Exercícios Derivada

Seja a função $f(x) = x^2$, determinar a inclinação da reta tangente ou $f'(x)$ da função no ponto $(2, 4)$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$f'(2) = \frac{(2+h)^2 - (2)^2}{h}$$

$$f'(2) = \frac{4 + 4h + h^2 - 4}{h}$$

$$f'(2) = \frac{4h + h^2}{h}$$

$$f'(2) = \frac{\cancel{h}(4+h)}{\cancel{h}}$$

$$f'(2) = 4 + h$$

$$f'(2) = 4 + 0$$

$$f'(2) = 4$$

In [ ]:

Derivada do Produto

Regra: $(f.g)'(x) = f'(x)g(x) + f(x)g'(x)$

In [ ]:

Considere as funções: $f(x) = x^3 + 2x$ e $g(x) = 3x + 1$

Determinar $f.g'(x)$

$$f.g(x) = (x^3 + 2x).(3x + 1)$$

$$f.g'(x) = (3.x^{3-1} + 2).(3x + 1) + (x^3 + 2x).(3 + 0)$$

$$f.g'(x) = (3x^2 + 2).(3x + 1) + (x^3 + 2x).(3 + 0)$$

$$f.g'(x) = 9x^3 + 3x^2 + 6x + 2 + 3x^3 + 6x$$

$$\boxed{f.\,g'(x) = 12x^3 + 3x^2 + 12x + 2}$$

In [ ]:

Derivada do Quociente

Regra: $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

In [ ]:

Considere as funções: $f(x) = x^3 + 2x$ e $g(x) = 3x + 1$

Determinar: $\left(\frac{f}{g}\right)'(x)$

In [ ]:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x^3+2x).(3x+1) - (x^3+2x).f'(3x+1)}{(3x+1)^2}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{(3x^2+2).(3x+1) - (x^3+2x).(3)}{(3x+1)^2}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{(3x^2+2).(3x+1) - (x^3+2x).(3)}{(3x+1)^2}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{(9x^3+3x^2+6x+2) - (9x^2+6)}{9x^2+6x+1}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{9x^3+3x^2+6x+2-9x^2-6}{9x^2+6x+1}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{9x^3+3x^2+6x+2-9x^2-6}{9x^2+6x+1}$$

$$\left(\frac{f}{g}\right)'(x) = \frac{9x^3-6x^2+6x-4}{9x^2+6x+1}$$

In [ ]:

Pontos Críticos

Ache os pontos críticos e classifique-os

$f(x) = 3x^4 - 8x^3 - 18x^2 + 12$

In [ ]:

$f'(x) = 12x^3 - 24x^2 - 36x + 0$

$\boxed{f'(x) = 12x^3 - 24x^2 - 36x}$

In [ ]:

$12x^3 - 24x^2 - 36x = 0$

$12x(x^2 - 2x - 3) = 0$

$12x = 0 \rightarrow \boxed{x = 0}$

In [ ]:

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$\boxed{x = 3}$

$\boxed{x = -1}$

Fazer segunda derivada para encontrar os pontos de máximo e mínimo

$f''(x) = 12x^3 - 24x^2 - 36x$

$\boxed{f''(x) = 36x^2 - 48x - 36}$

$f''(0) = 36.0^2 - 48.0 - 36$

$\boxed{f''(0) = -36} \rightarrow máximo$

In [ ]:

$f''(3) = 36.3^2 - 24.3^2 - 36$

$f''(3) = 324 - 21 - 36$

$\boxed{f''(3) = 8} \rightarrow mínimo$

In [ ]:

$f''(-1) = 36.(-1)^2 - 48.(-1) - 36$

$f''(-1) = 36 + 48 - 36$

$\boxed{f''(-1) = 48} \rightarrow mínimo$

In [ ]:

Regras de L'Hôspital

Limites do tipo $\frac{0}{0}$ ou $\frac{\infty}{\infty}$

Exemplo:

$$\lim_{x\to\infty} \frac{x^2-1}{2x^2+1} = \lim_{x\to\infty} \frac{1-\dfrac{1}{x^2}}{2+\dfrac{1}{x^2}} = \lim_{x\to\infty} \frac{1-\dfrac{1}{\infty}}{2+\dfrac{1}{\infty}} = \frac{1-0}{2+0} = \frac{1}{2}$$

In [ ]:

Exemplo:

$$\lim_{x\to o} \frac{\sin x}{x} = \frac{\sin x'}{x'} = \frac{\cos x}{1} = \cos x = 1$$

In [ ]:

Regra da Cadeia

$$[f(g(x))]' = f'(g(x)).\, g'(x)$$

In [ ]:

Exemplo:

$j(x) = (x^2+2x)^2 = 2.(x^2+2x).\,(2x+2)$

$(2x^2+4x).\,(2x+2) = 4x^3+4x^2+8x^2+8x$

$j'(x) = 4x^3+12x^2+8x$

In [ ]:

Exemplo:

$r(x) = \sqrt{x^3+2}$

$r'(x) = (x^3+2)^{\dfrac{1}{2}} = \frac{1}{2}.\,(x^3+2)^{\dfrac{1}{2}-1}.\,(3x^2)$

$r'(x) = \frac{1}{2}.\,(x^3+2)^{-\dfrac{1}{2}}.\,(3x^2)$

$$r'(x) = \frac{1}{2}.\frac{1}{\sqrt{(x^3+2)}}.(3x^2) = \frac{3x^2}{2\sqrt{(3x^2+2)}}$$

In [ ]:

Exemplo:

$$g'(x) = \left(\frac{2x+1}{3x^2+2x}\right)^2$$

$$g'(x) = 2.\left(\frac{2x+1}{3x^2+2x}\right).\left(2.\frac{(3x^2+2x)-(2x+1)(6x+2)}{(3x^2+2x)^2}\right)$$

$$g'(x) = \left(\frac{4x+2}{3x^2+2x}\right).\left(\frac{6x^2+4x-(12x^2+4x+6x+2)}{(3x^2+2x)^2}\right)$$

In [ ]: