GAME THEORY APPLIED TO LEGGED ROBOTICS: A VARIANT OF THE DOLICHOBRACHISTOCHRONE PROBLEM

Paul Muench, PhD
David Bednarz, PhD

US Army RDECOM/TARDEC
Warren, MI

ABSTRACT

We model the scenario between a robotic system and its operating environment as a strategic game between two players. The problem will be formulated as a game of timing. We will treat disturbances in a worst case scenario, i.e., as if they were placed by an opponent acting optimally. Game theory is a formal way to analyze the interactions among a group of rational players who behave strategically. We believe that behavior in the presence of disturbances using games of timing will reduce to optimal control when the disturbance is suppressed. In this paper we create a model of phase space similar to the dolichobrachistochrone problem. We discretize phase space to a simple grid where Player P is trying to reach a goal as fast as possible, i.e., with minimum cost. Player E is trying to maximize this cost. To do this, E has a limited number of "chips" to distribute on the grid. How should E distribute his resources and how should P navigate the grid? Rather than treating disturbances as a random occurrence, we seek to treat them as an optimal strategy

INTRODUCTION

Force does not exist for mobility, but mobility for force. Alfred Thayer Mahan

Mobility is, by its nature, a means to an end; it does not exist for its own sake, but as a service to the larger force. Legged mobility stands as a nascent industry today due in part to the advances in control theory and computing power over the past few decades, but mainly it is due to the fact that the US government has poured such a vast amount of research into it. As this excitement inevitably winds down, it is the rather mundane problem of practical service to the military force that will ultimately determine the fate of the industry.

The most likely first application for a legged robot is in dismounted logistics. The dismounted soldier is clearly overburdened; some type of off-loading robot that approaches the mobility of a dismounted soldier bearing a full march load would be of particular value to our military forces.

Does this off-loading robot need to be a legged robot? The short answer is no. Unless we can show some quantifiable advantage in mobility, the additional cost of a legged platform is unjustifiable. Thus, the question becomes: How do we quantify mobility? Traditionally, we have defined mobility by looking at a list of things like: length of gap crossing, height of step obstacles, and terramechanics, i.e., soft soil mobility, to name a few. These discrete metrics do capture some of what we mean by mobility, but there is no standard way to quantify what each of these metrics is worth to us, i.e. to compare the relative value of one metric vs. the other. We have complicated the problem further now because we want add the capabilities of a legged robot to the list. Legged mobility is about narrow passageways and mountainous trails, where typically wide wheel based vehicles cannot traverse. Should we merely add these “tall and narrow” metrics to our rather ad-hoc definition of mobility in order to skew the decision matrix towards a legged platform?

Rather than merely collecting a long list of metrics, we seek a general framework for mobility in the context of its value to the larger force. If we can show a quantifiable advantage to the force, we can begin to justify increased expenditures to achieve that

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advantage. Advantage implies an opponent, and to be explicit, we'll call that opponent: terrain. We make no distinction between terrain that is natural or man-made. An opposing force uses the natural terrain to their advantage anyway, so you might as well lump them together and treat terrain as the unified opponent to the mobility of the force.

Game Theory is positively the right way to think strategically about an interaction between opposing players. Furthermore, an optimal strategy is one that encompasses the strategy of both opposing players. In other words, mobility and terrain are two sides of the same coin. It is useless to talk about my mobility, unless I also talk about your ability to prevent my maneuver. Over the last few years [1], we have studied the legged robotics problem as a control problem, i.e., the operation of the mechanism as a balance between efficiency and speed. This treats the problem in a vacuum, as if no one cares whether or not you are strolling through their territory. We seek now to put this problem into the perspective of a game of strategy.

MOBILITY AS A GAME OF PURSUIT

I do not propose to write an all-encompassing theory of mobility. What we do here is provide a starting point. In 1951, when the RAND corporation was interested in applying the progress made during WWII, Rufus Isaacs developed a theory of pursuit and evasion [2]. These games of pursuit and evasion play a key role in our approach to mobility. Mobility is about progress from point A to point B. The mobility player, whom we call player P, seeks to minimize the time it takes to progress from point A to the goal. The opposing terrain player, whom we call player E, seeks to maximize this time. So we have subsumed a game of kind, where the results are go or no-go, with a game of degree where there is a continuous payoff, i.e. the time it takes to progress from point A to the goal.

And as a Game of Limited Resources

Clearly, we could create an untenable situation for the mobility player P, by giving E an unlimited amount of resistance. We get around this problem by limiting E's resources, both by limiting E's intensity of resistance and limiting E's total amount of resistance.

DISCRETE DOLICHOBRACHISTOCHRONE

Isaacs proposed a game theory variant to the classic brachistochrone problem or curve of quickest descent. A pursuer P seeks to reach the y-axis in minimum time, while the evader E seeks to maximize the time to reach the y-axis.

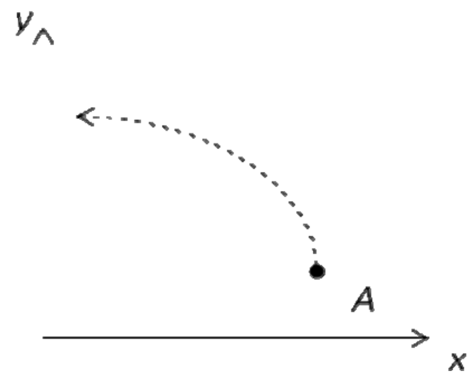

Figure 1: Dolichobrachistochrone.

In the discretized version of the problem, we set up a 4x4 grid as in Figure 2, giving P two choices at each node:

1) Stay at the same speed (horizontal) and head directly towards the goal (or)
2) Pick up speed (vertical drop) and forgo the goal for now:

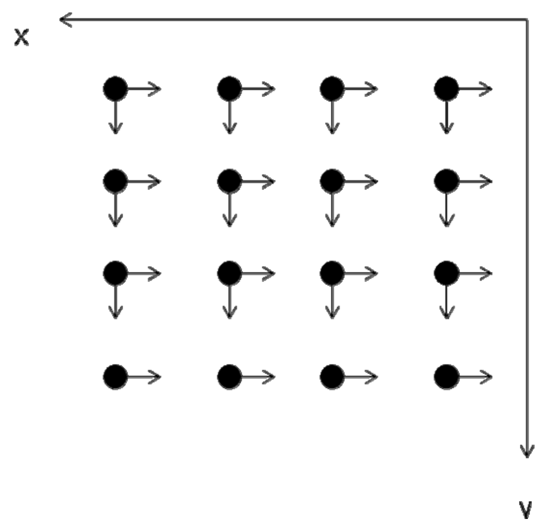

Figure 2: Discrete Dolichobrachistochrone.

The maximizing player E is trying to delay the inevitable. To do this, E has a limited number of "chips" to place on the 4x4 game board. With 4 chips there are

$$\binom{16}{4} = \frac{16!}{4!(16-4)!} = 1820 \tag{1}$$

different ways to place them on the grid. At each spot where E places a grid, he will be able to double the time cost at that node.

To start off, setup the y-dimension grid:

$$Y = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \tag{2}$$

Conservation of Energy yields:

$$v^2 = 2gh \tag{3}$$

Ignoring Constants:

$$V = sqrt(Y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{3} & \sqrt{3} \\ \sqrt{4} & \sqrt{4} & \sqrt{4} & \sqrt{4} \end{bmatrix} \tag{4}$$

This gives us a time cost between nodes (d=rt)

$$T = D / V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} & 1/\sqrt{4} \end{bmatrix}$$

Using the following node numbering scheme

$$\begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{bmatrix} \tag{6}$$

E playing no chips – lower bound

Without E playing any chips, the problem reduces to the discrete brachistochrone. We obtain the minimum path tree from dynamic programming [3]:

$$f(i) = \begin{bmatrix} 3.8284 & 3 & 2 & 1 \\ 2.8284 & 2.1213 & 1.4142 & 0.7071 \\ 2.3094 & 1.7321 & 1.1547 & 0.5774 \\ 2 & 1.5 & 1 & 0.5 \end{bmatrix} \tag{7}$$

The numbers at each grid location in equation (7) denote the optimal time cost from that grid location to the goal line--the column to the right--shown in Figure 2. As we can see in equation (7), from our starting point at A on the upper left, the optimal cost is 3.8284. From this point forward in the paper, we focus on the time cost from position A; we ignore the costs from other locations.

We now show the optimal choices of P for every location on the grid. Choosing a horizontal move is denoted by placing the number one at that grid location. Vertical is denoted by placing the number two at that grid location. For every grid location, we show the choices of P and also the locations where E will place his chips. We denote a chip placement by a number one at that grid location for E and a zero where E does not place a chip. For the lower bound, E has no chips:

$$P = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} E = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

$$Value = f(1) = 3.8284$$

E playing unlimited chips – upper bound

If E had unlimited chips, we would merely scale the problem by a factor of two.

$$P = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} E = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{9}$$

$$Value = f(1) = 7.6569$$

E playing 4 chips – sequential game

For the sequential version of this game: E is given 4 chips, which he places on the board. P sees where E has played those chips, and chooses his path accordingly. For this version, there are 4 solutions that yield the same value, or time cost, of 5.7236:

$$P=\begin{bmatrix}2&1&1&1\\2&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} E=\begin{bmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \quad (10)$$

$$P=\begin{bmatrix}2&1&1&1\\2&2&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} E=\begin{bmatrix}1&1&0&0\\1&0&1&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$P=\begin{bmatrix}2&1&1&1\\2&2&2&1\\1&1&1&1\\1&1&1&1\end{bmatrix} E=\begin{bmatrix}1&1&0&0\\1&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$P=\begin{bmatrix}2&2&1&1\\2&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix} E=\begin{bmatrix}1&0&1&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$Value = f(1) = 5.7236$$

E playing 4 chips – simultaneous game

In the simultaneous version of this game: E plays his 4 chips and P chooses his path through the grid simultaneously, without seeing each other's play. The choices for E are still the same, he has 1820 different ways to play his chips. However, P must choose his path now without a priori knowledge of E's choice.

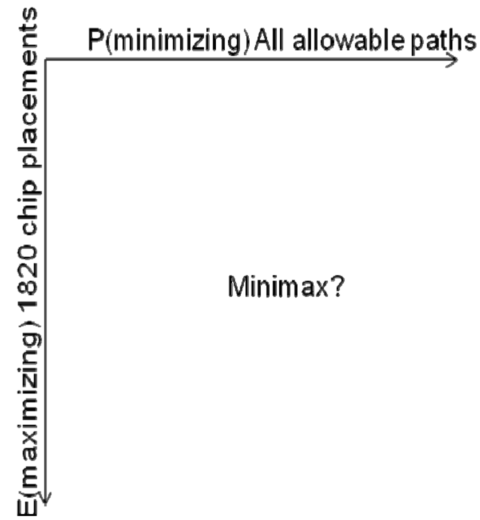

Figure 3: Simultaneous Play of the Discrete Dolichobrachistochrone.

The game matrix shown in Figure 3 is made up of a spreadsheet of time scores for each of the 35 possible paths P, the minimizing column player, could take through the grid against each of the 1820 different combinations E, the maximizing row player, could place his chips on the grid. We solve this game matrix by techniques as in [4].

For P to reach the goal line they must pass through one of the following nodes--note the node numbering sequence from Equation (6): Through node number 13 there is one path only:

$$(1,5,9,13) \quad (11)$$

Through node number 14, there are four possible paths:

$$\begin{gathered}(1,2,6,10,14)\\(1,5,6,10,14)\\(1,5,9,10,14)\\(1,5,9,13,14)\end{gathered} \quad (12)$$

Through node number 15, there are ten possible paths:

$$\begin{aligned}&(1,2,3,7,11,15)\\&(1,2,6,7,11,15)\\&(1,2,6,10,11,15)\\&(1,2,6,10,14,15)\\&(1,5,9,13,14,15)\\&(1,5,9,10,14,15)\\&(1,5,9,10,11,15)\\&(1,5,6,7,11,15)\\&(1,5,6,10,11,15)\\&(1,5,6,10,14,15)\end{aligned} \tag{13}$$

Through node number 16 there are twenty paths:

$$\begin{aligned}&(1,2,3,4,8,12,16) && (1,5,9,13,14,15,16)\\&(1,2,3,7,8,12,16) && (1,5,9,10,14,15,16)\\&(1,2,6,7,11,12,16) && (1,5,9,10,11,15,16)\\&(1,2,3,7,11,15,16) && (1,5,9,10,11,12,16)\\&(1,2,6,7,8,12,16) && (1,5,6,7,8,12,16)\\&(1,2,6,7,11,12,16) && (1,5,6,7,11,12,16)\\&(1,2,6,7,11,15,16) && (1,5,6,7,11,15,16)\\&(1,2,6,10,11,12,16) && (1,5,6,10,11,12,16)\\&(1,2,6,10,11,15,16) && (1,5,6,10,11,15,16)\\&(1,2,6,10,14,15,16) && (1,5,6,10,14,15,16)\end{aligned} \tag{14}$$

Thus there are 35 total paths to get from point A to the goal. Similar to the familiar rock, paper, scissors game, you want to employ a mixed strategy to keep your opponent guessing.

For E the mixed strategy is this:

$$E=\begin{bmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \quad \text{with probability } 0.5779$$

$$E=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix} \quad \text{with probability } 0.0327$$

$$E=\begin{bmatrix}1&1&0&0\\1&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix} \quad \text{with probability } 0.3893$$

Similarly for player P you want to vary the path you take through the grid:

$$\begin{aligned}&(1,5,9,13) && \text{with probability } 0.2412\\&(1,2,6,10,14) && \text{with probability } 0.3411\\&(1,2,3,7,11,15) && \text{with probability } 0.4177\end{aligned}$$

For this simultaneous game, we see that the value, the expected time cost, is higher than the sequential game.

$$Value = 5.9673 \tag{15}$$

Since P can no longer see E's move before he makes his play, it makes sense that E obtains an advantage.

CONCLUSION

As difficult as it is to create a general theory of mobility, it is worth the effort. We see evidence of this mobility game going on in Afghanistan. The Taliban uses the terrain to their advantage. They know that our immobility will constrain our vehicles to the road, and thus they know exactly where to place their IEDs. While we have really only scratched the surface of the problem, we hope in future work to apply this framework to a more practical application.

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