Video Signals
2. Image Definition and Processing

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1 Image Definition

- **Image:** Defined as a continuous function $f(x, y)$ over space.
- The function provides **brightness level** (luminance) or **color info**.
- Visualized as a "layer of rectangles" (grid structure).
- **Analog State:** Originally continuous in amplitude and space.
- To become **Digital**, it undergoes:
  - **Sampling:** Discretization of Space (determines resolution/number of pixels).
  - **Quantization:** Discretization of Amplitude (determines bit depth).
- **Pixel:** "Picture element". Represents brightness at a specific point.

2 Mathematical Representation

The image is represented as a matrix/grid function $f(x, y)$:

$$f(x,y) \approx \begin{bmatrix} x_{0,0} & \cdots & x_{0,N} \\ \vdots & \ddots & \vdots \\ x_{M,0} & \cdots & x_{M,N} \end{bmatrix} \tag{1}$$

- $f(x, y)$: Intensity value at spatial coordinates x, y (Row, Column).

3 Data Size Calculations

The general formula for calculating image data size is:

$$\text{Size} = L \times N \times M \times c \tag{2}$$

Where:
- L: Number of bits per pixel (Bit depth).
- $N \times M$: Resolution (Height $\times$ Width).
- c: Number of color components.

Example 1: Static Image

- Resolution: 1024×1280.
- Color Depth: 8 bits $\times$ 3 channels (RGB).
- Calculation:

$$1024 \times 1280 \times 8 \times 3 = 31,457,280 \text{ bits} \tag{3}$$

- In Bytes: $31,457,280/8 \approx 3,932,160$ bytes.
- In Megabytes: ≈ 3.75 MB.

Example 2: Video Data Rate

- Resolution: 2160×3840 (4K).
- Frame Rate: 50 fps.
- Calculation:

$$\text{Total bits/sec} = 2160 \times 3840 \times 24 \text{ bits} \times 50 \text{ fps} \tag{4}$$

- Result: $\approx 9,953,280,000$ bits/s ≈ 9.95 Gbps.

4 Illuminance and Histograms

- **Illuminance:** A physical measure related to the human perception of brightness.
- **Weber's Threshold:** Approximately 2% contrast is required for the eye to distinguish brightness differences.
- **Histogram Definition:** A graph/plot showing how many pixels fall into each intensity value.

Histogram Properties

1. **Many-to-One Mapping:** A histogram is unique for an image, but different images can share the same histogram.
2. **No Spatial Info:** It contains no information about *where* the pixels are located.
3. **Contrast Indicator:** If the histogram occupies only a small part of the range $[0, 255]$, the image has **low contrast**.

Conclusion: Equalization is required to fix low contrast.

5 Histogram Equalization (HE)

- **Goal:** To spread (stretch) histogram values across the full range.
- **Output Distribution:** Ideally results in a uniform (constant) distribution over $[0, L-1]$.
- **Transformation Formula:**

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) \tag{5}$$

- Uses the cumulative distribution function (CDF) to map input levels r to output levels s.

6 Adaptive Histogram Equalization (AHE)

- **Purpose:** Improves contrast **locally** rather than globally.
- **Use Case:** Necessary when contrast varies significantly across different parts of the scene.

Approaches

1. **Sliding Window:**
   - Computes a local histogram for a window centered on every pixel.
   - Computationally expensive ("passive" approach).
2. **Tiling (Block-based):**
   - Image is divided into a grid of small tiles.
   - Mappings are calculated for each tile independently.
   - **Bilinear Interpolation:** Required to blend mappings and remove artifacts at tile boundaries.

Issues and Solutions

- **Noise Amplification:** In "flat" (homogeneous) regions, standard AHE over-amplifies noise because the local histogram is narrow.
- **Solution (Clipping):** A "clip limit" is applied to restrict the contrast enhancement.
- **CLAHE:** Contrast Limited Adaptive Histogram Equalization.

7 Point Operations: Noise Reduction

- **Image Averaging:** Reduces noise by averaging multiple frames of the same static scene.
- **Process:**
  - Capture N frames.
  - Average the pixel values: preserves signal, cancels random noise.
  - Result: A "better picture" with improved Signal-to-Noise Ratio (SNR).

Mean Squared Error (MSE)

Used to measure the error/difference between the original image f and the processed image $\hat{f}$:

$$MSE = \frac{1}{N} \sum_{x,y} [f(x, y) - \hat{f}(x, y)]^2 \tag{6}$$

8 High Dynamic Range (HDR) Imaging

- **Definition:** Capturing scenes with a large brightness range (dynamic range) that exceeds standard sensor limits.
- **Problem:** Standard images lose detail in deep shadows or bright highlights.
- **Method (Exposure Bracketing):**
  - Capture multiple images at different exposures (e.g., underexposed for highlights, overexposed for shadows).
  - **Composition:** Select the best pixels from each exposure to form one HDR image.
- **Tone Mapping:** Required to compress the HDR range back to a displayable range.

9 Image Subtraction

- **Definition:** Computing the pixel-wise difference between two images.
- **Application:** Detecting motion or changes in light values.

$$d(x, y) = f_1(x, y) - f_2(x, y) \tag{7}$$

- $d(x, y)$: The difference image (shows changes).
- f_1, f_2: The two input frames.