4 Img Spatial Processing

Fourier Transform

Fourier Series (Periodic signal)

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

Fourier Transform

- Signal Type: Aperiodic
- Frequency Type: Continuous

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

- Relation from Periodic and Aperiodic fn

$$\mathrm{Rect}_T(t) = \begin{cases} 1 & |t| \le \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$$

$$\mathrm{Rect}_T(t) = \sum_{n=-\infty}^{\infty} \mathrm{Rect}_T\left(t - \frac{n}{T}\right)$$

$$\mathrm{Rect}_T(t) = \sum_{n=-\infty}^{\infty} \mathrm{Rect}_T(t - nT)$$

$$\mathrm{Rect}_T(t) = \sum_{n=-\infty}^{\infty} \mathrm{Rect}_T(t - nT) * \delta(t - nT)$$

$$\mathrm{Rect}_T(t) = \mathrm{Rect}_T(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

Q. Then Rect is equivalent to $\frac{1}{T}\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$

Fourier Series Filter (Periodic Fn)

- $g(t) = \cos(t)$

- Q. Remove $\frac{\pi}{2}$ → shift $\frac{\pi}{2}$ → $\cos\left(t - \frac{\pi}{2}\right)$
  - $\sin(t)$ Q. $\sin$ shift

Sampling Filter (Aperiodic)

$$f_s(t) = \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - nT)$$

$$F_s(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} F\left(\omega - \frac{2\pi n}{T}\right)$$

where $\frac{2\pi}{T}$ is sampling frequency

ω_{max} is max frequency of the signal

$\omega_{max} \le \frac{\omega_s}{2}$ or $\omega_s \ge 2\omega_{max}$ (Nyquist criterion)

Low Pass Gaussian Filter

- Gaussian Filter
- Remember Fourier Transform

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}}$$

$$F(\omega) = e^{-\frac{\omega^2}{2\sigma^2}}$$

- Low Pass Filter
  - σ is cut off frequency
  - σ is cut off frequency
  - σ is cut off frequency

Q. Gaussian Filter

- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Hi Pass Filter (High Pass)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Q. Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Hi Pass Filter

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Hi Pass Filter (Low Pass)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Fourier Transform Properties

Linearity

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

Scaling

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Shifting

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Modulation

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

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- σ is cut off frequency

Convolution

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
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- σ is cut off frequency

Fourier Transform Pairs

Dirac Delta Function

- Signal Type: Aperiodic
- Frequency Type: Continuous

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \ne 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\int_{-\infty}^{\infty} f(t)\, \delta(t)\, dt = f(0)$$

$$\int_{-\infty}^{\infty} f(t)\, \delta(t - t_0)\, dt = f(t_0)$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1$$

$$\delta(t - t_0) \xrightarrow{\mathcal{F}} e^{-j\omega t_0}$$

$$\delta(t - t_0) * \delta(t - t_1) = \delta(t - t_0 - t_1)$$

$$\delta(t - t_0) * \delta(t - t_1) * \delta(t - t_2) = \delta(t - t_0 - t_1 - t_2)$$

$$\delta(t - t_0) * \delta(t - t_1) * \delta(t - t_2) * \delta(t - t_3) = \delta(t - t_0 - t_1 - t_2 - t_3)$$

Gaussian Function

- Signal Type: Aperiodic
- Frequency Type: Continuous

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2\sigma^2}}$$

$$F(\omega) = e^{-\frac{\omega^2}{2\sigma^2}}$$

- Low Pass Filter
  - σ is cut off frequency
  - σ is cut off frequency
  - σ is cut off frequency

Rectangular Function

- Signal Type: Aperiodic
- Frequency Type: Continuous

$$\mathrm{Rect}_T(t) = \begin{cases} 1 & |t| \le \frac{T}{2} \\ 0 & |t| > \frac{T}{2} \end{cases}$$

$$\mathrm{Rect}_T(t) = \sum_{n=-\infty}^{\infty} \mathrm{Rect}_T\left(t - \frac{n}{T}\right)$$

$$\mathrm{Rect}_T(t) = \sum_{n=-\infty}^{\infty} \mathrm{Rect}_T(t - nT)$$

$$\mathrm{Rect}_T(t) = \sum_{n=-\infty}^{\infty} \mathrm{Rect}_T(t - nT) * \delta(t - nT)$$

$$\mathrm{Rect}_T(t) = \mathrm{Rect}_T(t) * \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\mathrm{Rect}_T(t) \xrightarrow{\mathcal{F}} \frac{1}{T}\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)$$

Sinc Function

- Signal Type: Aperiodic
- Frequency Type: Continuous

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Discrete Fourier Transform (DFT)

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Discrete Fourier Transform

- Signal Type: Periodic
- Frequency Type: Discrete

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Discrete Fourier Transform

- Signal Type: Periodic
- Frequency Type: Discrete

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\mathrm{sinc}(\omega) = \frac{\sin(\pi \omega)}{\pi \omega}$$

$$\mathrm{sinc}(t) \xrightarrow{\mathcal{F}} \mathrm{Rect}_T(\omega)$$

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
- σ is cut off frequency
- σ is cut off frequency

Gaussian Filter

Gaussian Filter

- Gaussian Filter
- σ is cut off frequency
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