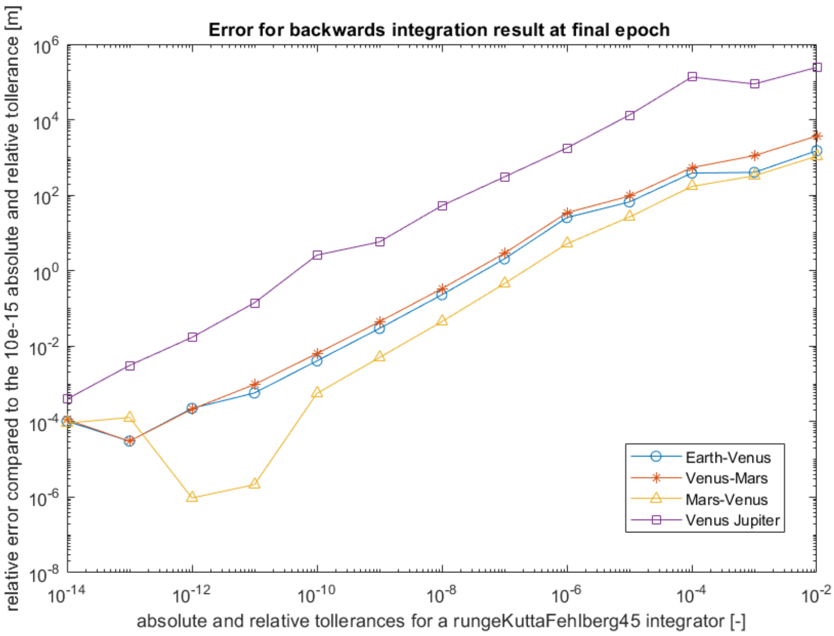

Figure 2: Benchmark run RKF4(5), Cowell, Results for backwards integration at final Epoch, comparing 1E-2 till 10E-14 tolerances compared to 1E-15 tolerance.

4 Integrator and propagator selection

Now with a acceptable benchmark the investigation for the best integrator and propagator for the given requirements can be begin. As described by the assignment it starts with an investigation in to the different integrators. The available integrators within TUDAT are listed below.

- Euler
- RungeKutta4
- RungeKuttaFehlberg4(5)
- RungeKuttaFehlberg5(6)
- RungeKuttaFehlberg7(8)
- RungeKutta8(7)Dormand Prince
- BulirschStoer
- AdamsBashforthMoulton

For the fixed step size integrators(first two) I used a step size of 1E3,1E4,1E5 and 1E6. For all the variable step size integrators except RungeKutta8(7)Dormand Prince a range between 1E-5 till 1E-15 absolute and relative tolerance, whereby these 2 where always equal to each other. For the RungeKutta8(7)Dormand Prince I used a range of tolerances between 1E-5 till 1E-13 because during initial analysis it was found that the integrator is not stable at very low tolerances, as also supported by the theory. For the BulirschStoer an extra variable is investigated next to the tolerances, namely, the number of integrations used for a single extrapolation. This variable ranged from 2 till 12 with increments of two and will be denoted from now as different integraters by calling them, for example, BS2 for the BulirshStoer with 2 integrations per extrapolation. During initial analysis it was found that the AdamBashforthMoulton was not performing as it is supposed to, as it performs very poorly and this is probably due to an error in the base code of TUDAT, said by D. Dirkx, Profesor of this course. Therefore only the variable step size AdamBashforthMoulton with standard settings is included and other settings are excluded due to this reasoning.

The results are given in the figure below. It shows the epoch where the position error is maximum over the full propagation. Figure 3 shows each leg individual and they all 4 show the same trend, only for the last leg the error increases one order of magnitude. I suspect the relative high mass of Jupiter compared to the other planets is the cause of this shift. To be able to have a better analysis the first leg is plotted again, separately to have a closer look at the performance. Here you can see the earlier mentioned poor performance of the AdamBashforthMoulton (ABM). Also the Euler integrator has the expected poor performance from the theory. Roughly half of the integrators show a smooth curve indicating the integrators dominant error source is the truncation error. The other half with a more random behaviour has the rounding error as dominant error source. It is preferred to have an integrator with a dominant truncation error since otherwise your computation power is used less efficiently. Now looking at the requirements, the best performing integrators are: RungeKuttaFehlberg7(8), RungeKutta8(7)Dormand Prince, RungeKuttaFehlberg5(6).‘ The horizontal line at 1E-3 shows the approximate error of the benchmark and for all data around and under this line no valid conclusions can be drawn since it in within the uncertainty domain of the benchmark solution.

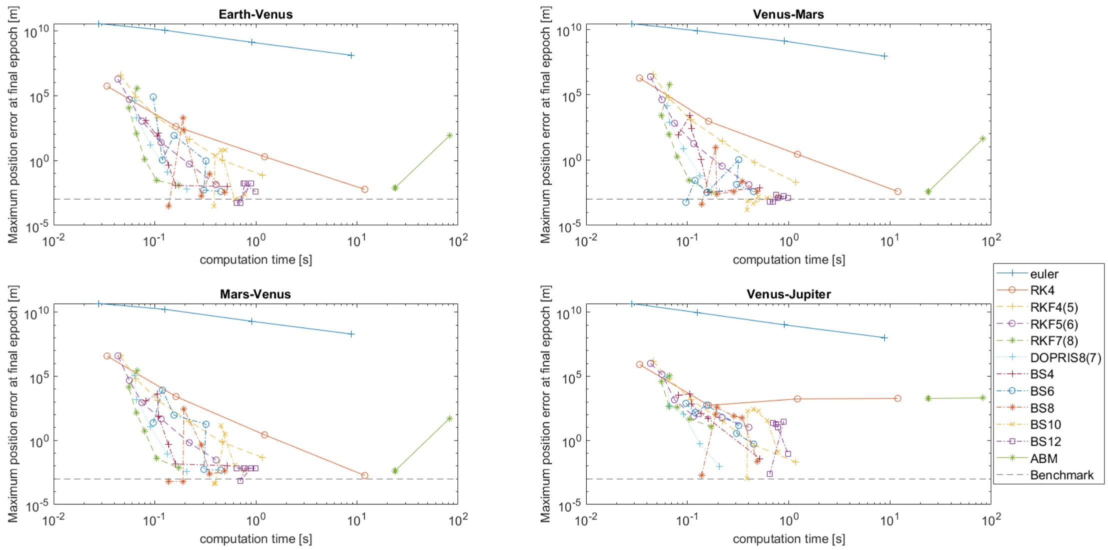

Figure 3: Maximum position error of the different integrators compared to the benchmark model for all four legs.

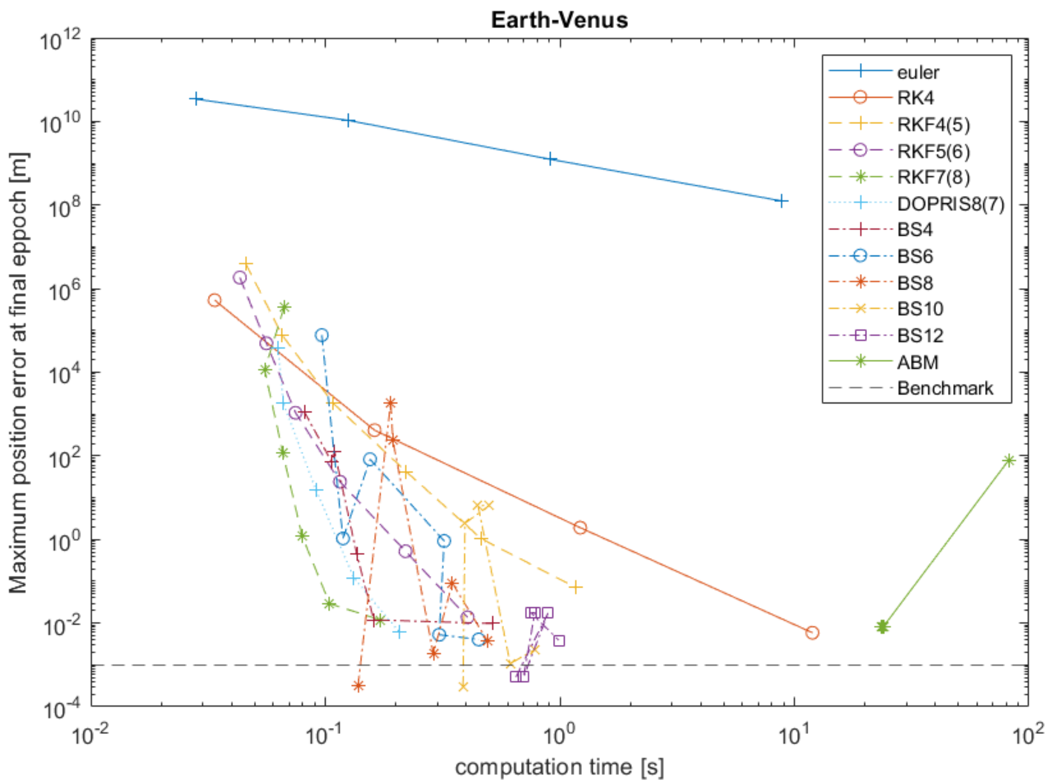

Figure 4: Maximum position error of the different integrators compared to the benchmark model for leg one

No the best propagator needs to be determined. For this analysis a fixed step size integrator needs to be used according to the assignment. Figure 4 shows the RungeKutta4 perform much better than the Euler, therefore the RungeKutta4 is selected for this analysis. A step size of 1E5 will be selected since for the Cowell propagator this resulted in an error of 1E2 meter which is well above the benchmark giving enough room for the varying error for the different propagators without getting close the the benchmark. The list below will show all the propagators used for this analysis.

- Encke
- Cowell
- Gauss Keplerian
- Gauss Modified Equinoctial
- Unified State Model Quaternions
- Unified State Model Modified Rodrigues Parameters
- Unified State Model exponential map

Since the trend is the same for all for leg, only the analysis of the first leg will be shown but the results are the same for all four legs. Now the different propagators are compared to the benchmark and the resulting maximum local position error is shown in figure 5. These errors are plotted against their computation time. The top 3 most accurate propagators are in order as follows; Gauss Modified Equinoctial, Gauss Keplerian and Encke. But accuracy is not the only requirement, the computation time is also a requirement and therefore the fasted 3 propagators; Unified State Model Modified Rodrigues Parameters, Unified State Model Quaternions and Cowell, will be used to do final comparison with the 3 best performing integrators for the current requirements.

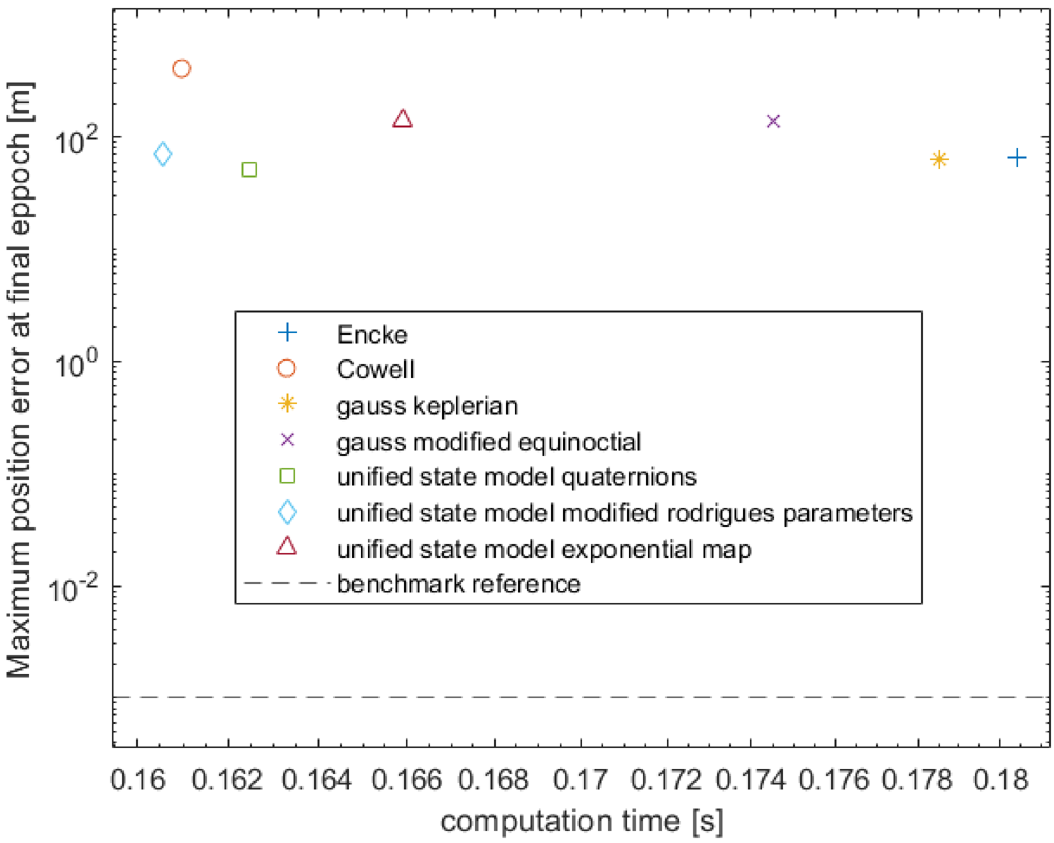

Figure 5: Maximum position error for different propagators compared to the computation time.

Nine combinations of the 3 best performing integrators and the 3 fasted propagators are compared to determine which combination of integrators and propagators is the best for the set of requirements of this simulation. The integrators with their used tolerances are listed below. These runs are than again compared to the benchmark and the maximum position error is determined. This can be seen in figure 6. For the current set of requirements there are several solutions possible. All the points within the black marked box in figure 6 are solutions for this problem. To find a single solution an extra requirement will be added; **REQ-HT-Time-2**: the computation time of a single run needs to be minimised. Now the run with an error smaller than 1 km and the lowest run time is the best solution for this simulator. Than doing this analysis for the different legs, shown in figure 7 8 and 9 for leg two, three and four respectively, the optimal solution is different for each leg, but the trend is still similar and a reasonable solution for all four legs would be the RungeKuttaFehlberg7(8) with an absolute and relative tolerance of 1E-9 and the Cowell propagator. With this combination all requirements are met for all legs. Also due to the large range of tolerances the figure 6 can be used as a good starting position for a new analysis with other values and requirements, or a new iteration on this analysis.

- RungeKuttaFehlberg7(8) - 1E-7 till 1E-13
- RungeKutta8(7)Dormand Prince - 1E-5 till 1E-11
- RungeKuttaFehlberg5(6) - 1E-7 till 1E-12
- Unified State Model Modified Rodrigues Parameters
- Unified State Model Quaternions
- Cowell

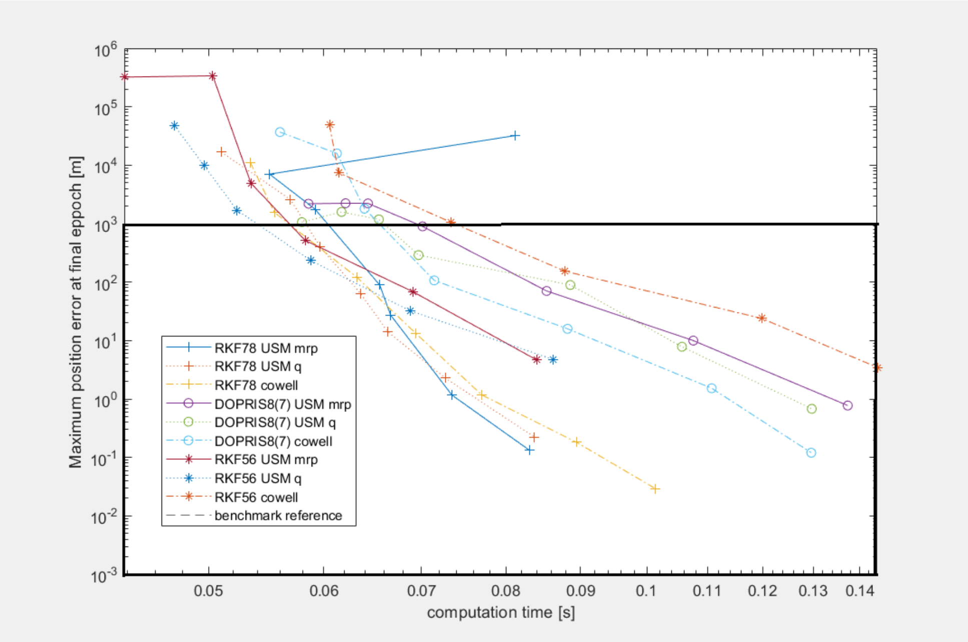

Figure 6: Maximum position error for a combination of integrators and propagators for the Earth-Venus leg where the black box is the solution for the set requirements.

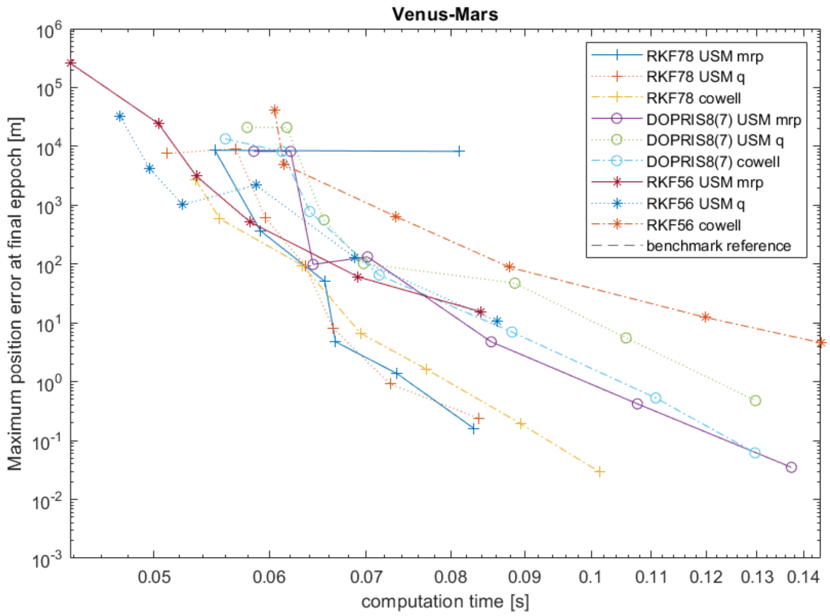

Figure 7: Maximum position error for a combination of integrators and propagators for the Venus-mars leg.

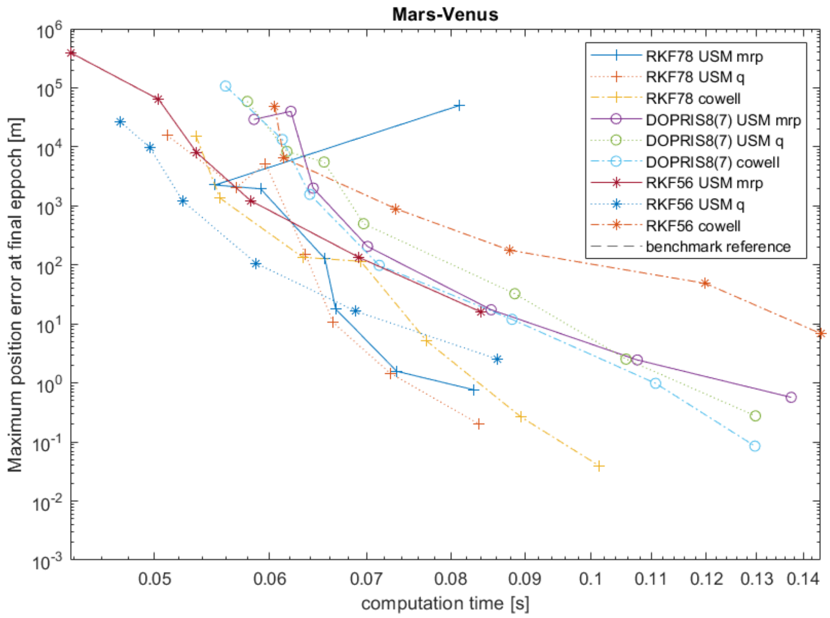

Figure 8: Maximum position error for a combination of integrators and propagators for the Mars-Venus leg.

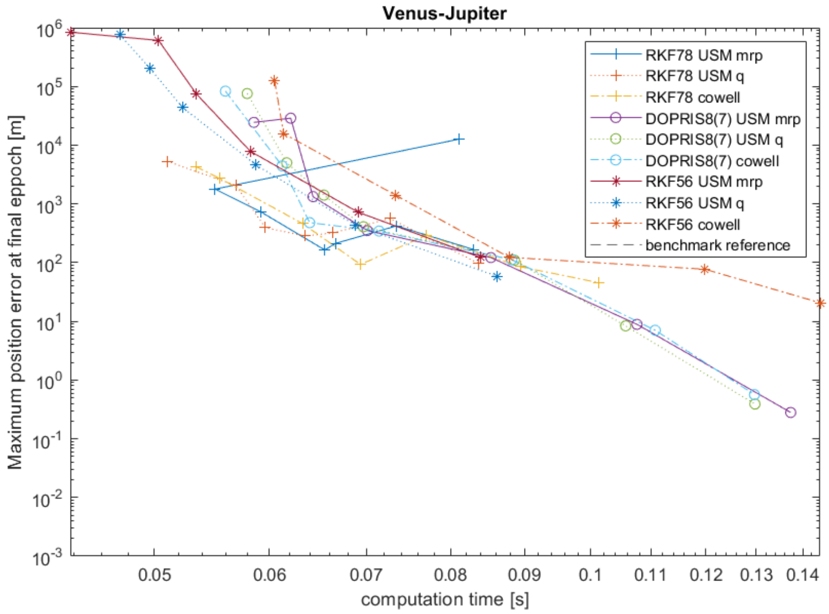

Figure 9: Maximum position error for a combination of integrators and propagators for the Venus-Jupiter leg.

5 Model improvements

In the beginning of this report the environmental model was explained. This environmental model is a very simplified version of the reality. In this section I will propose model improvements to reduce the error in the environment. First of all the solar radiation pressure will be added because the spacecraft is almost constant exposed to the sun and its pressure. Also the during the transfer the spacecraft swings by Venus two times which means the spacecraft comes closer to sun, intensifying the effect since the radiation pressure scales with an vector distance to the sun squared. Also the swing by by the planet in combination with the applied thrust can be propagated instead of only propagating the transfer in the sphere of influence. The Thrust can be modelled in a finite time instead of an instantaneous delta-V increase.