

Question 1)

- **Acquire around 1000 samples of the signals and show a plot of these signals**

The potential divider was built using $1\text{ M}\Omega$ resistors which formed a pseudo ground for the “L” and “R” signals. In the graph below ADC1 is “L” and ADC0 is “R”. Acquiring 1100 samples and discarding the first 100 for equally time spaced data we obtain

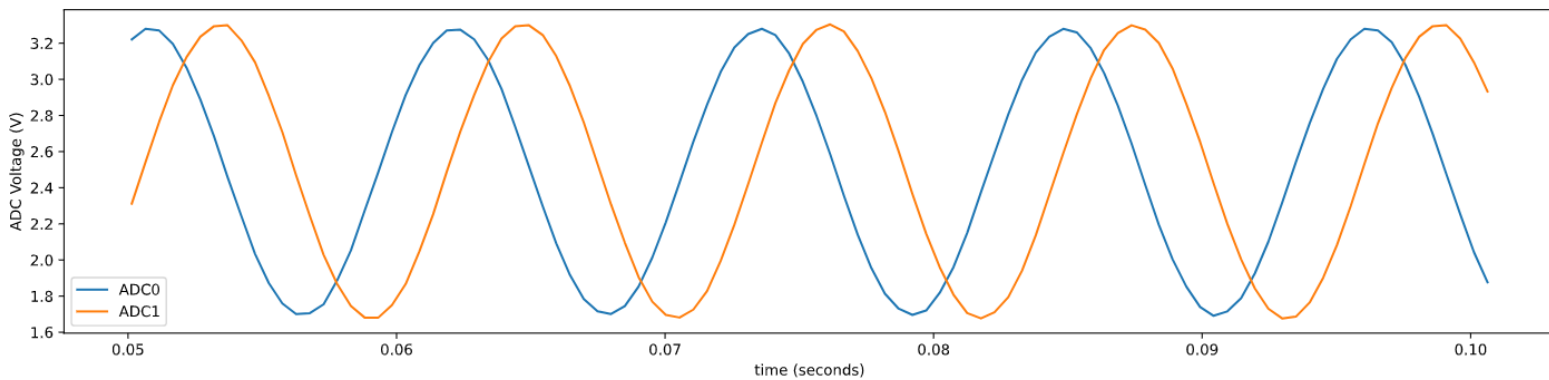


Figure 1: Signal plot of 88 Hz signals from the “L” and “R” channels using a pseudo ground of 2.5V

- **Determine the frequency and amplitude of the fundamental Fourier component of each signal using the fast Fourier transform (FFT); show the resulting graph of the FFT spectrum. [Hint: These frequencies should both be close to 88 Hz!]**

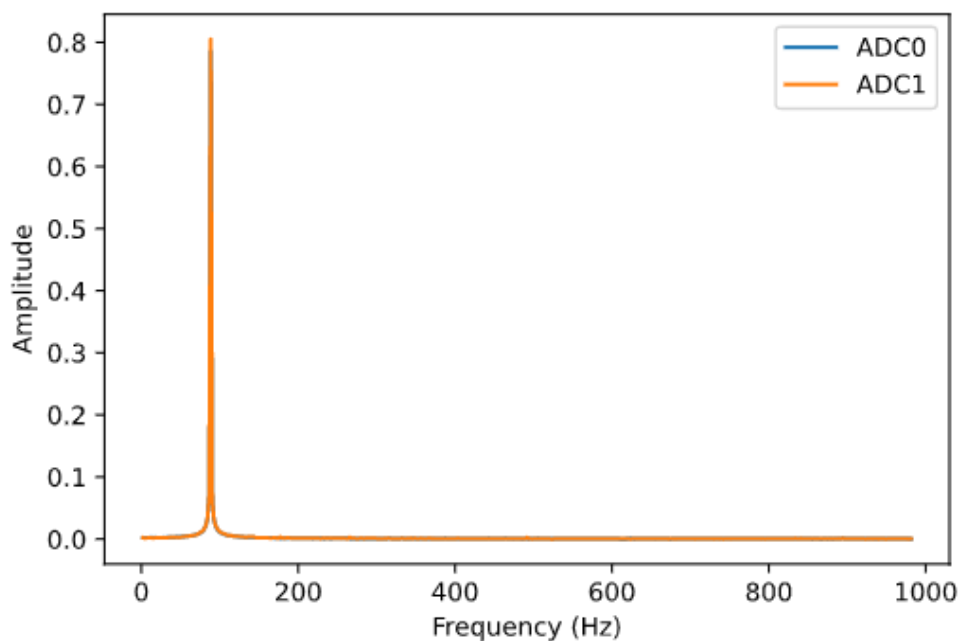


Figure 2: FFT plot of 88 Hz generated signal

From the FFT analysis we get the following results

Signal output	Arduino input	Fundamental frequency (Hz) $\pm 2\text{Hz}$	Fundamental frequency Amplitude (V) $\pm 0.005\text{V}$
"L"	ADC1	88	0.805
"R"	ADC0	88	0.785

Now use the audio file that you have been provided to generate your 'mystery signal'.

- **Acquire around 1000-2000 samples of the signals and show a plot of these signals.**

Sampling 2100 and discarding the first 100 we obtain the graph below with ADC0 being "L" and ADC1 being "R"

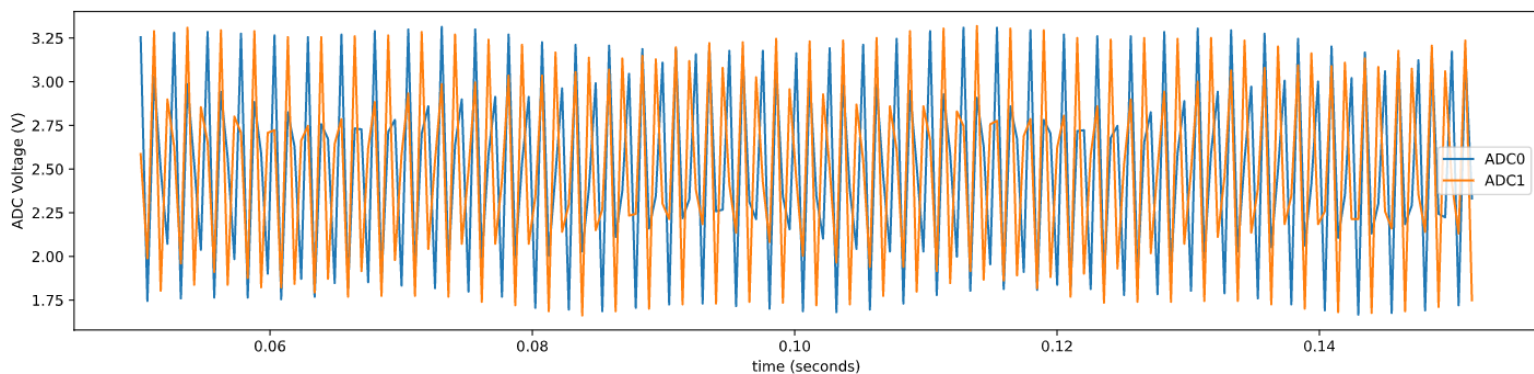


Figure 3: Signal plot of mystery signal using 2100 samples

- **Determine the frequency and amplitude of the largest-amplitude Fourier component of each signal using the FFT; show the resulting graph of the FFT spectra. [Hint: These frequencies should be the same, and somewhere in the range 600 - 800 Hz.]**

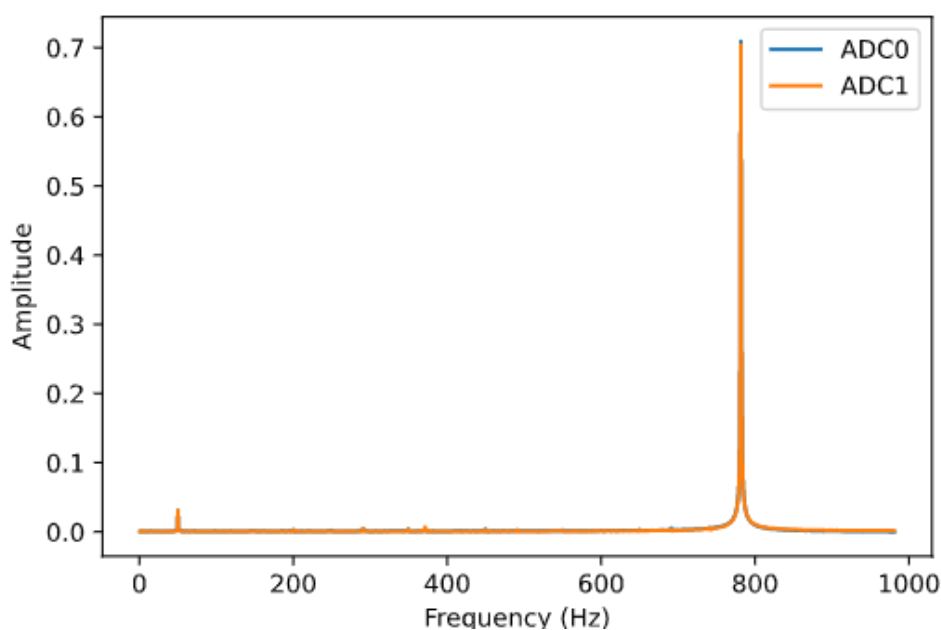


Figure 4: FFT plot of mystery signal using 2100 samples

From the FFT analysis we get the following results

Signal output	Arduino input	Fundamental frequency (Hz) ± 1Hz	Fundamental frequency Amplitude (V) ± 0.005V
"L"	ADC0	782	0.709
"R"	ADC1	782	0.704

- **Calculate how many samples per 'cycle' of this high-frequency mystery signal you expect to measure at the maximum sampling rate of your Arduino**

To calculate how many samples per cycle we first calculate how long it takes for one sample to be taken,

$$\text{Time for one sample} = \frac{\text{Measuring time}}{\text{Number of samples}}$$

. Then we can calculate the number of samples per "cycle" using

$$\text{Samples per cycle} = \frac{1}{\text{Fundamental frequency} \cdot \text{Time for one sample}}$$

. In the case of my circuit, we obtain

$$\text{Samples per cycle} = 2.51$$

- **Determine the maximum voltage gain that you would expect to be able to apply equally to both signals before they saturate, nominally exceeding the 0-5 V limit of both the amplifier output and the Arduino analog inputs. Explain why you would not want to apply a larger gain than this.**

We know the maximum voltage that the Arduino can measure is 5 volts but we are offset from ground by 2.5 V hence we can use

$$\text{Gain} = \frac{V_{out}}{V_{in}} = \frac{5}{3.209} = 1.56$$

Note that gain does not have any units.

Exceeding saturation would mean that the waveform would not be transmitted in its original shape. This is caused by clipping of the signal at the high and low voltages as we exceed the range at which the Arduino can measure. Hence, we lose information about what the original waveform was transmitting.

- **The signal that we are interested in is encoded as the difference between the two waveforms. Explain why you might not expect to be able to extract this signal reliably from the sampled dataset.**

The random error (noise) of the two signals means that we get compounding errors in the extracted waveform. As the extracted waveform will be small (the two signals have similar amplitudes) then the percentage error on the extracted waveform will be large, this means that the extracted waveform is very susceptible to random fluctuations and hence will be unreliable.

Question 2)

- *Using the `frequency_generator.py` code and your Arduino, demonstrate that each of your unity-gain buffers functions as intended; producing a voltage-gain of 1, and phase-shift of zero.*

Producing a signal of 88 Hz, taking 2100 samples and discarding the first 100 for equally time spaced data we produce the graphs below where the prebuffer signal is ADC0 and postbuffer signal ADC1

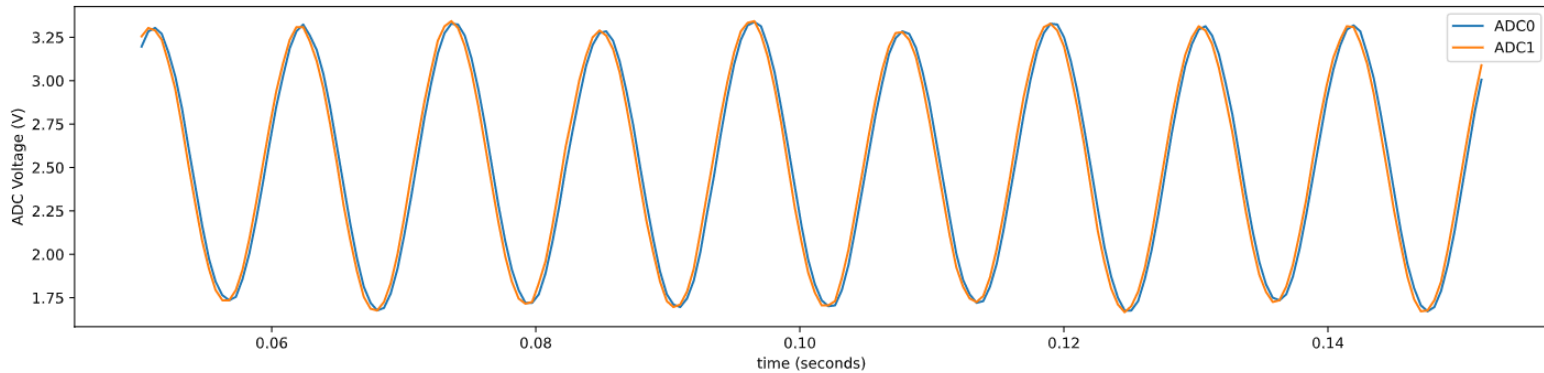


Figure 5: Signal plot of 88 Hz signal of the first unitary gain buffer

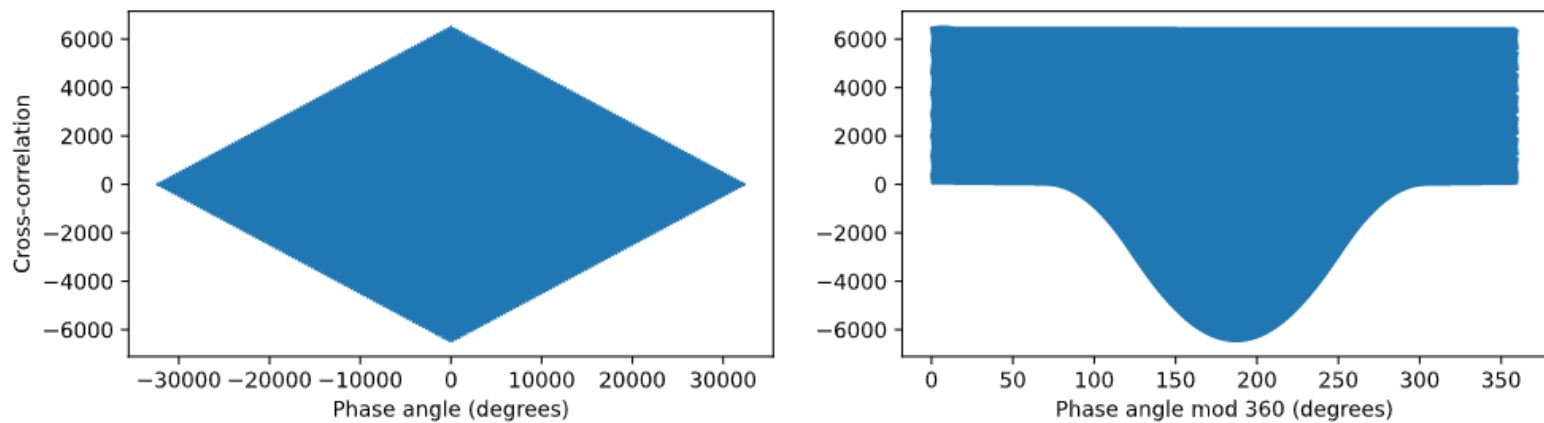


Figure 6: Phase shift plots for the 88 Hz signal for the first unitary gain buffer

Similar graphs were obtained for the second unitary buffer but are not included. The results from the two buffers are summarised in the table below:

Signal output	Arduino input	Fundamental frequency (Hz) ± 1 Hz	Fundamental frequency Amplitude (V) ± 0.005 V
(1 st buffer) "L"	ADC0	88	0.776
(1 st buffer) "L" after buffer	ADC1	88	0.776
(2 nd buffer) "L"	ADC0	88	0.776
(2 nd buffer) "L" after buffer	ADC1	88	0.776

Further we can calculate the phase shift using cross correlation and also calculate the gain in each case with results summarised in the following table

Op amp	Voltage Gain	Phase shift (degrees) ± 2 degrees
1 st buffer	1.00	8
2 nd buffer	1.00	8

We can see from this that the Voltage gain is just 1 V in each case. The phase shift is given as 8 degrees however when measuring the unbuffered “L” signal using both measuring ports of the Arduino we find that the phase shift is again not quite zero.

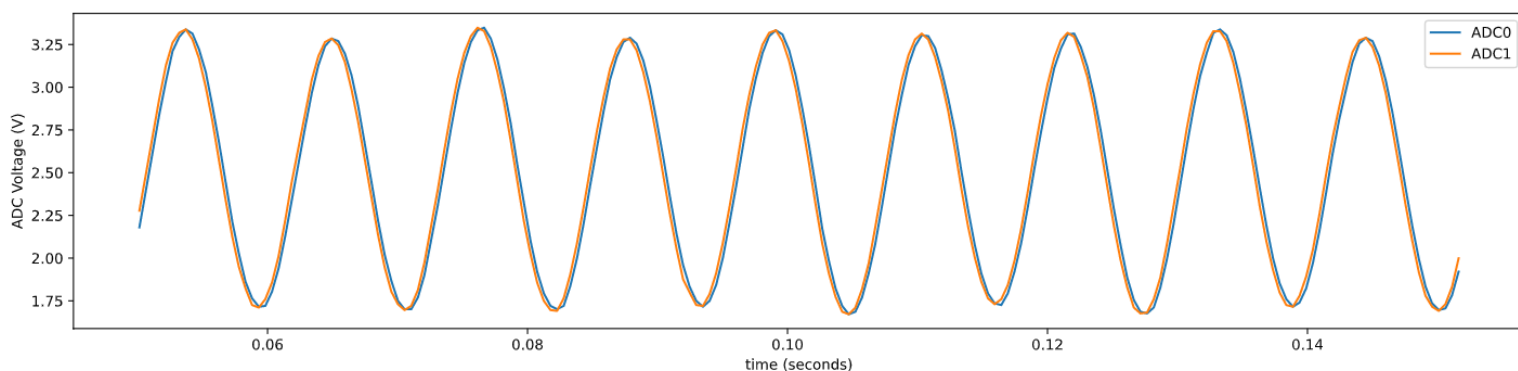


Figure 7: Measuring “L” of 88 Hz signal using both ports of the Arduino

In this case the phase shift is given by 6 ± 2 degrees we can see that the phase shift of the signals between the op amp lie within experimental error of the unbuffered signal phase shift. The reason that this signal is offset from 0 is that the Arduino has a finite sampling speed and can only take one measurement at a time hence we get an apparent phase shift of 6 ± 2 degrees between measurement of both signals. Thus we have demonstrated that the unitary buffers work as intended.

Question 3)

Use components from your Laboratory Kit, including a second op-amp chip, to construct a differential amplifier with a voltage gain of 1.

- **Connect the inputs to voltages supplied by the fixed-voltage outputs of your Arduino such that $V_1 = 3.3\text{ V}$ and $V_2 = 5\text{ V}$. Make a suitable measurement to determine a value for V_{out} with an uncertainty. Comment on the result.**

Building the circuit as described in the question using resistor values of $10\text{ k}\Omega$ to obtain a gain of 1, we obtain the following signal plot where ADC0 is V_{out} and ADC1 is connected to ground

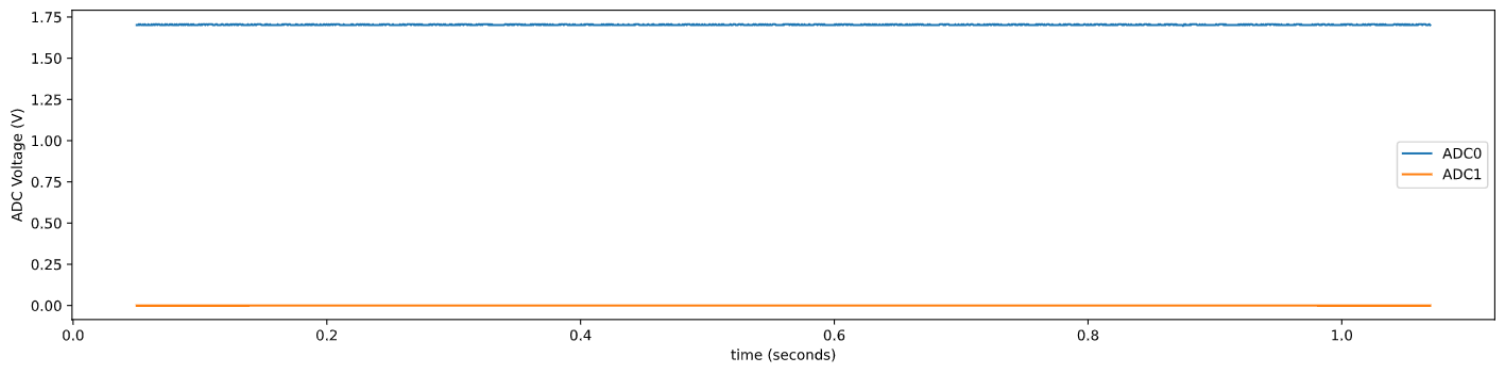


Figure 8: Test of differential amplifier with inputs $V_1 = 3.3\text{ V}$ and $V_2 = 5.0\text{ V}$

We take the time average of the voltage to get the mean and standard error of the signal and obtain the following results:

Signal output	Arduino input	Average voltage (V)
V_{out}	ADC0	1.705 ± 0.005

The average value seems stable across the time range and it is what we expect as using the equation in the question we get

$$V_{out} = R_2/R_1(V_2 - V_1) = (5.0 - 3.3) = 1.7\text{ V}$$

Which is what we find experimentally to within experimental error. The calculated standard error ($\pm 0.00005\text{ V}$) is less than the resolution of the Arduino so the error on the Arduino dominates this term hence the error in the table is just the error due to the resolution.

Question 4)

- **Acquire 1000-2000 samples of the V_1 and V_{out} signals using your Arduino, and show a plot of these signals. Ensure that it is made clear which signal is which.**

Connecting the circuit as in the question using 2100 readings (discarding the first 100) using the “L” channel signal at 88 Hz we obtain the graph below. Here ADC1 is the output from the differential amplifier and ADC0 is in the input to the differential amplifier.

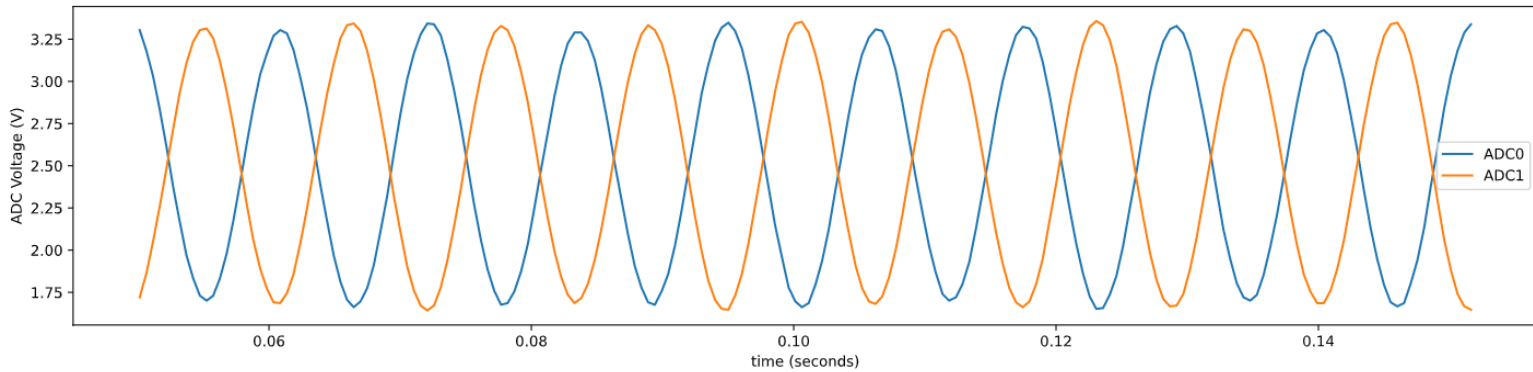


Figure 9: 88 Hz input signal and output signal of a differential amplifier with the second input set to 5V

- **Determine the phase-shift between the pair of signals, and comment on the result**

The phase shift at 88 Hz is given as -173 ± 2 degrees. The negative sign indicates the V_{out} lags V_1 . This is a phase shift lag of around 180 degrees, this is what we expect from rearranging the equation in question 3, fixing V_2 to be 5 V

$$V_{out} + V_1 = 5 \text{ v}$$

This is what we observe happening in figure 9 as the two waves centred around 2.5 V always add to 5 V to within experimental error. This is acting as a signal inverter causing a low input signal to be a high output signal and a high input signal to be a low output signal. The difference from 180 degrees can be explained as in question 2 by the finite sampling speed of the Arduino causing a small apparent phase shift of around 6 ± 2 degrees so this matches the expected 180 degrees to within experimental error.

Question 5)

- Acquire 1000-2000 samples of the V_1 and V_2 signals using your Arduino, and show a plot of these signals. Ensure that it is made clear which signal is which. Determine the frequency and amplitude of the largest-amplitude Fourier component of each signal, and show the FFT spectra - do the results match what you found previously?

Using 2100 samples (discarding first 100) with ADC0 being “L” though the unitary buffer and ADC1 being “R” though the unitary buffer we get the following plots.

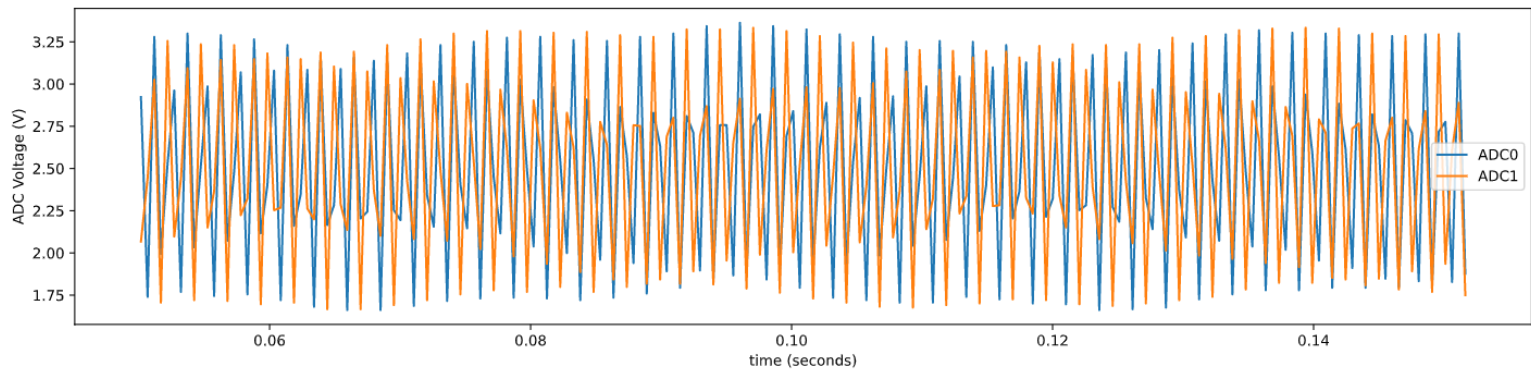


Figure 10: Inputs into the differential amplifier from the mystery signal

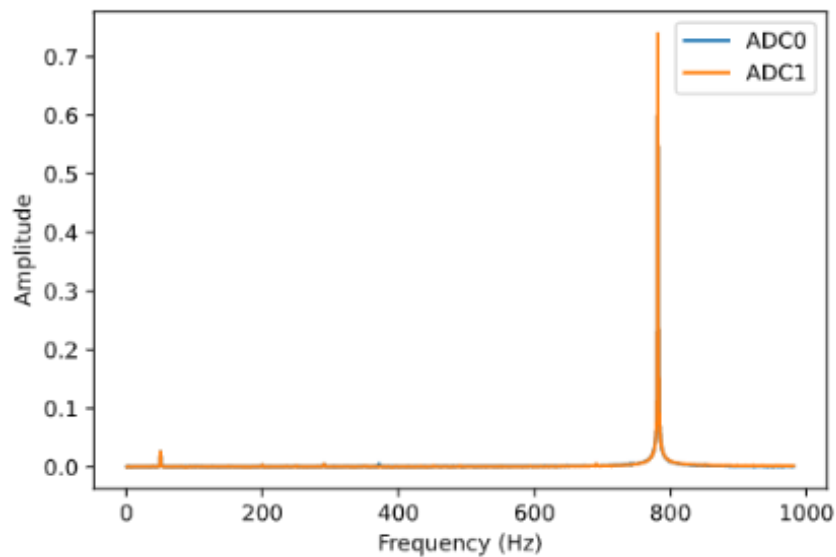


Figure 11: FFT plot of inputs to the differential amplifier

We obtain the following results:

Signal output	Arduino input	Fundamental frequency (Hz) ± 1 Hz	Fundamental frequency Amplitude (V) ± 0.005 V
“L” though unitary buffer	ADC0	782	0.737
“R” though unitary buffer	ADC1	782	0.740

Comparing the results from the table in question 1 we find that the fundamental frequencies match what we expect. We get a very slight gain in Fourier amplitude possible due to the buffer not quite being unitary gain due to internal resistance of the op-amp.

We can also compare graphs with question 1 and find that they match well as we expect.

- ***Acquire and plot 1000-2000 samples of the V_{out} signal. Take the FFT of the acquired signal and show the resulting graph of the FFT spectrum. Comment on the result. [Hint: Remember that the resolution of the Arduino analog input is ~ 5 mV, and your differential amplifier only has unity voltage gain.]***

We take 2100 samples (discarding the first 100) and obtain the following graphs where ADC0 is V_{out} and ADC1 is kept at ground.

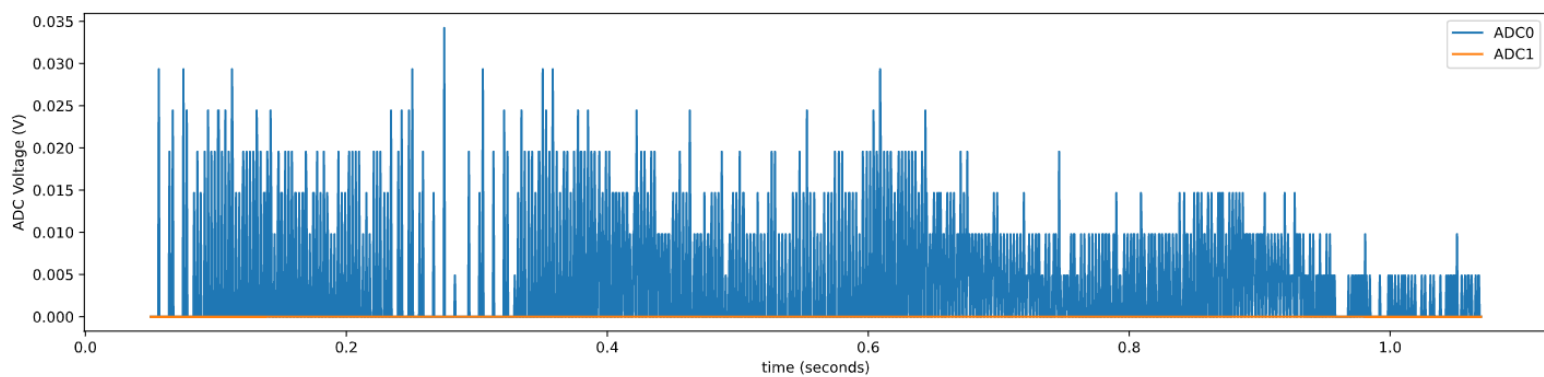


Figure 12: Signal plot of voltage output for differential amplifier of gain one using the mystery signal

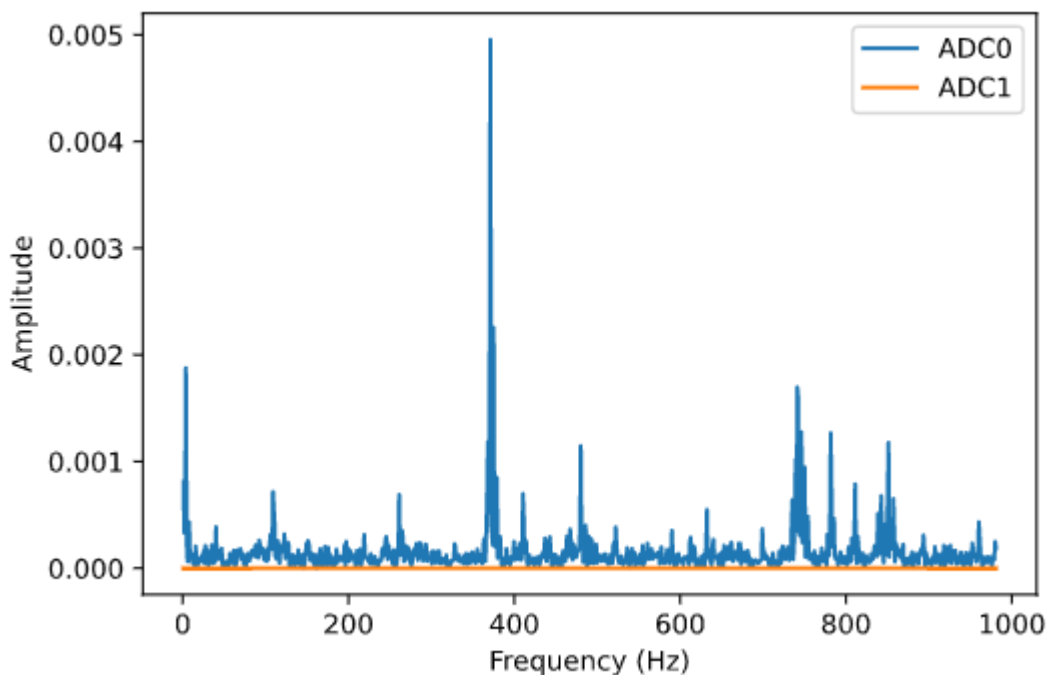


Figure 13: FFT plot of voltage output for differential amplifier of one gain using the mystery signal

We obtain the following results:

Signal output	Arduino input	Fundamental frequency (Hz) $\pm 1\text{Hz}$	Fundamental frequency Amplitude (V) $\pm 0.005\text{V}$
V_{out}	ADC0	371	0.005

We see from the signal plot that the resulting signal is discretised in units of the minimum resolution of the Arduino. This can be explained by the fact that the difference between the two original waveforms is very small (in addition to only having a gain of one) and we only have a finite resolution of 5mV from the Arduino. Hence the resulting waveform amplitude moves in discrete steps.

Question 6)

- **Acquire and plot 1000-2000 samples of the V_{out} signal. Take the FFT of the acquired signal and show the resulting graph of the FFT spectrum. State any values determined, and comment on the result.**

We want the gain to be 10 so we pick R2 to be 100 K Ω and R1 to be 10 K Ω .

Taking 2100 reading (discarding the first 100) we obtain the following graphs where ADC0 is V_{out} and ADC1 is connected to ground.

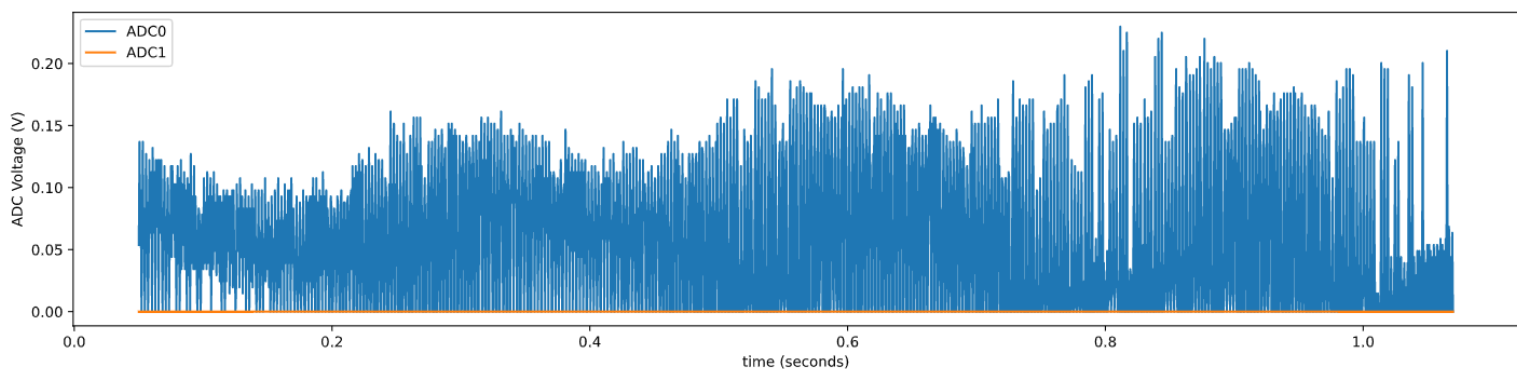


Figure 14: Output of differential amplifier with gain 10 using the mystery signal

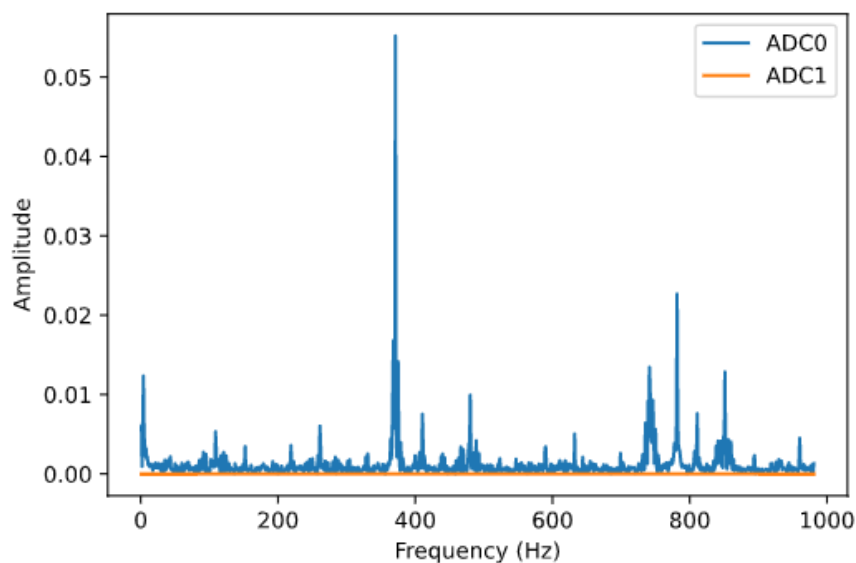


Figure 15 : FFT plot of the output of differential amplifier with gain 10 using the mystery signal

From the FFT of the signal we obtain the following results.

Signal output	Arduino input	Fundamental frequency (Hz) \pm 1Hz	Fundamental frequency Amplitude (V) \pm 0.005V
V_{out}	ADC0	371	0.055

The fundamental frequency matches that of the gain one plot in question 5 and the fundamental frequency amplitude is around 10 times bigger than in question 5 which is what we expect with a circuit with gain 10. The signal appears to have a lot of variation potentially due to noise and no clear pattern can be observed with the number of samples collected. Considering the FFT plot we see that the carrier frequency at around 780 Hz is still being transmitted with this peak being larger than the peak at the very low frequencies (whereas they are similar when considering gain of 1).

Question 7)

- ***State the overall designed gain of your instrumentation amplifier as both the voltage gain, and in dB units***

Following the circuit diagram in the question we used the resistor values:

Resistor label	Resistor value (K Ω)
R	22
R_{gain}	10
R1	10
R2	100

To calculate gain we use the formula below

$$Gain = \left(1 + \frac{2R}{R_{gain}}\right) \cdot \frac{R_2}{R_1}$$

Using the values from the table and the equation above we get that Gain = 54.0

To get Decibel gain we use

$$dB_{gain} = 20 \cdot \log_{10}(Gain)$$

Using the gain that we calculated we get Decibel gain = 34.6 dB

- **Acquire and plot 1000-2000 samples of the V_{out} signal. Take the FFT of the acquired signal and show the resulting graph of the FFT spectrum.**

Obtaining 2100 samples (discarding the first 100) we get the graphs below with ADC0 being the V_{out} signal and ADC1 being held at ground.

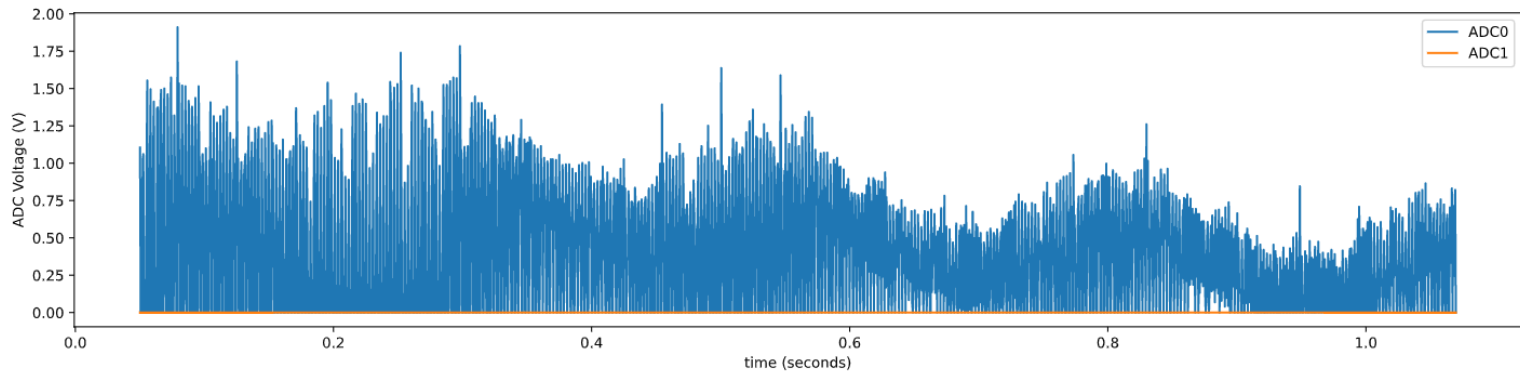


Figure 16: Plot of the output voltage for the instrumentation amplifier with the mystery signal as the input with gain 54.0

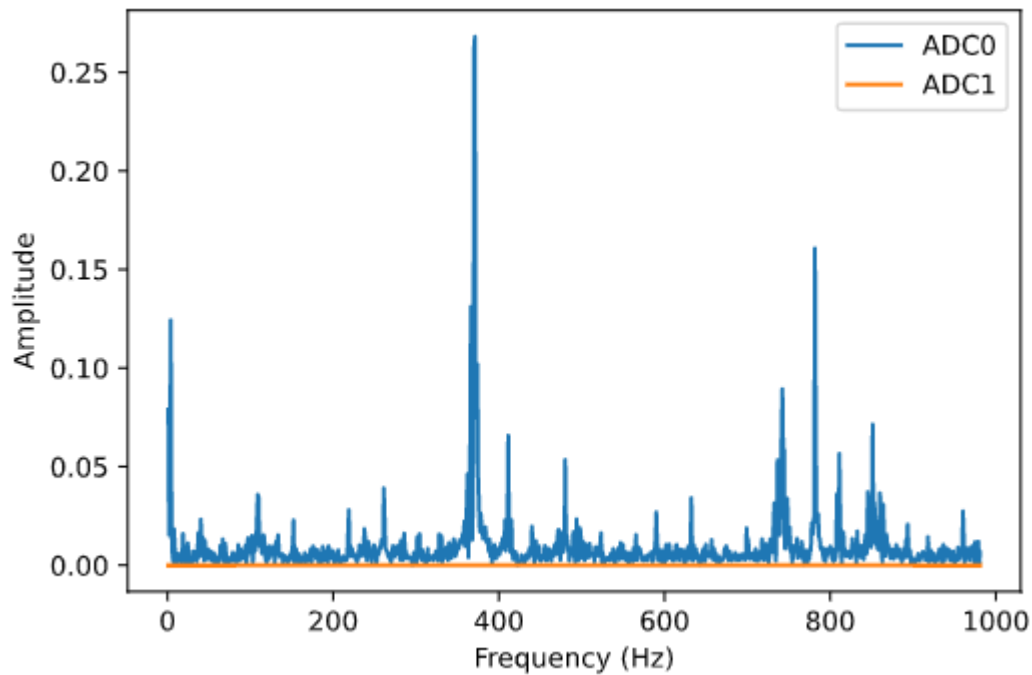


Figure 17: FFT plot of the signal described in figure 16

We obtain the following results

Signal output	Arduino input	Fundamental frequency (Hz) ± 1 Hz	Fundamental frequency Amplitude (V) ± 0.005 V
V_{out}	ADC0	371	0.268

- ***Increase the number of samples to around 20000. Determine the FFT and plot the resulting spectrum so as to clearly show the main frequency components of the signal.***

Increasing the number of samples to 20100 (discarding first 100) we obtain the following graphs with ADC0 being the V_{out} signal and ADC1 being held at ground.

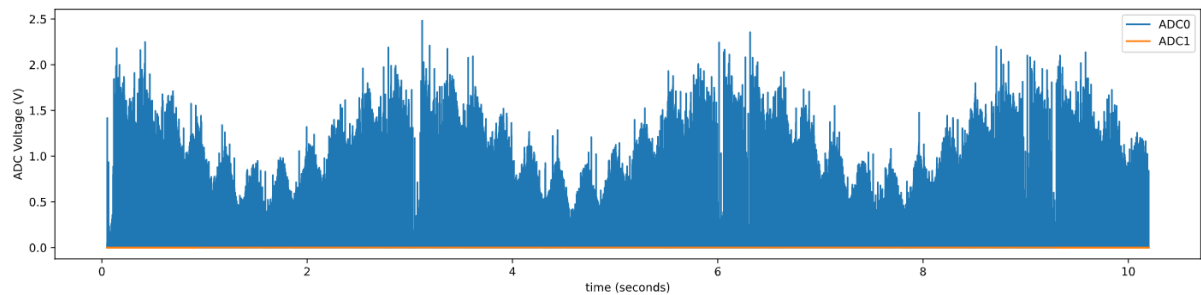


Figure 18: output of instrumentation amplifier with mystery signal for 20000 samples

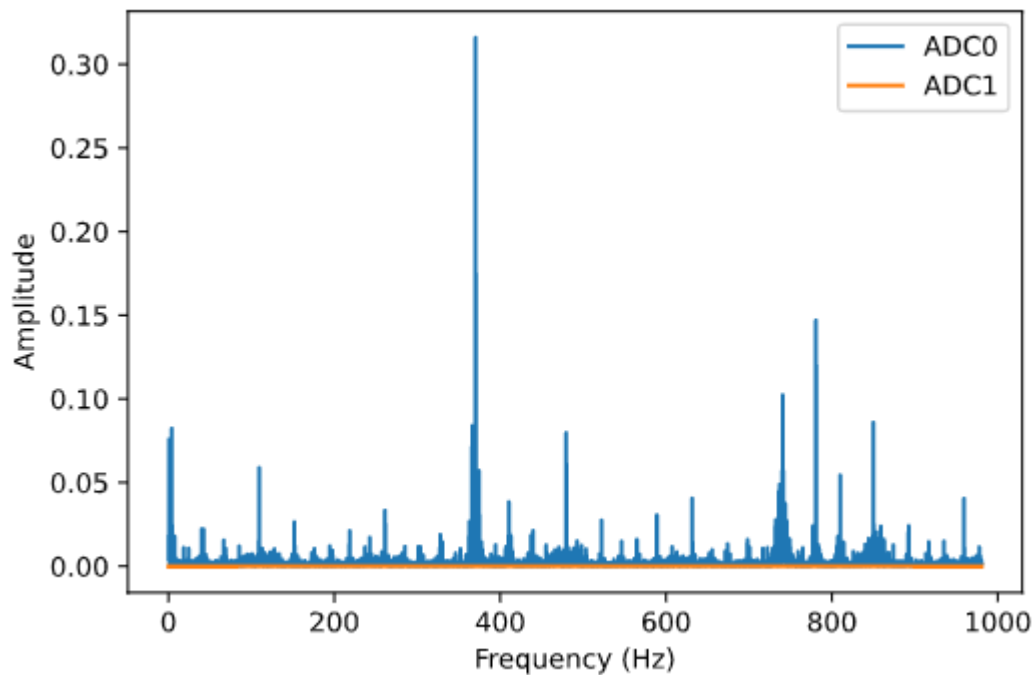


Figure 19: FFT plot of output of instrumentation amplifier with mystery signal for 20000 samples

We obtain the following results

Signal output	Arduino input	Fundamental frequency (Hz) $\pm 0.1\text{Hz}$	Fundamental frequency Amplitude (V) $\pm 0.005\text{V}$
V_{out}	ADC0	370.3	0.316

Question 8)

- ***Explain what an appropriate value for the corner frequency for this filter would be.***

We want the corner frequency to be at least a factor of 100 less than the carrier frequency with which the PWM signal is encoded. We see from question 7 that the carrier frequency (fundamental frequency) is around 370 Hz so we require our corner frequency to be 4 or less. With the resistor and capacitor values that we have access to a suitable corner frequency would 1.6 Hz formed from a 100 K Ω resistor and 1 μ F capacitor.

- ***Acquire and plot around 50000 samples. Comment on the result.***

Obtaining 50100 samples (discarding first 100) we obtain the following graphs with ADC0 being the output from the low pass filter with corner frequency 1.6 and ADC1 being held at ground.

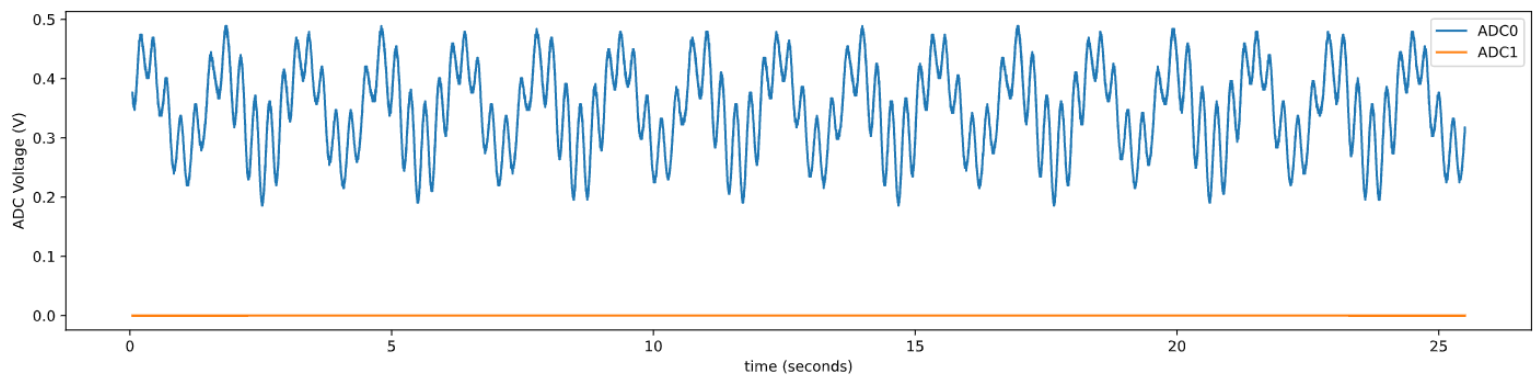


Figure 20: Output of the low-pass filter over 25 seconds

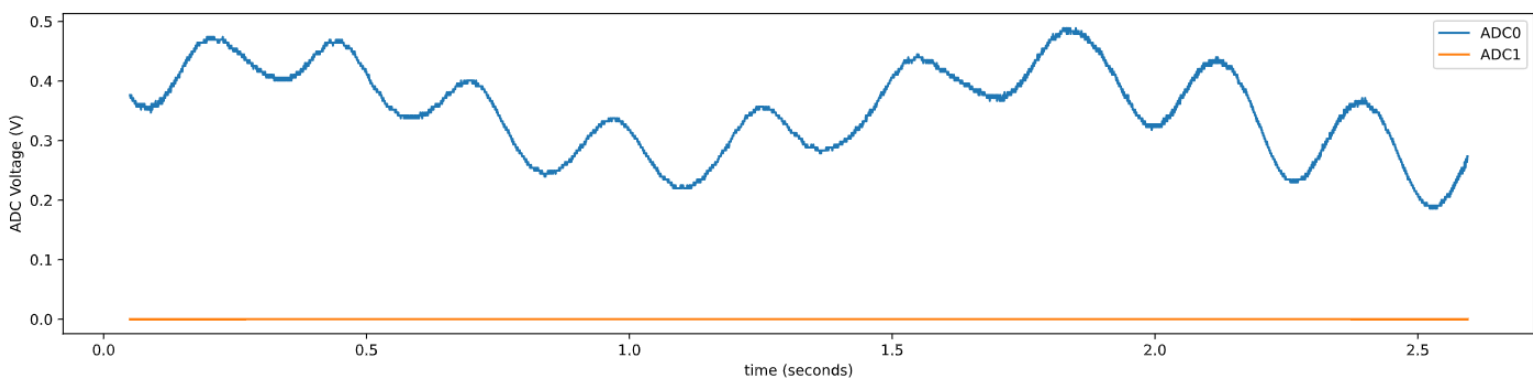


Figure 21: Output of the low-pass filter magnified for time range over 2.5 seconds

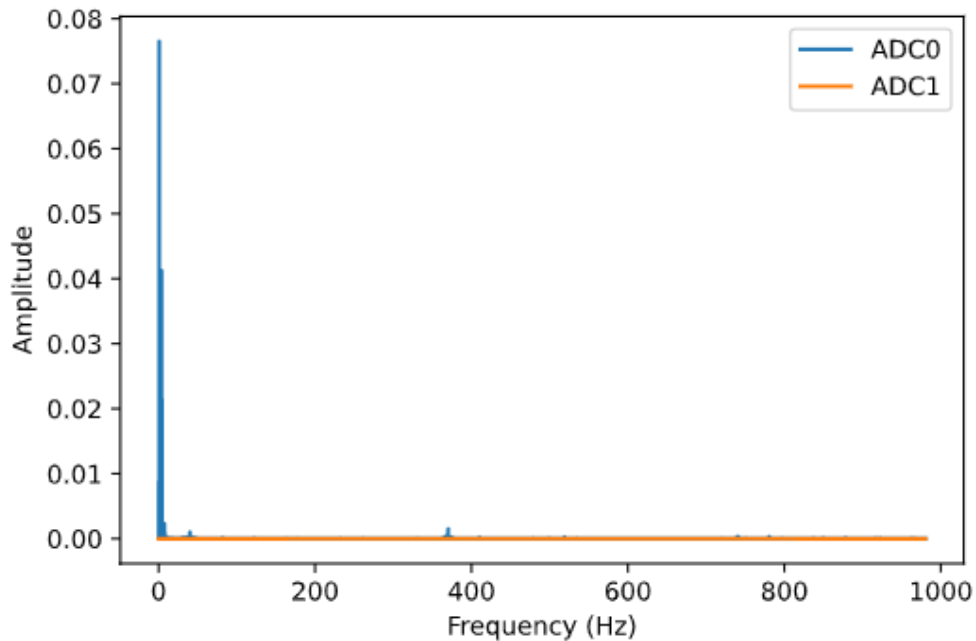


Figure 22: FFT plot of output of low pass filter

We can see the largest contribution to the FFT plot occurring when the frequency is very small this is to be expected as we are using a small corner frequency. There is still some leaking of the signal at 380 Hz (fundamental frequency of question 7) and a negligible contribution at around 780 Hz (fundamental frequency of question 1). The large frequencies do not play a significant role in the shape of the resulting waveform as seen in figure 21 which is also what we see in the FFT plot.

- **Determine approximate frequency and amplitude values of the two main frequency-components which make up the extracted signal; justify your answer.**

We can generate a magnified plot of the FFT spectrum

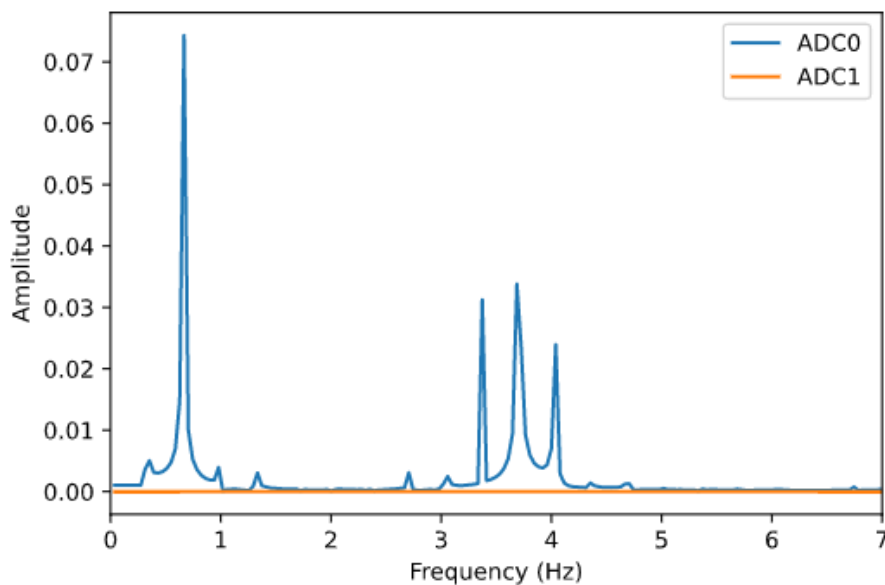


Figure 23: Magnified FFT plot showing two main frequency components

From this FFT plot we can read off the approximate frequency and amplitude

Approximate Fundamental frequency (Hz) \pm 0.2 Hz	Approximate Fundamental frequency Amplitude (V) \pm 0.005V
0.6	0.07
3.6	0.035

We can check these small frequencies to be correct by estimating the low frequencies found in figure 18, which is around 0.3 Hz for the lowest visible frequency oscillations and around 4 Hz for the next fastest set of oscillations, this is close to what we read off the FFT plot showing that these are the two main frequency components which make up the extracted signal.