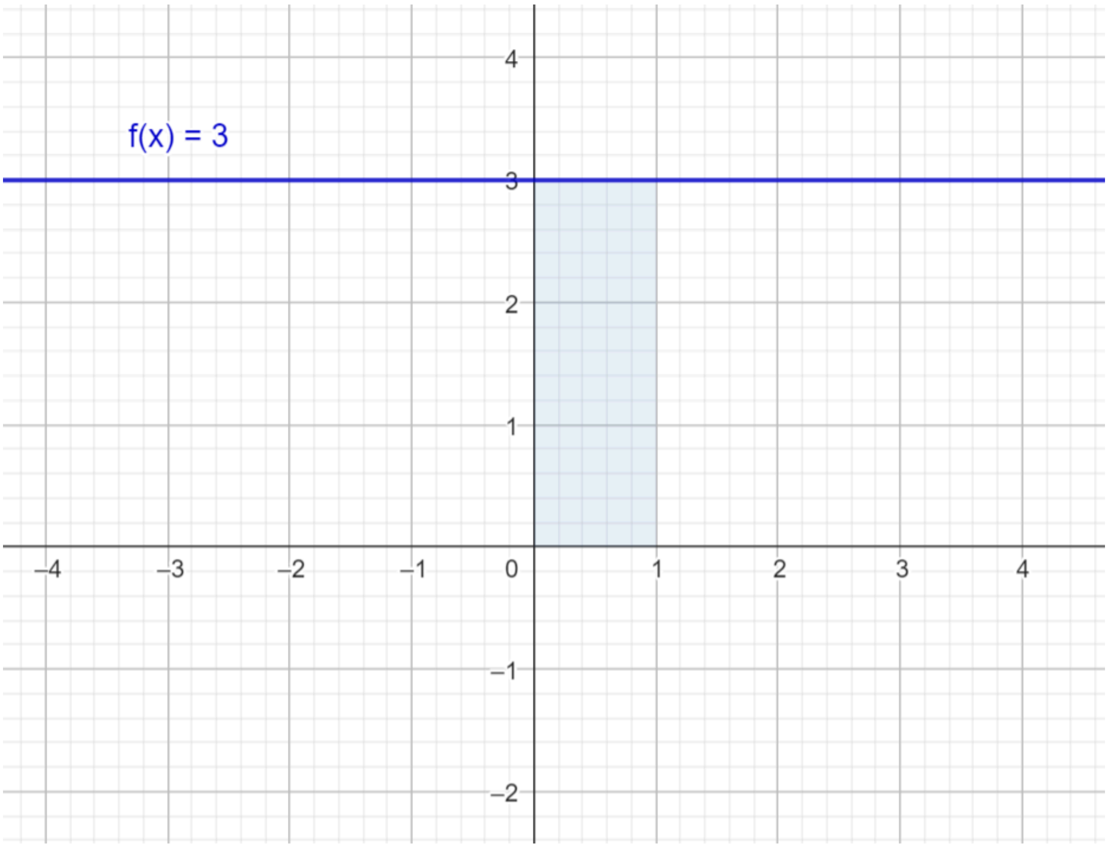
Integral

Exemplos

Calcular $\int_0^1 3d(x)$

In [ ]:

$$\int_0^1 3d(x) = 1.3 = 3\,u.\,a$$

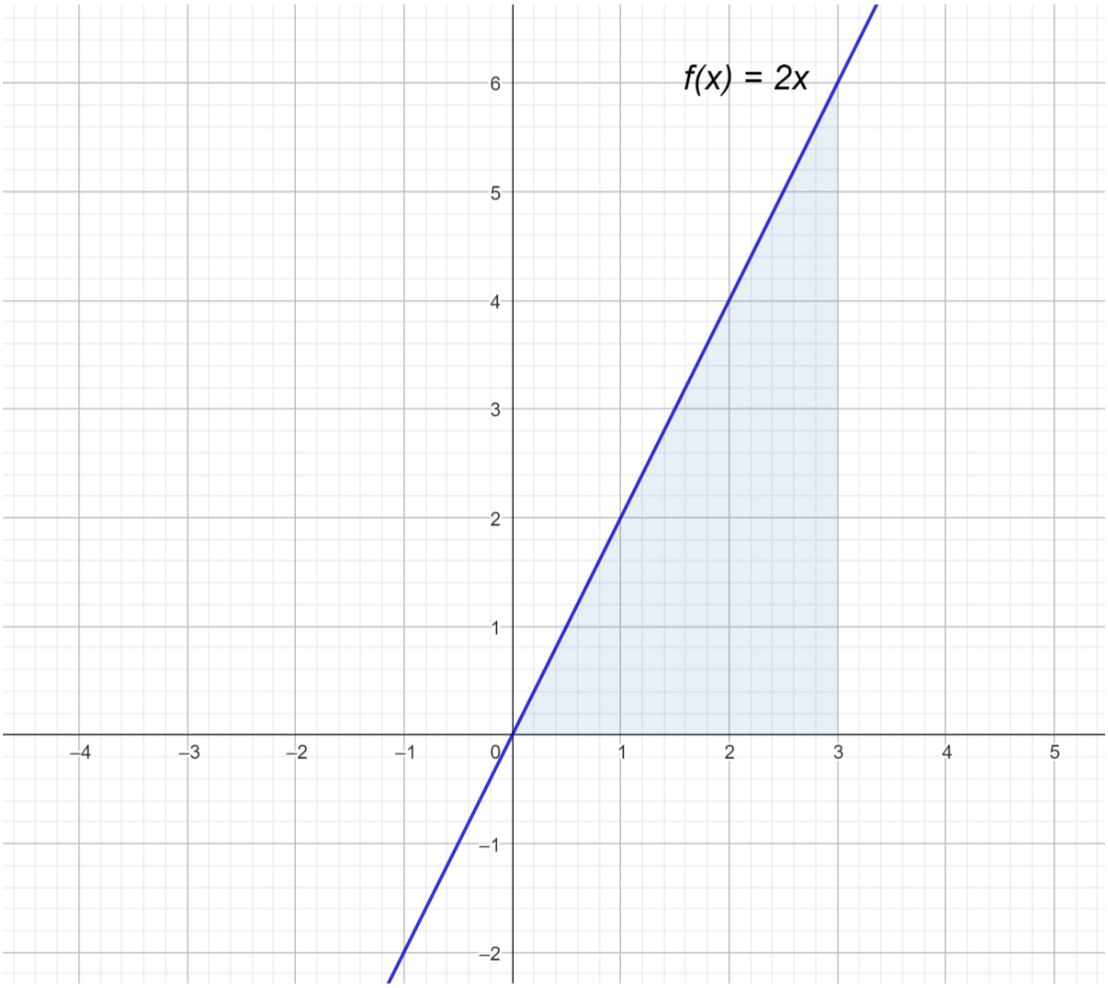


In [ ]:

Calcule $\int_0^3 2x\,dx$

In [ ]:

$$\int_0^3 2x\,dx = \frac{3.6}{2} = 9\,u.\,a$$

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In [ ]:
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Calcule $\int_{-1}^{1} x^2 dx$

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In [ ]:
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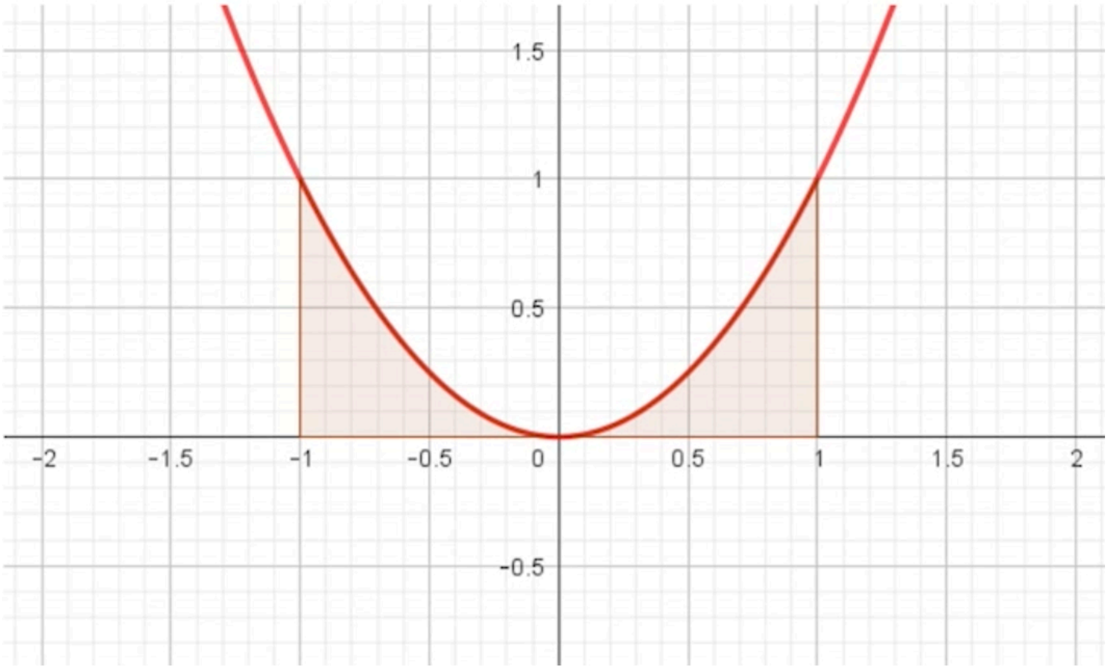
$$\int_{-1}^{1} x^2 dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3} = \frac{(1)^3}{3} - \frac{(-1)^3}{3} = \frac{1}{3} - \frac{(-1)}{3} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$



In [ ]:

Calcule $\int_0^2 x^3 - 2x^2 + 3dx$

In [ ]:

$$\int_0^2 x^3 - 2x^2 + 3dx = \frac{x4}{4} - 2\frac{x^3}{3} + 3x = \frac{(2)^4}{4} - 2\frac{(2)^3}{3} - [0] = 4 - \frac{16}{3} + 6 = 10 -$$

In [ ]:

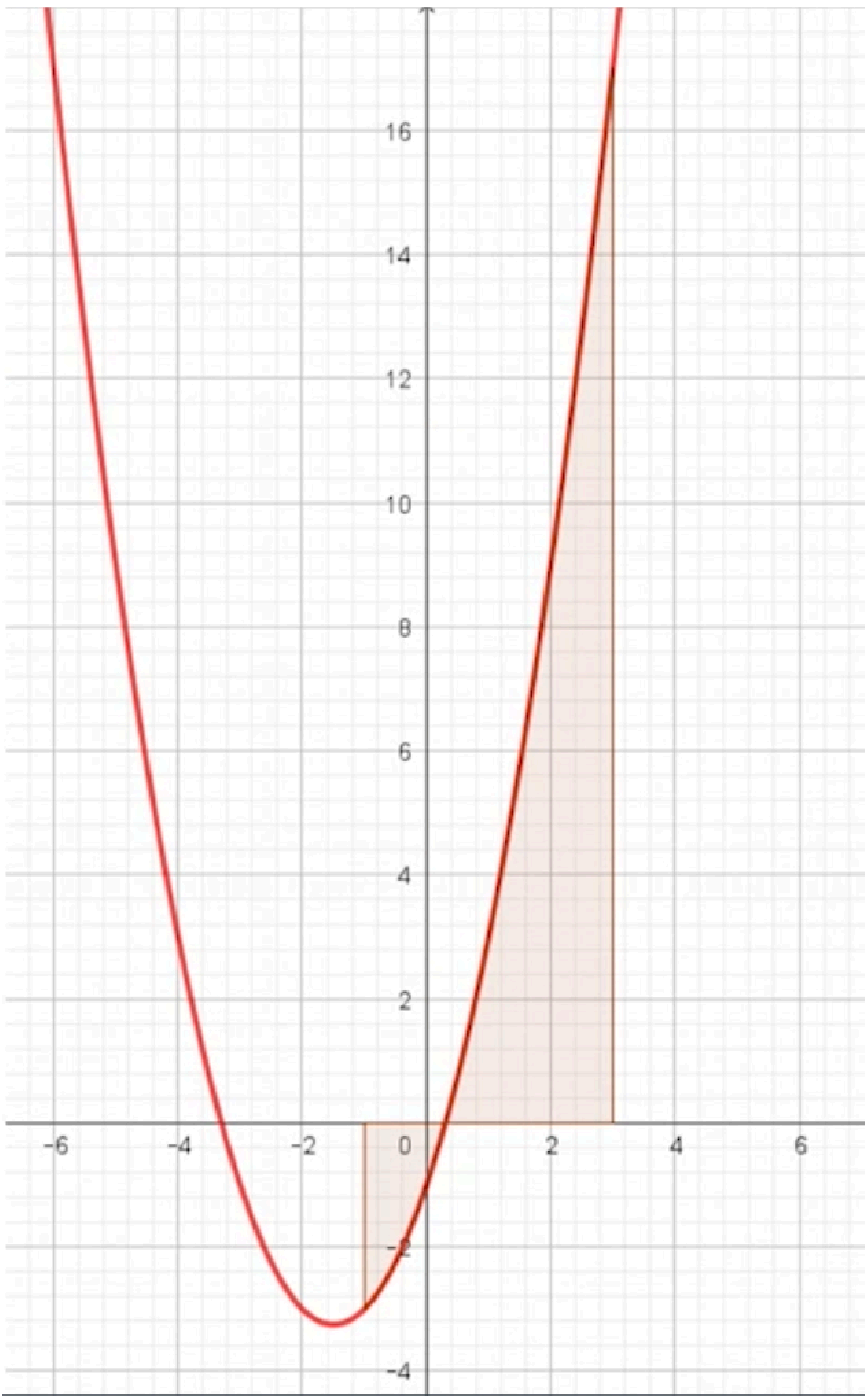
Calcule $\int_{-1}^{3} x^2 + 3x - 1\,dx$

In [ ]:

$$\int_{-1}^{3} x^2 + 3x - 1\,dx = \frac{x^3}{3} + 3\frac{x^2}{2} - x$$

$$= \frac{(3)^3}{3} + 3\frac{(3)^2}{2} - (3) - \left[\frac{(-1)^3}{3} + 3\frac{(-1)^2}{2} - (-1)\right]$$

$$= \frac{27}{3} + \frac{27}{2} - 3 + \frac{1}{3} - \frac{3}{2} - 1 = \frac{28}{3} + \frac{24}{2} - 4 = \frac{56 + 72 - 24}{6} = \frac{104}{6} = \frac{52}{3}$$



In [ ]:

Integrais Indefinidas

Calcule $\int x dx$

In [ ]:

Calcule $\int x dx = \frac{x^{1+1}}{1+1} + C = \frac{x^2}{2} + C$

In [ ]:

Calcule $\int \sin x dx$

$\int \sin x dx = -\cos x + C$

In [ ]:

Integral por Substituição Quando Não se Conhece os Limites

Seja a funçõ composta F(x)=f(g(x)), a sua derivada é obtida por meio da regra da cadeia:

$$F'(x) = f'(g(x)).\, g'(x)$$

e a integral de $f'(g(x)).\, g'(x)$ é:

$$\int f'(g(x)).\, g'(x) dx = f(g(x)) + C$$

Fazendo a mudança da variável $u = g(x)$, então:

$$\int f'(g(x)).\, g'(x) dx = f(g(x)) + C = f(u) + C = \int f(u) du$$

In [ ]:

Exemplos:

Calcular $\int 2xe^{x2} dx$

$u = x^2$, $\quad du = \frac{d(x^2)}{dx} = 2xdx$

$\int e^{x2}.2xdx$

$\int \, e^u .\, du$

$e^u + C$

$e^{x^2} + C$

In [ ]:

Calcule $\int x^3 \cos{(x^4+2)} dx$

In [ ]:

$u = x^4 + 2$

$\frac{du}{dx} = \frac{d(x^4+2)}{dx}$

$\frac{du}{dx} = 4x^3$

$du = 4x^3 dx$

$\frac{du}{4} = x^3 dx$

$$\int \cos{(x^4+2)}.\, x^3 dx$$

$$\int \cos{(u)}.\, \frac{du}{4}$$

$$\frac{1}{4} \int \cos{(u)}.\, du$$

$\frac{1}{4}.\sin{u} + C$

$$\boxed{\frac{1}{4}.\sin{(x^4+2)} + C}$$

In [ ]:

Integral por Substituição Quando se Conhece os Limites

Exemplo: calcular $\int_1^2 2x(x^2+1)^3 dx$

Método 1

Fazer a mudança de variável e calcular os novos limites de integração

In [ ]:

$u = x^2 + 1$

$\frac{du}{dx} = 2x \rightarrow du = 2xdx$

$u_1 = (1)^2 + 1 = 2$

$u_2 = (2)^2 + 1 = 5$

In [ ]:

$$\int_2^5 u^3 . du$$

$$\frac{u^4}{4}\Big|_5^2$$

$$\frac{(5)^4}{4} - \frac{(2)^4}{4}$$

$$\frac{625}{4} - \frac{16}{4}$$

$$\boxed{\frac{609}{4}}$$

In [ ]:

Método 2

Fazer a mudança de variável para a integral indefinida e depois substituir os limites de integração.

$$\int_2^5 u^3 . du$$

$$\frac{u^4}{4} + C$$

$$\frac{(x^2+1)^4}{4}\Big|_1^2 = \frac{(2^2+1)^4}{4} - \frac{(1^2+1^4)}{4} = \frac{(5)^4}{4} - \frac{(2)^4}{4} = \frac{625}{4} - \frac{16}{4} = \frac{609}{4}$$

In [ ]:

Integração por Partes

Seja o produto entre duas funções $f(x).\, g(x)$, então:

$$\frac{d}{dx}[f(x).\, g(x)] = f'(x).\, g(x) + f(x).\, g'(x)$$

Integrando ambos os lados:

$$\int \frac{d}{dx}[f(x).\,g(x)]dx = \int f'(x).\,g(x)dx + \int f(x).\,g'(x)dx$$

Rearranjando os termos:

$$\int f'(x).\,g(x)dx = f(x).\,g(x) - \int f(x).\,g'(x)dx$$

ou

$$\int u\,dv = u\,v - \int v\,du$$

In [ ]:

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Exemplos:

Calcular $\int x.\cos x\,dx$

$u = x$

$du = dx$

$dv = \cos x dx$

$\int dv = \int \cos x\,dx \rightarrow v = \sin x$

$\int x\ \cos x\,dx = x.\sin x - \int \sin x\,dx$

$\int x\ \cos x\,dx = x\ \sin x - (-\cos x) + C$

$\int x\ \cos x\,dx = x\ \sin x + \cos x + C$

In [ ]:

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Calcular $\int x\,e^x dx$

$u = x$

$du = dx$

$dv = e^x dx$

$\int dv = \int e^x dx \rightarrow v = e^x$

$\int x\,e^x dx = x.\,e^x - \int e^x dx$

$$\int x\, e^x dx = x\, e^x - e^x + C$$

In [ ]:

Calcular $\int_0^5 x\, e^{-x} dx$

$$u = x$$

$$du = dx$$

$$dv = e^{-x} dx$$

$$\int dv = \int e^{-x} dx \rightarrow v = -e^{-x}$$

$$\int x\, e^{-x} dx = x.\, (-e^{-x})\Big|_0^5 - \int_0^5 -e^{-x} dx$$

$$\int x\, e^{-x} dx = -x\, e^{-x}\Big|_0^5 + (-e^{-x})\Big|_0^5$$

$$\int x\, e^{-x} dx = -5e^{-5} + 0 - e^{-5} - (-e^0)$$

$$\int x\, e^{-x} dx = -5e^{-5} - e^{-5} + 1$$

$$\int x\, e^{-x} dx = -6e^{-5} + 1$$

In [ ]: