Exercises
Functions

1. Using library functions, define a function

   ```
   halve :: [a] -> ([a], [a])
   ```

   that splits an even-lengthed list into two halves. For example:

   ```
   *Main> halve [1,2,3,4,5,6]
   ([1,2,3],[4,5,6])
   *Main>
   ```

2. Define a function ***third***

   ```
   third :: [a] -> a
   ```

   that returns the third element in a list that contains at least this many elements using:

   (a) ***head*** and ***tail***;

   (b) list indexing ***!!***;

   (c) pattern matching.

3. Consider a function ***safetail*** that behaves in the same way as ***tail***, except that ***safetail*** maps the empty list to the empty list, whereas tail gives an error in this case. Define ***safetail*** using:

   (a) a conditional expression;

   (b) guarded equations;

   (c) pattern matching.

4. In a similar way to ***&&*** in this section's slides, show how the disjunction operator || can be defined in four different ways using pattern matching.

   Hint: the library function ***null*** :: [a] $->$ Bool can be used to test if a list is empty.

5. Show how the meaning of the following curried function definition can be formalised in terms of lambda expressions:

   ```
   mult :: Int -> Int -> Int -> Int
   mult x y z = x*y*z
   ```

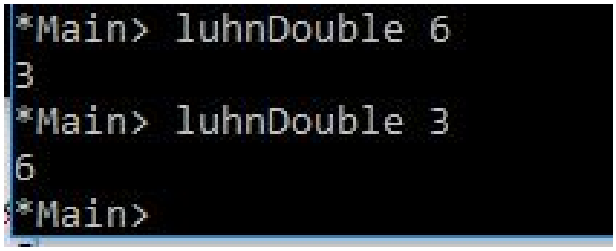
6. The ***Luhn Algorithm*** is used to check bank card numbers for simple errors such as mistyping a digit and proceeds as follows:

   - consider each digit as a separate number;
   - moving left, double every other number from the second last;
   - subtract 9 from each number that is now greater than 9;
   - add all the resulting numbers together;
   - if the total is divisible by 10, the card number is valid.

   Define a function

   ```
   luhnDouble :: Int -> Int
   ```

   that doubles the a digit and subtracts 9 if the number is greater than 9. For example

   Using ***luhnDouble*** and the integer remainder function ***mod***, define a function

   ```
   luhn :: Int -> Int-> Int-> Int-> Bool
   ```

   that decides if a four digit number is valid. For example: