

FROM IMPRESSIONISM TO EXPRESSIONISM: AUTOMATICALLY IDENTIFYING VAN GOGH'S PAINTINGS

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ABSTRACT

Curators, art historians, and connoisseurs are often interested in determining the authorship of paintings. Machine learning and image processing techniques can assist in this task by providing non-invasive, automatic, and objective methods. In this work, we study the automatic identification of Vincent van Gogh's paintings using a Convolutional Neural Network that extracts discriminative visual patterns of a painter directly from images, and a machine learning classifier allied with a fusion method in the final decision process. We divide each painting into non-overlapping patches, classify them individually, and then aggregate the outcomes for the final response. We find out that using the patch with highest confidence score leads to the best result, outperforming the traditional voting scheme. We also contribute with a new and public dataset for van Gogh painting identification.

Index Terms— Painter attribution, CNN-based authorship attribution, data-driven painting characterization

1. INTRODUCTION

The authorship of a painting influences its artistic, social, historic, and monetary values [1]. Art specialists often employ methods such as optical microscopy, UV light, X-ray radiography, and infrared reflectography to determine authenticity [2]. However, some of these methods are potentially invasive and thus have the downside of interfering with the painting. With the recent advances in machine learning and image processing techniques, it is only natural that these fields join forces with curators, art historians, and connoisseurs in order to explore non-invasive ways for finding discriminative patterns in paintings directly from high resolution images, and hence making the authentication an automatic and objective process.

In this sense, we have experienced an increasing number of research efforts studying how to automatically attribute the author to a painting or to classify its style in the last few decades. In particular, a number of these studies analyze Vincent van Gogh's works. Van den Herik and Postma [3] employed engineered features with (shallow and non-convolutional) neural networks for preprocessing and classifying paintings, whereas Berezhnoy et al. [4] experimented with LAB color space and Gabor filters, and Li et al. [5] performed statistical hypothesis testing for distinguishing van Gogh from his contemporaries based on brush stroke analysis. In a more extensive study, Johnson et al. [6] carried out an investigation simultaneously with multiple researchers for identifying van Gogh's paintings. The

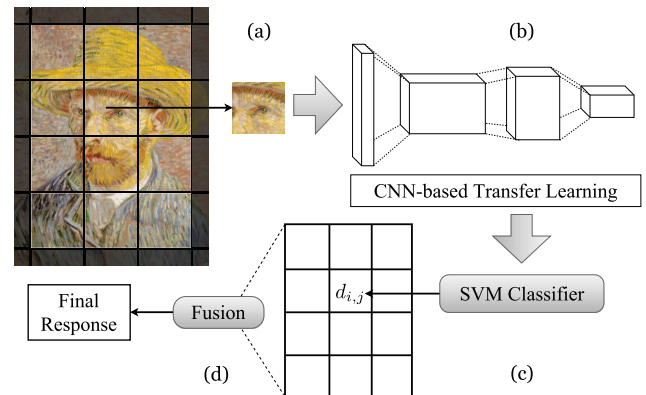


Fig. 1: Pipeline of the proposed method: (a) Patch extraction; (b) Feature extraction; (c) Patch classification; and (d) Evidence fusion.

teams evaluated different wavelet approaches, along with Hidden Markov Model (HMM), Support Vector Machine (SVM) [7], and Multidimensional Scaling (MDS).

In terms of other painters, Lyu et al. [8] studied Pieter Bruegel's drawings using wavelets and MDS, while Shamir et al. [9] analyzed automatic recognition of nine painters using a number of image transformations, such as Chebyshev, wavelets, and Fast Fourier. More recently, Gatys et al. [10] worked with a Convolutional Neural Network (CNN) to create artistic images based on content and style reconstructions, generating new examples of computerized art. Khan et al. [11] used state-of-the-art image processing techniques to classify paintings within 91 artists, and, to the best of our knowledge, this work provided the only public dataset for painting identification to this day, although without standardization with respect to image density (in Pixels per Inch).

Inspired by studies analyzing van Gogh's works, in this paper, we introduce a new and public painting dataset focused on van Gogh's paintings, which is further described in Section 3. In total, it contains 333 RGB density-normalized images, with an associated JPEG quality of at least 75%. Differently from [11], this is the first dataset related to paintings that is public and, at the same time, takes into account the density problem. Using images that are not density-normalized may introduce a classification bias in automatic techniques, and they might end up learning to differentiate densities rather than painters. We understand that other factors can impose additional biases, such as JPEG quality or different acquisition methods. However, since all the images used in this work are gathered from the Internet, we expect that there is enough randomness in them, so such influences are minimized.

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The authors thank the financial support of Capes DeepEyes project and Microsoft Research.

With our study, we do not wish to supplant art specialists, but to contribute with an automatic solution for identifying van Gogh’s paintings, so we can improve current results by combining existing methods. Similarly to [6], we analyze each painting by splitting it into smaller patches, then classifying each patch, and finally combining these intermediate results to generate a final classification response for the corresponding painting. By employing a Support Vector Machine (SVM) classifier on the patches, we are able to innovate and thus contribute with different aggregation methods, exploring the distance of the separating hyperplane as a measure of confidence for issuing the final decisions when combining different patch classifications.

Even though it is important to explore characterization techniques based on feature engineering (sometimes even incorporating the domain-knowledge of specialists), it is equally important to verify the potential of Convolutional Neural Networks (CNNs) for the problem of painting authorship, considering the recent success of such characterization method. Additionally, the process of learning the most important features of a painter directly from the images is non-trivial, since training a CNN from scratch requires a massive amount of data. We overcome this difficulty by using the concept of transfer learning, in which we employ a network that has been previously trained for extracting discriminative visual patterns in a very large dataset of natural images (in this case, ImageNet [12]), and we adapt it to extract the most important characteristics of a painter in our specific task. For a more thorough discussion, Razavian et al. [13] studied the generalization capabilities of previously trained CNNs in different recognition tasks, such as image classification, image retrieval, and object detection.

Therefore, the main objective of our method, which is further described in Section 2, is to learn intrinsic characteristics in paintings in an automatic fashion directly from images. In Section 3, we define the experimental setup, then we describe our results in Section 4, and, finally, we present the respective conclusions in Section 5.

2. METHOD

Given a set of paintings from a painter of interest, and a number of images from other painters considering some sampling rules (*e.g.*, similar epochs, style, *etc.*), the proposed method’s training pipeline consists of dividing each image into smaller patches, extracting their discriminative visual features using a CNN, training a patch classifier, and then using the relevant classification scores for a final aggregated response.

The testing pipeline is similar. Once a new testing painting comes in, it goes through the same steps as in the training, but instead of training a classifier, it uses the trained model. The responses for all patches in an image are then combined for the final response. Figure 1 depicts the testing pipeline.

2.1. Patch extraction

Dividing each input image into non-overlapping patches is critical since dealing with the entire painting at once would represent a huge computational burden, leading to a representation model with potentially billions of parameters to be estimated. Images have different sizes and are larger than the input of our selected CNN. If we decided to downsize every painting, then we would lose the important density normalization. Moreover, by splitting the input images, we are able to employ concepts such as *divide and conquer*, and we can capture finer details in paintings.

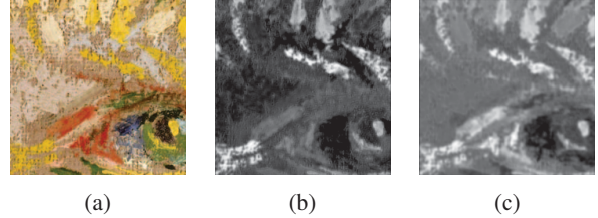


Fig. 2: Features emphasizing brush strokes: (a) Input patch from painting *Self-Portrait with Straw Hat* (F365v, JH1354); (b) An output from the first layer; and (c) An output from the second layer.

Therefore, the first step of the pipeline is to extract patches from the images, which is done in a convolutional fashion. Square blocks of 224×224 pixels are selected from the images without overlap, *i.e.*, with a step size of 224 pixels. In case these blocks do not perfectly fit the image sizes, the borders are discarded, *i.e.*, only the central part of the image is considered.

2.2. Feature extraction

Given that fully training a Convolutional Neural Network for painter attribution is difficult, especially due to the demanding amount of data, we make use of a network that was previously trained on millions of natural images, which makes it highly capable of extracting complex visual patterns. Therefore, in the feature extraction step, a general purpose CNN is used. Such network has been developed and trained using the ImageNet dataset [12].

Specifically, the VGG (Visual Geometry Group, University of Oxford) network [14] obtained state-of-the-art results in the ImageNet Large Scale Visual Recognition Challenge 2014 (ILSVRC-2014). Based on this work, we have used the network configuration comprising 19 weight layers, and 144 million parameters, which were optimized on 1.3 million images. This is an interesting network because it is very deep, while keeping a reasonable number of parameters, which is due to the small receptive fields used. This model takes 224×224 -pixel images in the RGB color space as input. As we focus on transfer learning, we strip out the last fully-connected and softmax layers, thus using the third-to-last layer results as a final feature vector, with 4,096 features.

Note that, among many layers and filters, some of the extracted characteristics are closely related to the brush strokes in the painting, as Figure 2 depicts. Detecting and describing brush strokes is precisely what Li et al. [5] explored, however, they custom-tailored a manual brush stroke extraction method for that purpose, while in our work these patterns are automatically extracted from the data.

2.3. Classification and fusion

In our method, the classification is done using SVM with a linear kernel [15]. The principle of SVM is mapping objects onto a sufficiently higher dimension in which there potentially exists a linear separation between classes. We opted for using a linear SVM as it has been shown to be efficient and effective for high-dimensional data (4096-d in our case) [14, 16], although any other classifier could be used as well.

We use this classifier to predict the class of each patch in an image. Then, we analyze different fusion methods for the final decision-making process. The patch extraction and classification are the *divide* stage of the proposed method, whereas aggregation is the *conquer* stage.

For each classified patch, we calculate its signed distance from the separating hyperplane. The magnitude could be essentially interpreted as a confidence score, in which higher value means higher confidence. Using such distances, we evaluated five different approaches for combining the results into one final decision:

- (a) **Mode** – The sign of each patch indicates the respective class, and the sign with more voting patches is the final class of the painting. The main idea is to classify the painting following the majority opinion of patches. This is the traditional fusion scheme explored in many machine learning algorithms in the literature, such as [6].
- (b) **Sum** – The sign resulting from the sum of all distances indicates the class. This can be interpreted as the overall confidence of every patch, since each one equally contributes with its corresponding certainty.
- (c) **Far** – The sign of the patch with the highest absolute distance indicates the class. Using the farthest patch from the separating hyperplane means that we classify each painting according to its top confidence patch. It is important to note, however, that this technique might be susceptible to outliers.
- (d) **Mean** – We calculate the mean distance of all positive patches and the mean distance of all negative patches. The sign of the higher absolute value indicates the class. This is analogous to the Sum method, but is less biased by the number of patches in each class, since we calculate the respective averages.
- (e) **Median** – Similar to the Mean method, but using the median distance for each group, which makes it more robust to outliers.

3. EXPERIMENTAL SETUP

To validate our method, we collected a novel dataset for Vincent van Gogh’s painting authorship attribution, and split it into a training set and a test set, for fair comparison with other methods. We also specify the metrics used for performance evaluation, and how we generated the classification model.

3.1. Dataset

Given that there is no open dataset concerning paintings that normalized the image densities, we decided to gather one with such characteristics, and, to this end, we collected images and metadata from Wikimedia Commons [17]. We crawled more than 200 categories of paintings that were related to van Gogh according to some style and chronological criteria. For instance, several categories from the 19th and 20th centuries were explored, including movements such as Impressionism, Post-Impressionism, Neo-Impressionism, and Expressionism. In this process, we gathered over 27,000 images.

The next step was to clean up the collected data. Basically, we analyzed the metadata provided by Wikimedia for each image, thus removing entries that missed important information, such as artist, painting identification, and the real dimensions of each painting.

Afterwards, we calculated the density (in Pixels Per Inch) for each image and kept only those with 196.3 PPI (the same value used in [6]) or higher. Also, the maximum allowed difference of PPI in height and width was 5%, that is,

$$\frac{\text{abs}(\text{PPI}_{\text{height}} - \text{PPI}_{\text{width}})}{\min(\text{PPI}_{\text{height}}, \text{PPI}_{\text{width}})} \leq 5\% \quad (1)$$

and the minimum associated JPEG quality was 75%. Finally, all images were converted to the lossless Portable Network Graphics

(PNG) format, and downsized with Lanczos filter so that the smaller density was 196.3 PPI.

The resulting dataset comprises a total of 124 RGB images for van Gogh paintings, and a total of 207 RGB images for non-van Gogh paintings. Once again, we recall that the non-van Gogh images represent paintings related to the painter in terms of chronology and style, as we discussed previously. Moreover, we also collected two paintings that, according to the corresponding Wikimedia pages, have their authorship still under debate, namely, *Kingfisher (F28, JH1191)*, and *Portrait of a woman (F215b, JH1205)*. The dataset is publicly available¹ and it is named **VGDB-2016**.

For a fair comparison between different methods, we initially split the paintings into a training and a test set, forming a standard validation protocol, which is also publicly available. This split was done randomly, preserving 80% of images for training, and 20% for testing. It is important to mention that no painting in the training set appears in the testing set and vice-versa. The produced sets are summarized in Table 1, along with the number of generated patches that are used in our method.

Class	Training		Test	
	Images	Patches	Images	Patches
van Gogh	99	15,895	25	3,927
non-van Gogh	165	31,513	42	8,611
Total	264	47,408	67	12,538

Table 1: Summaries for the training and test sets.

3.2. Evaluation metrics

Given a traditional confusion matrix, in which positive refers to van Gogh class, and negative refers to non-van Gogh class, we can define both precision = $TP/(TP + FP)$ and recall = $TP/(TP + FN)$. For performance evaluation, we consider the F_β -score:

$$F_\beta = (1 + \beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(\beta^2 \cdot \text{precision}) + \text{recall}} \quad (2)$$

This metric is especially useful when dealing with a dataset that is not balanced, such as ours. In order to give both precision and recall the same emphasis, we set $\beta = 1$, resulting in the F_1 -score, which is the harmonic mean between these two measures: $F_1 = 2 \cdot \text{precision} \cdot \text{recall}/(\text{precision} + \text{recall})$.

Given our previously defined test set, we calculate three simple baseline approaches, so we can better determine that our results are not simply due to chance. If a classifier always predict as positive (van Gogh), it yields an F_1 -score = 54.3%. Conversely, if a classifier always predict as negative (non-van Gogh), it yields an F_1 -score = 0.0%. Lastly, a random classifier would result in F_1 -score = 42.7%.

3.3. Classification model

In order to develop the final SVM patch classifier, we first randomly split the patches in the training set into three folds. Then, we optimize the parameters of the classifier using the F_1 -score as performance measure. Specifically, the best parameters found were penalty $C = 2^{-9}$ and equal class weights, despite the difference in class frequencies. With these parameters in hand, the SVM is trained again using all the patches from the training set. Then, we use this classifier to predict the class of each patch of a painting in the test set.

¹<https://dx.doi.org/10.6084/m9.figshare.3370627>

4. RESULTS AND DISCUSSIONS

Assuming a traditional confusion matrix, in which positive refers to van Gogh class, and negative refers to non-van Gogh class, the results for each method are presented in Table 2. Given that the methods Sum, Mean and Median had the same outcomes, they are presented in a single matrix, named *SMM*. The respective F_1 -scores are 88.0% for Mode, 90.6% for *SMM*, and 92.3% for the Far method.

		Mode		SMM		Far	
		+	-	+	-	+	-
True Class	+	22	3	24	1	24	1
	-	3	39	4	38	3	39

Table 2: Resulting confusion matrix for each aggregation method. The rows and columns represent, respectively, the true and the predicted classes. *SMM* stands for Sum, Mean and Median.

We now further discuss the Far method, which achieved the best result in our experiments. Even though it proved to be a good overall fusion technique for the painting attribution problem, it can still be influenced by outliers. Considering that, in a real-world setting, the number of non-van Gogh paintings is extremely larger than the amount of paintings by van Gogh, our False Positive Rate (FPR) = $FP/(FP + TN) = 3/(3 + 39) = 7.1\%$ may seem too high. However, it is important to note that our negative samples are among the hardest, given that they are closely related to van Gogh.

To report the four wrongly classified paintings, we first convert the score distances to probabilities using Platt normalization for SVMs [18], then we present the paintings along with the highest probability for the positive and negative classes, in this order. They are *Portrait of a Girl (WV570)* by Paula Modersohn-Becker with probabilities (99.7%, 98.3%), *The Road to Evordes (206)* by Ferdinand Hodler with (99.95%, 97.01%), *The Palazzo Contarini, Venice (W1767)* by Claude Monet with (99.4%, 96.9%), and *Self-portrait with Bandaged Ear (F527, JH1657)* by Vincent van Gogh with (99.92%, 99.97%). Note that, in the four misclassified cases, the probabilities of the chosen patches are similar, showing that a more refined analysis would be needed to separate these cases.

Our initial intuition on the discriminative capabilities of a CNN-based transfer learning approach is corroborated in Figure 3, where we plot the kernel density estimations of the distances between patches in the test set and the SVM’s separating hyperplane. Specifically, the distances of van Gogh patches were in the interval $[-4.0, +6.3]$, and the distances of non-van Gogh patches were in the interval $[-5.8, +3.5]$. The confounding extremes for each class (-4.0 for van Gogh, and $+3.5$ for non-van Gogh) could also help explain the higher number of false positives.

Considering these two distributions, in comparison with [5], we adopt a statistical hypothesis test, namely, Welch’s unequal variances t-test [19]. Setting the null hypothesis that the two groups have the same average, we obtain the corresponding p-value of 0.0, which enables us to state that the difference between the expected distances from the separating hyperplane is statistically significant.

Finally, we evaluated the two paintings in our dataset with authors still under debate using the Far method, since it produced the best result. The probabilities of the patches with highest values for van Gogh and non-van Gogh classes, respectively, in the *Kingfisher (F28, JH1191)* painting were (98.8%, 89.7%), and in the *Portrait of a woman (F215b, JH1205)* painting were (99.9%, 90.2%). Therefore, the prediction for each of these images using our approach is that they were painted by Vincent van Gogh.

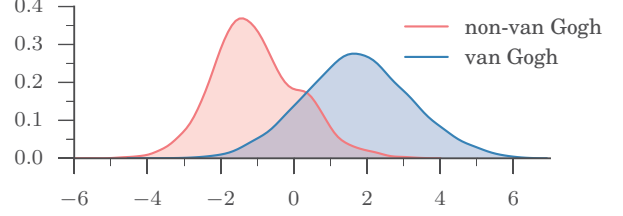


Fig. 3: Patch distance distribution.

5. CONCLUSIONS

Automatically finding patterns in paintings and identifying authorship are still open problems. In our study, we proposed a new and public painting dataset for artist identification, which improves on previous works that either did not disclose the data used, or did not properly handle density normalization. With this contribution, we wish to encourage open science and reproducible research, and expect that other scientists build upon such data.

Extending upon works such as [13], we show that painter attribution is yet another task that previously trained CNNs are capable of generalizing upon. Specifically, we employed a CNN that achieved top results in the ImageNet dataset in order to extract distinctive visual information directly from images. These networks are known to perform well at detecting textures and shapes, which is essentially what we aim for at first, when assessing paintings of any author, and the obtained results indicate that using such technique is promising.

We also evaluated some patch aggregation methods, and demonstrated that taking advantage of scores related to the classifier confidence improves on the traditional voting scheme, which was used in previous works [6]. Our best result was reached by defining the painting label with the patch that had the highest confidence, *i.e.*, the largest distance to the decision hyperplane. Besides, considering we did not apply any data augmentation or weight optimization technique, the results found were very promising, and represent only the first step towards a more robust, data-driven characterization and classification approach for painting authorship attribution.

Given the set of techniques presented in this paper, along with the corresponding discussions, we envision a computational tool in which art experts could input a painting whose authorship is under debate, and this tool would display the most distinctive patches going in favor and against van Gogh style, along with the corresponding scores. This way, specialists could combine such computational method with their extensive knowledge in order to evaluate paintings, knowing that the highest absolute score is usually a good indicator of authorship, while manually handling possible outliers. Moreover, provided that the necessary dataset is available, our work could be easily extended to other authors, in order to verify whether a painting has been portrayed by a painter of interest.

For future work, one could experiment with data augmentation, such as multi-resolution analysis and density variance, horizontal and vertical flips, and general affine transformations (*e.g.*, [20, 21]). However, such techniques may distort the discriminative brush stroke characteristics. With enough data, it would also be interesting to optimize the weights in a pre-trained CNN and, ultimately, train a novel architecture from scratch. Alternatively, another option could include investigating CNN learning from few samples [22, 23]. The use of Extreme Value Theory (EVT) [24] in the fusion step also holds promise. Finally, multi-class and open set approaches [25] to better handle the particularities between distinct painters and the relatively small number of negative samples are also of interest. ■

6. REFERENCES

- [1] G. E. Newman and P. Bloom, "Art and authenticity: The importance of originals in judgments of value," *Journal of Experimental Psychology*, vol. 141, no. 3, pp. 558, 2012.
- [2] J. Ragai, "The scientific detection of forgery in paintings," *Proceedings of the American Philosophical Society*, vol. 157, no. 2, pp. 164–175, 2013.
- [3] H. J. van den Herik and E. O. Postma, "Discovering the visual signature of painters," in *Future Directions for Intelligent Systems and Information Sciences*, N. Kasabov, Ed., vol. 45 of *Studies in Fuzziness and Soft Computing*, pp. 129–147. Physica-Verlag HD, 2000.
- [4] I. Berezhnuy, E. Postma, and J. van den Herik, "Computer analysis of van Gogh's complementary colours," *Pattern Recognition Letters*, vol. 28, no. 6, pp. 703–709, 2007, Pattern Recognition in Cultural Heritage and Medical Applications.
- [5] J. Li, L. Yao, E. Hendriks, and J. Z. Wang, "Rhythmic brushstrokes distinguish van Gogh from his contemporaries: Findings via automated brushstroke extraction," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 34, no. 6, pp. 1159–1176, 2012.
- [6] C. R. Johnson, E. Hendriks, I. J. Berezhnuy, E. Brevdo, S. M. Hughes, I. Daubechies, J. Li, E. Postma, and J. Z. Wang, "Image processing for artist identification," *IEEE Signal Processing Magazine*, vol. 25, no. 4, pp. 37–48, 2008.
- [7] C. Cortes and V. Vapnik, "Support-vector networks," *Machine Learning*, vol. 20, no. 3, pp. 273–297, 1995.
- [8] S. Lyu, D. Rockmore, and H. Farid, "A digital technique for art authentication," *Proceedings of the National Academy of Sciences*, vol. 101, no. 49, pp. 17006–17010, 2004.
- [9] L. Shamir, T. Macura, N. Orlov, D. M. Eckley, and I. G. Goldberg, "Impressionism, expressionism, surrealism: Automated recognition of painters and schools of art," *ACM Transactions on Applied Perception*, vol. 7, no. 2, pp. 8:1–8:17, 2010.
- [10] L. A. Gatys, A. S. Ecker, and M. Bethge, "A neural algorithm of artistic style," *CoRR*, vol. 1508.06576, 2015.
- [11] F. S. Khan, S. Beigpour, J. Weijer, and M. Felsberg, "Painting-91: A large scale database for computational painting categorization," *Machine Vision and Applications*, vol. 25, no. 6, pp. 1385–1397, 2014.
- [12] O. Russakovsky, J. Deng, H. Su, J. Krause, S. Satheesh, S. Ma, Z. Huang, A. Karpathy, A. Khosla, M. Bernstein, A. C. Berg, and L. Fei-Fei, "ImageNet Large Scale Visual Recognition Challenge," *Intl. Journal of Computer Vision*, pp. 1–42, 2015.
- [13] A. S. Razavian, H. Azizpour, J. Sullivan, and S. Carlsson, "CNN features off-the-shelf: An astounding baseline for recognition," in *IEEE Conference on Computer Vision and Pattern Recognition Workshops*, 2014, pp. 512–519.
- [14] K. Simonyan and A. Zisserman, "Very deep convolutional networks for large-scale image recognition," *CoRR*, vol. 1409.1556, 2014.
- [15] R. Fan, K. Chang, C. Hsieh, X. Wang, and C. Lin, "Liblinear: A library for large linear classification," *Journal of Machine Learning Research*, vol. 9, pp. 1871–1874, 2008.
- [16] G. Chiachia, A. X. Falcao, N. Pinto, A. Rocha, and D. Cox, "Learning person-specific representations from faces in the wild," *IEEE Transactions on Information Forensics and Security*, vol. 9, no. 12, pp. 2089–2099, 2014.
- [17] Wikimedia, "Wikimedia Commons," <https://commons.wikimedia.org>, [Online; accessed 2015-03].
- [18] J. C. Platt, "Probabilistic outputs for support vector machines and comparisons to regularized likelihood methods," *Advances in large margin classifiers*, vol. 10, no. 3, pp. 61–74, 1999.
- [19] B. L. Welch, "The generalization of 'Student's' problem when several different population variances are involved," *Biometrika*, vol. 34, no. 1/2, pp. 28–35, 1947.
- [20] A. Krizhevsky, I. Sutskever, and G. E. Hinton, "ImageNet classification with deep convolutional neural networks," in *Advances in Neural Information Processing Systems 25*, F. Pereira, C. J. C. Burges, L. Bottou, and K. Q. Weinberger, Eds., pp. 1097–1105. Curran Associates, Inc., 2012.
- [21] D. Ciresan, U. Meier, and J. Schmidhuber, "Multi-column deep neural networks for image classification," in *IEEE Intl. Conference on Computer Vision and Pattern Recognition*, 2012, pp. 3642–3649.
- [22] R. Wagner, M. Thom, R. Schweiger, G. Palm, and A. Rothermel, "Learning convolutional neural networks from few samples," in *Intl. Joint Conference on Neural Networks*, 2013, pp. 1–7.
- [23] D. Menotti, G. Chiachia, A. Pinto, W. R. Schwartz, H. Pedrini, A. X. Falcao, and A. Rocha, "Deep representations for iris, face, and fingerprint spoofing detection," *IEEE Transactions on Information Forensics and Security*, vol. 10, no. 4, pp. 864–879, 2015.
- [24] W. J. Scheirer, A. Rocha, R. J. Micheals, and T. E. Boulton, "Meta-recognition: The theory and practice of recognition score analysis," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 33, no. 8, pp. 1689–1695, 2011.
- [25] W. J. Scheirer, L. P. Jain, and T. E. Boulton, "Probability models for open set recognition," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 36, no. 11, pp. 2317–2324, 2014.