

Portfolio Optimization Methods

Programming Skills Assessment - 15.06.2021

Introduction

The code for the project is stored in two separate scripts: **PortfolioOptimizerSPDR.py** and **TasksSPDR.py**. First file contains the *PortfolioOptimizer* superclass with corresponding subclasses of specific portfolios, while the second file contains a script used to run and evaluate the strategies.

The *PortfolioOptimizer* class was constructed in an object oriented manner with the use of inheritance. The motivation for such a solution was supported by code reproducibility and efficiency. More details can be found in the aforementioned scripts.

Task 1

Given 10 indices, implement the following asset allocation strategies:

1. 60% World Equities, 40% Global Bonds
2. Equal Risk Contribution
3. Minimum Variance Portfolio
4. Equal Weight Portfolio

The data of index levels has a monthly frequency. Thus during the strategy construction, I assumed a 12 month rebalance period, which is an industry standard and good starting point. I assumed also that weights of particular portfolio constituents cannot be negative (long only) and that those must sum up to 1 (no cash in portfolio). Furthermore, I employed a benchmark (World Equities Index) to compute several benchmark-related performance metrics for all considered portfolios and assumed that the risk free rate is equal to 0.

Strategy 1

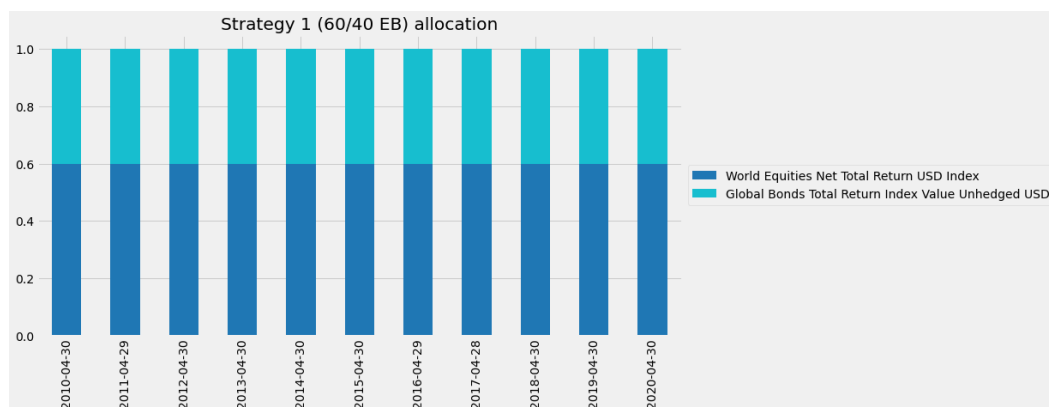
The first strategy is a simple 60/40 portfolio. It is crucial to set timely rebalances in order to avoid the increased/decreased exposure to any asset. The below table shows the performance metrics of strategy and benchmark. As we can observe the cumulative return of the portfolio is smaller than the corresponding cumulative return of benchmark.

| | Portfolio | Benchmark |
|--------------------------|-----------|-----------|
| Cumulative Return | 1.574645 | 2.045950 |
| CAGR | 0.081998 | 0.097262 |
| Annualised Vol | 0.099237 | 0.144401 |
| Sharpe Ratio | 0.265279 | 0.224195 |
| Sortino Ratio | 0.416751 | 0.315465 |
| Jensen's Alpha | 0.001625 | NaN |
| Beta | 0.639337 | NaN |
| Information Ratio | -0.095193 | NaN |
| Treynor Ratio | 0.011887 | NaN |

The Compound Annual Growth Rate (CAGR) is also lower, but given significantly lower Annualised Volatility this translates to a higher value of Sharpe Ratio than the benchmark. It means that the 60/40 portfolio has a better risk-return ratio. Sortino Ratio, which takes into account only the downside risk magnifies the difference between strategies.

I also calculated the Jensen's alpha and beta that comes from CAPM. The first metric is very small, and the second is at the expected level that relates to the significant share of World Equities in our portfolio. The Information Ratio shows the relationship between average excess return and tracking error. The metric's value is negative and low, which indicates rather poor tracking. There is also a low value of Treynor Ratio that is another variation of Sharpe Ratio, but this time utilizing beta apart from standard deviation of returns in the denominator.

Below is the graph presenting the portfolio allocation on rebalancing dates. Please note that this graph always shows 60/40 weight allocation, which is only true on specified dates. The share of particular assets in a portfolio will vary between consecutive rebalance dates, given the natural changes in market valuation.

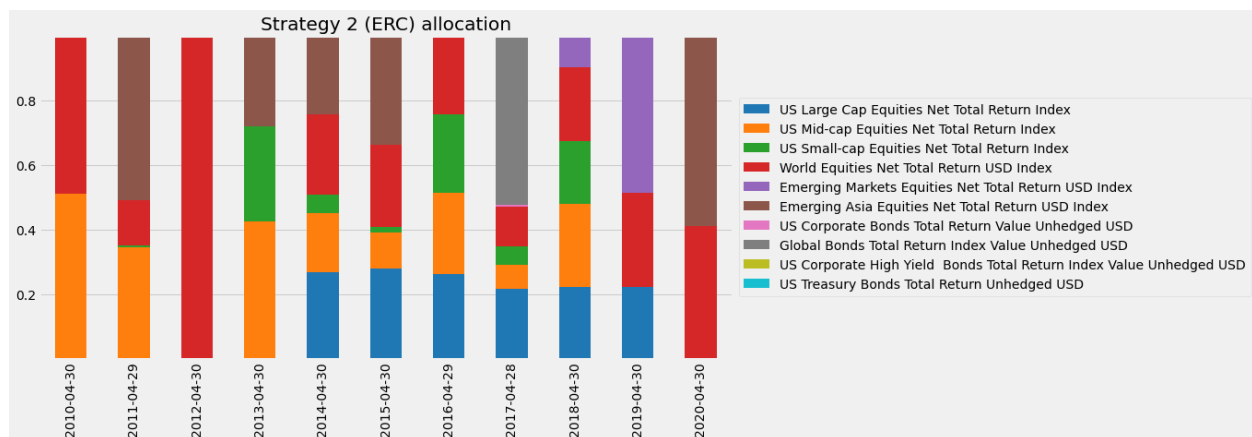


Strategy 2

The second portfolio is an Equal Risk Contribution strategy which focuses on risk allocation. This strategy aims to set the weights on levels which contribute to the same share of risk among portfolio constituents.

| | Portfolio | Benchmark |
|--------------------------|-----------|-----------|
| Cumulative Return | 1.905381 | 2.045950 |
| CAGR | 0.092950 | 0.097262 |
| Annualised Vol | 0.161067 | 0.144401 |
| Sharpe Ratio | 0.197705 | 0.224195 |
| Sortino Ratio | 0.281565 | 0.315465 |
| Jensen's Alpha | -0.000107 | NaN |
| Beta | 0.995123 | NaN |
| Information Ratio | -0.007287 | NaN |
| Treynor Ratio | 0.009238 | NaN |

We can observe that in this case the cumulative return is higher than in a 60/40 portfolio, but still lower than in the benchmark itself. What is worrying is also the high value of Annualised Volatility, which results in lower Sharpe Ratio. The beta of the portfolio is nearly equal to 1 which means that the portfolio return moves in the same manner as the market-portfolio (benchmark) return.



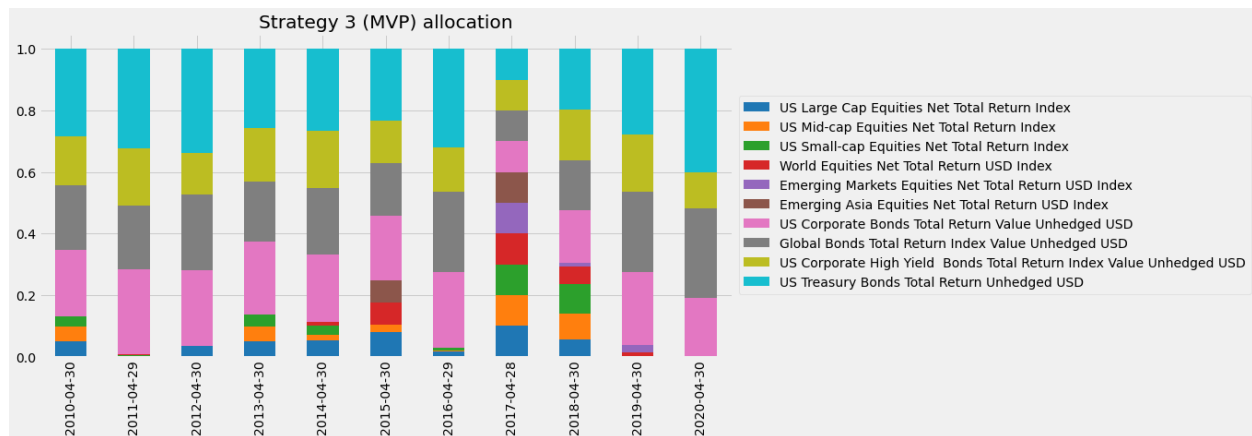
On the graph above showing the portfolio allocation on rebalance dates, we can observe that at some periods the portfolio is allocated fully to one index, while at others it consists of several assets with significantly different weights. This behavior may explain the high volatility of a portfolio.

Strategy 3

The third strategy is a Minimum Variance strategy that aims to minimize the overall variance of a portfolio during the weight optimization phase.

| | Portfolio | Benchmark |
|--------------------------|-----------|-----------|
| Cumulative Return | 1.011151 | 2.045950 |
| CAGR | 0.059954 | 0.097262 |
| Annualised Vol | 0.069048 | 0.144401 |
| Sharpe Ratio | 0.276027 | 0.224195 |
| Sortino Ratio | 0.467168 | 0.315465 |
| Jensen's Alpha | 0.003786 | NaN |
| Beta | 0.183650 | NaN |
| Information Ratio | -0.099353 | NaN |
| Treynor Ratio | 0.029958 | NaN |

This time also the cumulative return is lower than the World Equities Index benchmark, however the volatility is much lower, which results in a higher Sharpe Ratio. Beta of a portfolio is relatively low which means that portfolio returns are not very sensitive to market movements.



On the above graph we see that portfolio weights do not change dramatically between adjacent rebalance periods. Hence we can infer that the portfolio allocation is rather stable, which may help to preserve the goal of minimum variance along with significant share of bonds indices.

The interesting thing is an Equally Weighted Portfolio on 28-04-2017 date. This indicates the convergence problem of a solver given the fact that EWP is an initial guess for every optimization problem in this assessment. Such behavior may require further investigation and increased precision of computation.

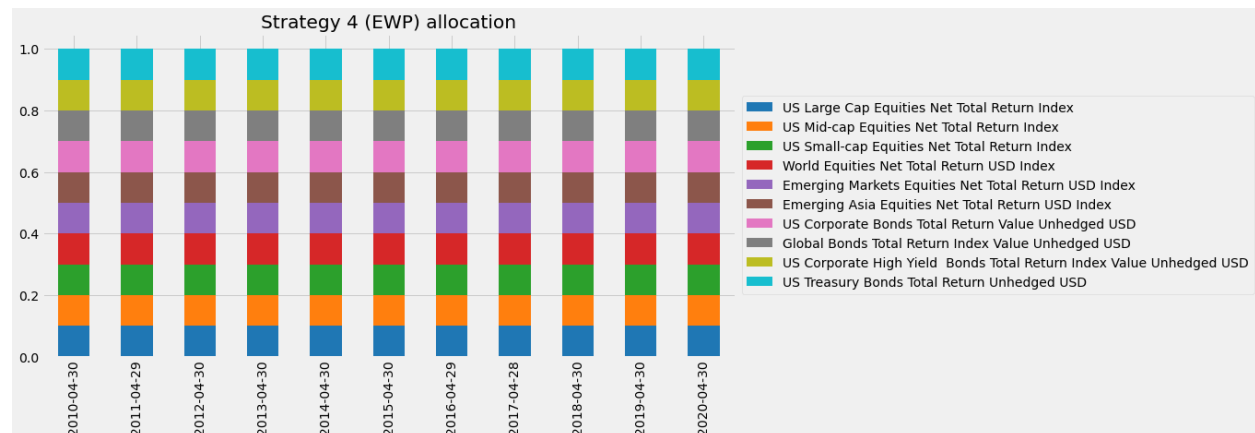
Strategy 4

The last strategy in this task was an EWP portfolio, which aims to preserve the equal weights of portfolio constituents during strategy lifetime. All discrepancies and deviations from the target are regulated during rebalancing periods.

| | Portfolio | Benchmark |
|--------------------------|-----------|-----------|
| Cumulative Return | 1.659420 | 2.045950 |
| CAGR | 0.084923 | 0.097262 |
| Annualised Vol | 0.099748 | 0.144401 |
| Sharpe Ratio | 0.272719 | 0.224195 |
| Sortino Ratio | 0.366684 | 0.315465 |
| Jensen's Alpha | 0.001749 | NaN |
| Beta | 0.653145 | NaN |
| Information Ratio | -0.086629 | NaN |
| Treynor Ratio | 0.012023 | NaN |

We can observe that again the cumulative return of a portfolio and its CAGR are lower than corresponding benchmark values. However, the volatility of a portfolio is lower which results in a higher Sharpe Ratio. The Jensen's alpha is again very low, while beta is at the similar level as in 60/40 portfolio.

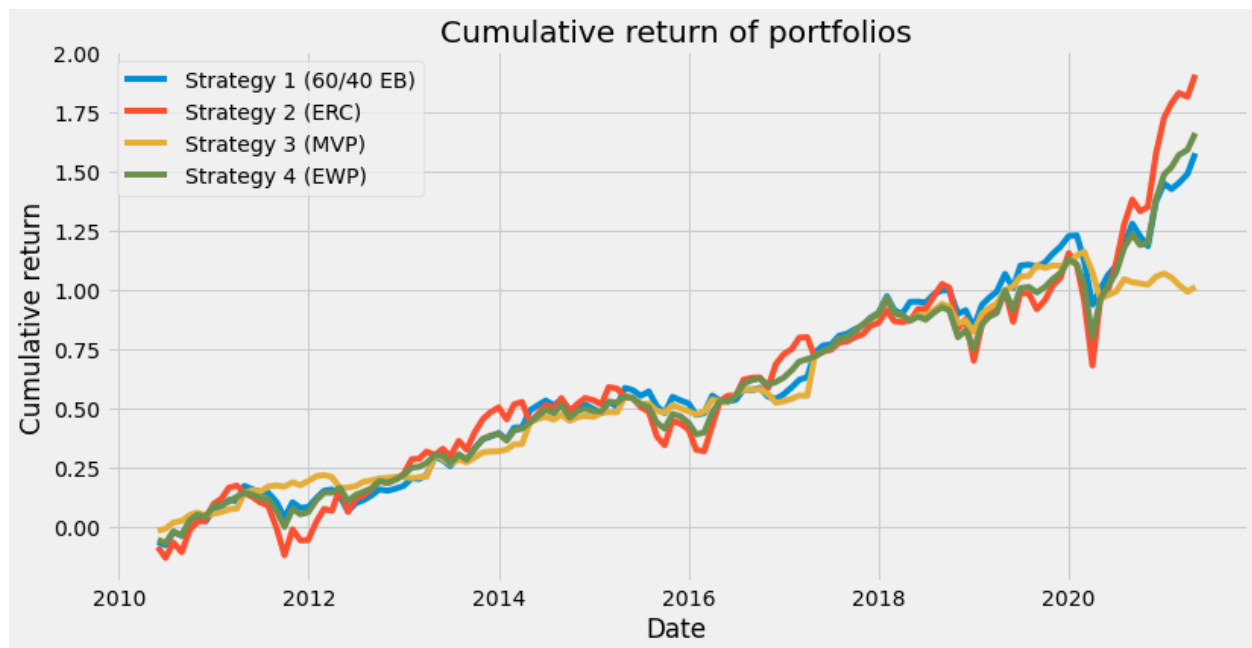
The allocation barplot below is presented for information purposes to show that each asset has 0.1 weight in the portfolio during rebalancing dates.



Conclusion

Each of the strategies presented above has a lower cumulative return than the World Equities Index benchmark. However, the majority of portfolios achieved lower Annualised Volatility which in fact resulted in higher Sharpe Ratios than benchmark. This may be a valuable premise for more risk aware investors looking for better tradeoff between portfolio volatility and its return, than the one offered by market index.

The only portfolio that can compete with the benchmark in terms of cumulative and annualised return is ERC portfolio. However, this strategy is characterized by high volatility and lower Sharpe than the benchmark.



On the plot above we can observe the cumulative returns of particular strategies over time. It is visible that ERC strategy is the most volatile one and obtains the highest returns. On the other hand MVP is characterized by low variance and low sensitivity to increased market volatility. It preserves the capital and does not record any major drawdowns.

The interesting fact is an increasing difference between the portfolios after a crisis triggered by Covid-19 pandemic in early 2020. This phenomenon can be an interesting topic for further research.

Task 2

Given 26 stocks, implement the following strategies:

1. 5% Target Return strategy
2. Equal Weight Portfolio

For this task, the constraints and assumptions remain the same: assets' weights must sum up to 1 (no cash left in portfolio), positive weights (long only) and rebalance every 12 months. Hence the same as above, the first year is used as a train set for weight optimization and the purchase decision is made after a first year in order to avoid a look ahead bias.

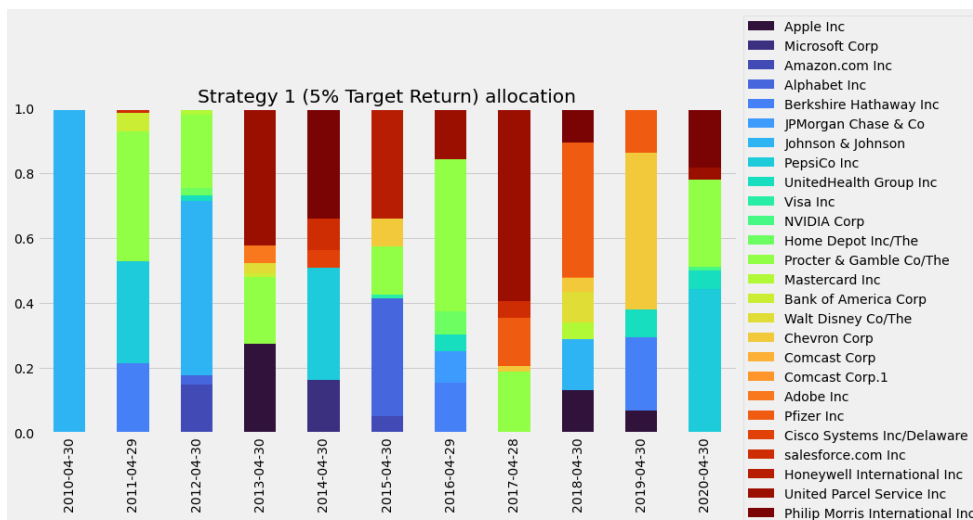
Taking into account the fact that the provided stock universe can be proxied by large cap US stocks, I chose US Large Cap Equities Index as a benchmark for strategy performance evaluation.

Strategy 1

The first strategy is a 5% target return strategy, which may be obtained through the mean-variance optimization. The optimization problem is constrained to the sum of weights equal to 1, as well as annualised mean portfolio return equal to target return. Given the aforementioned assumptions, we can minimize the annualised portfolio volatility.

| | Portfolio | Benchmark |
|--------------------------|-----------|-----------|
| Cumulative Return | 5.686227 | 3.124016 |
| CAGR | 0.171562 | 0.125322 |
| Annualised Vol | 0.238003 | 0.139601 |
| Sharpe Ratio | 0.241540 | 0.287936 |
| Sortino Ratio | 0.512569 | 0.403970 |
| Jensen's Alpha | 0.004766 | NaN |
| Beta | 1.019409 | NaN |
| Information Ratio | 0.090630 | NaN |
| Treynor Ratio | 0.016279 | NaN |

The first thing to note is a high cumulative return and CAGR, but also very high volatility. All the metrics are well above the same for the benchmark. The result is the lower Sharpe Ratio, but Sortino Ratio stays higher which may indicate that the volatility of our portfolio has a smaller share on the downside. The beta value of 1 indicates close relationship to market movements and high systematic risk.



The strategy allocation graph shows that weights are allocated mainly to a few stocks, including for example 100% allocation in Johnson & Johnson at the beginning of the investment period and significant share of United Parcel Service Inc in later periods. Majority of the stock universe has zero weights in several periods.

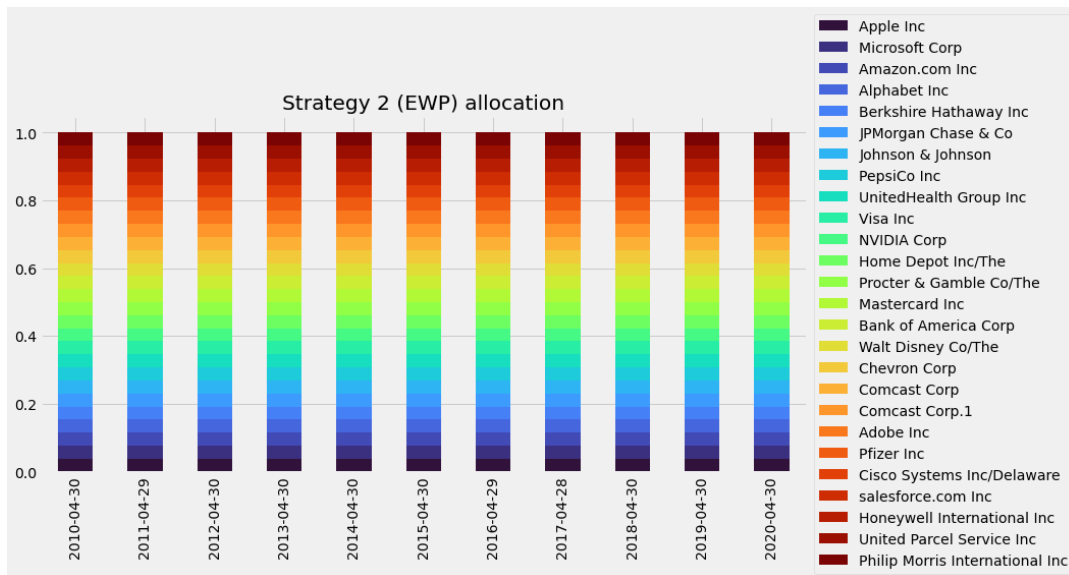
Strategy 2

The second strategy in this task is an EWP strategy known from the first part.

| | Portfolio | Benchmark |
|--------------------------|-----------|-----------|
| Cumulative Return | 7.124011 | 3.124016 |
| CAGR | 0.190733 | 0.125322 |
| Annualised Vol | 0.159630 | 0.139601 |
| Sharpe Ratio | 0.369560 | 0.287936 |
| Sortino Ratio | 0.648008 | 0.403970 |
| Jensen's Alpha | 0.004707 | NaN |
| Beta | 1.062023 | NaN |
| Information Ratio | 0.314333 | NaN |
| Treynor Ratio | 0.016035 | NaN |

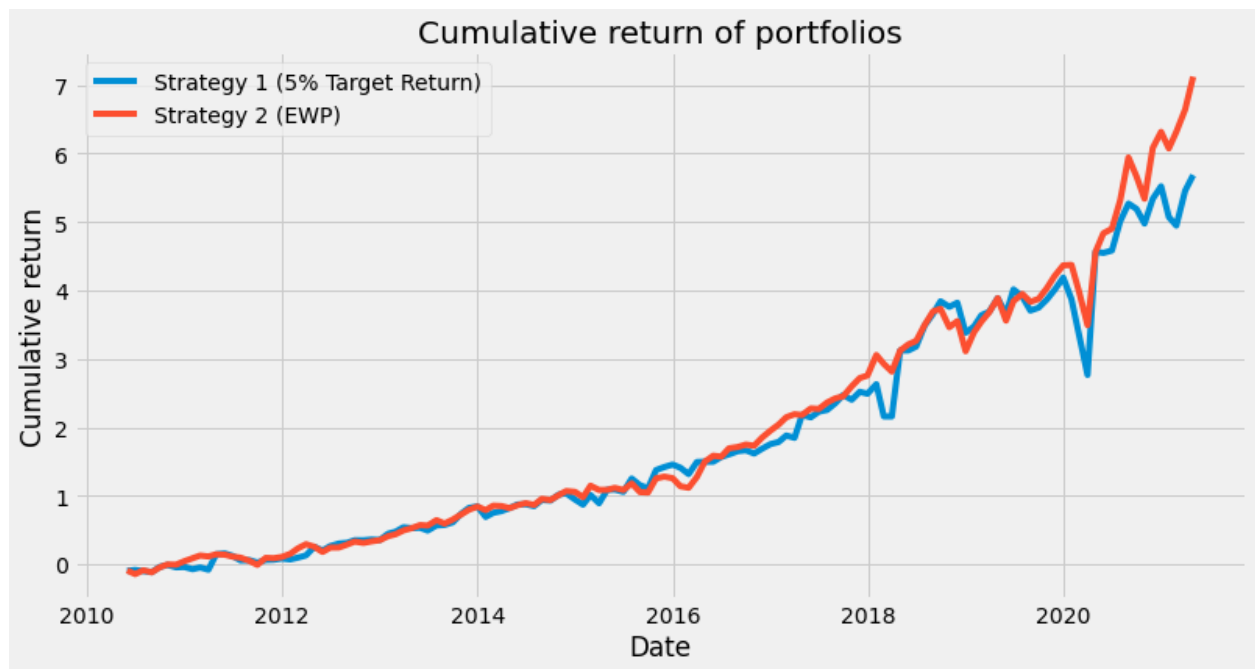
For the first time Sharpe Ratio is higher than 0.3 and a CAGR of a portfolio approaches nearly 20%. The Annualised Volatility is slightly higher than observed for benchmark, while the cumulative return over investment period is sevenfold the initial portfolio value, which is a very satisfactory result.

Below is the graph of portfolio weights on rebalancing dates for information purposes.



Conclusion

These two strategies present much better performance than the portfolios optimized in the first part of assessment over the same investment horizon and with similar assumptions.



As the best strategy the EWP can be considered. It manifests very high investment return, while keeping the volatility on market level. This time again we can observe the high drop in performance during Covid-19 shock, but after a short drawdown, both portfolios gained a significant momentum signal.