Demand System Asset Pricing

Counterfactuals and Applications to Liquidity Measurement, Return Decompositions, and Return Predictability

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Counterfactuals: Motivating questions

- We can use the asset demand system to compute counterfactuals.
- Examples of questions that can be explored:
  1. Have financial markets become more liquid over the last 30 years with the growing importance of institutional investors?
  2. How much of the volatility and predictability of asset prices is explained by institutional demand?
  3. Do large investment managers amplify volatility? Should they be regulated as SIFI?

Computing counterfactuals

- Recall the market clearing equation

$$ME(n) = S(n)P(n) = \sum_{i=1}^{I} A_i w_i(n, \mathbf{me}, \mathbf{x}, \epsilon).$$

- Taking logarithms implies

$$\mathbf{p} = \mathbf{f}(\mathbf{p}) = \log\left(\sum_{i=1}^{I} A_i \mathbf{w}_i(\mathbf{p})\right) - \mathbf{s}.$$

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- Market clearing defines an implicit function for log price:

$$\mathbf{p}_t = \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t).$$

- Indeed, asset prices are fully determined by shares outstanding, characteristics, the wealth distribution, the coefficients on characteristics, and latent demand.

Computing counterfactuals

- To solve for prices, we need to solve a high-dimensional non-linear system.
- In practice, this can be done quite easily starting from the market clearing condition in logarithms:

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- Given a price vector $\mathbf{p}_m$, Newton's method would update the price vector through

$$\mathbf{p}_{m+1} = \mathbf{p}_m + \left(\mathbf{I} - \frac{\partial \mathbf{f}(\mathbf{p}_m)}{\partial \mathbf{p}'}\right)^{-1} (\mathbf{f}(\mathbf{p}_m) - \mathbf{p}_m).$$

- For our application, this approach would be computationally slow because the Jacobian has a large dimension.

Computing counterfactuals

- Therefore, we approximate the Jacobian with only its diagonal elements

$$\begin{aligned}\frac{\partial \mathbf{f}(\mathbf{p}_m)}{\partial \mathbf{p}'} \approx & \operatorname{diag}\left(\min\left\{\frac{\partial f(\mathbf{p}_m)}{\partial p(n)}, 0\right\}\right) \\ = & \operatorname{diag}\left(\min\left\{\frac{\sum_{i=1}^{I} \beta_{0,i} A_i w_i(\mathbf{p}_m; n)(1 - w_i(\mathbf{p}_m; n))}{\sum_{i=1}^{I} A_i w_i(\mathbf{p}_m; n)}, 0\right\}\right),\end{aligned}$$

  where the minimum ensures that the elements are bounded away from one.

- We have found that this algorithm is fast and reliable, converging in fewer than 100 steps in applications.

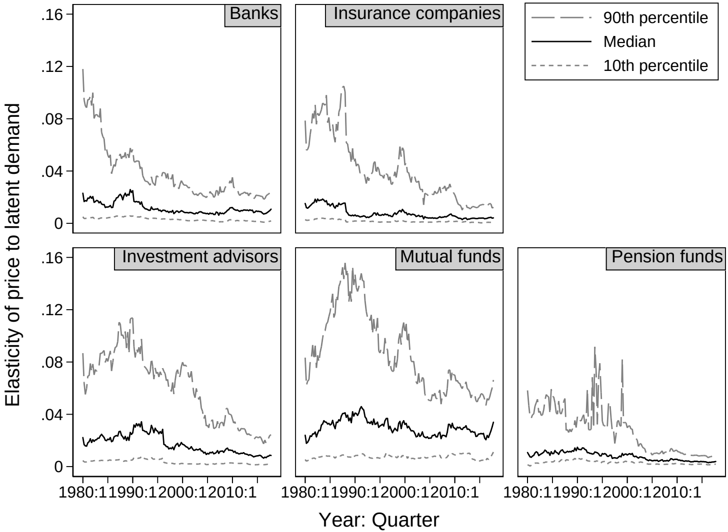
Liquidity measurement

- We define the coliquidity matrix for investor i as

$$\frac{\partial \mathbf{p}_t}{\partial \log(\epsilon_{i,t})'} = \left(\mathbf{I} - \sum_{j=1}^{I} A_{j,t}\beta_{0,j,t}\mathbf{H}_t^{-1}\mathbf{G}_{j,t} \right)^{-1} A_{i,t}\mathbf{H}_t^{-1}\mathbf{G}_{i,t}.$$

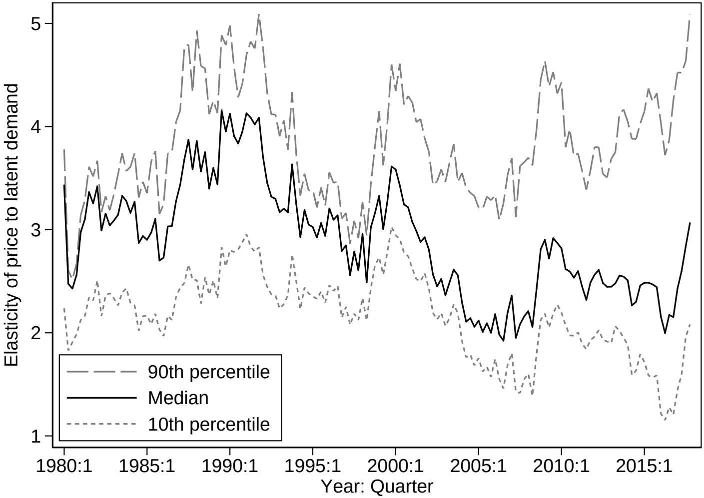
- We compute two measures of price impact
  - Price impact for each stock and institution via the diagonal elements of $\frac{\partial \mathbf{p}_t}{\partial \log(\epsilon_{i,t})'}$ and average by institutional type.
  - Aggregate price impact, defined as $\sum_{i=1}^{I} \frac{\partial \mathbf{p}_t}{\partial \log(\epsilon_{i,t})'}$, captures the price impact of systematic shocks to latent demand across all investors.

Price impact across stocks and institutions



Aggregate price impact across stocks

- Aggregate price impact: $\sum_{i=1}^{I} \partial p(n)/\partial \log(\epsilon_i(n))$.



Variance decomposition of stock returns

- We start with the definition of log returns:

$$\mathbf{r}_{t+1} = \mathbf{p}_{t+1} - \mathbf{p}_t + \mathbf{v}_{t+1},$$

where $\mathbf{v}_{t+1} = \log(\mathbf{1} + \exp\{\mathbf{d}_{t+1} - \mathbf{p}_{t+1}\})$.

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- The model implies that

$$\mathbf{p}_t = \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t)$$

1. $\mathbf{s}_t$: Shares outstanding.
2. $\mathbf{x}_t$: Asset characteristics.
3. $\mathbf{A}_t$: Assets under management.
4. β_t: Coefficients on characteristics.
5. ϵ_t: Latent demand.

Variance decomposition of stock returns

- We decompose the capital gain, $\mathbf{p}_{t+1} - \mathbf{p}_t$, as

$$\Delta\mathbf{p}_{t+1}(\mathbf{s}) + \Delta\mathbf{p}_{t+1}(\mathbf{x}) + \Delta\mathbf{p}_{t+1}(\mathbf{A}) + \Delta\mathbf{p}_{t+1}(\beta) + \Delta\mathbf{p}_{t+1}(\epsilon),$$

where:

$$\begin{aligned}\Delta\mathbf{p}_{t+1}(\mathbf{s}) =& \mathbf{g}(\mathbf{s}_{t+1}, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t) - \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t),\\ \Delta\mathbf{p}_{t+1}(\mathbf{x}) =& \mathbf{g}(\mathbf{s}_{t+1}, \mathbf{x}_{t+1}, \mathbf{A}_t, \beta_t, \epsilon_t) - \mathbf{g}(\mathbf{s}_{t+1}, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \epsilon_t),\\ & \dots\end{aligned}$$

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- We compute each of these counterfactual price vectors and decompose the cross-sectional variance of log returns as

$$1 = \frac{\mathrm{Cov}(\Delta\mathbf{p}_{t+1}(\mathbf{s}), \mathbf{r}_{t+1})}{\mathrm{Var}(\mathbf{r}_{t+1})} + \frac{\mathrm{Cov}(\Delta\mathbf{p}_{t+1}(\mathbf{x}), \mathbf{r}_{t+1})}{\mathrm{Var}(\mathbf{r}_{t+1})} + \dots.$$

Variance decomposition of stock returns

	% of variance
Supply:	
Shares outstanding	2.1
	(0.2)
Stock characteristics	9.7
	(0.3)
Dividend yield	0.4
	(0.0)
Demand:	
Assets under management	2.3
	(0.1)
Coefficients on characteristics	4.7
	(0.2)
Latent demand: Extensive margin	23.3
	(0.3)
Latent demand: Intensive margin	57.5
	(0.4)
Observations	134,328

Variance decomposition of stock returns in 2008

- The asset demand system can also be used to understand how much an investor contributes to the the fluctuations in a given stock.
- This provides a new perspective on the "dark matter" in financial markets.

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- The asset demand system can also be used to understand how much an investor contributes to the the fluctuations in a given stock.
- This provides a new perspective on the "dark matter" in financial markets.
- We provide an illustration during the financial crisis.
- We modify the variance decomposition as

$$\begin{aligned}\mathrm{Var}(\mathbf{r}_{t+1}) =& \mathrm{Cov}(\Delta\mathbf{p}_{t+1}(\mathbf{s}) + \Delta\mathbf{p}_{t+1}(\mathbf{x}) + \mathbf{v}_{t+1}, \mathbf{r}_{t+1}) \\ &+ \sum_{i=1}^{I} \mathrm{Cov}(\Delta\mathbf{p}_{t+1}(A_i) + \Delta\mathbf{p}_{t+1}(\beta_i) + \Delta\mathbf{p}_{t+1}(\epsilon_i), \mathbf{r}_{t+1}).\end{aligned}$$

Variance decomposition of stock returns in 2008

- Are large investment managers systemic?

AUM ranking	Institution	AUM ($ billion)	Change in AUM (%)	% of variance	
	Supply: Shares outstanding, stock characteristics & dividend yield			8.1	(1.0)
1	Barclays Bank	699	-41	0.3	(0.1)
2	Fidelity Management & Research	577	-63	0.9	(0.2)
3	State Street Corporation	547	-37	0.3	(0.0)
4	Vanguard Group	486	-41	0.4	(0.0)
5	AXA Financial	309	-70	0.3	(0.1)
6	Capital World Investors	309	-44	0.1	(0.1)
7	Wellington Management Company	272	-51	0.4	(0.1)
8	Capital Research Global Investors	270	-53	0.1	(0.1)
9	T. Rowe Price Associates	233	-44	-0.2	(0.1)
10	Goldman Sachs & Company	182	-59	0.1	(0.1)
	Subtotal: 30 largest institutions	6,050	-48	4.4	
	Smaller institutions	6,127	-53	40.7	(2.3)
	Households	6,322	-47	46.9	(2.6)
	Total	18,499	-49	100.0	

Predictability of stock returns

- Recall that

$$\mathbf{p}_T = \mathbf{g}(\mathbf{s}_T, \mathbf{x}_T, \mathbf{A}_T, \beta_T, \epsilon_T)$$

- Model ϵ_T as mean reverting and everything else as random walk.

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- Model ϵ_T as mean reverting and everything else as random walk.
- First-order approximation of expected long-run capital gain:

$$\begin{aligned} \mathbb{E}_t[\mathbf{p}_T - \mathbf{p}_t] \approx & \mathbf{g}(\mathbb{E}_t[\mathbf{s}_T], \mathbb{E}_t[\mathbf{x}_T], \mathbb{E}_t[\mathbf{A}_T], \mathbb{E}_t[\beta_T], \mathbb{E}_t[\epsilon_T]) - \mathbf{p}_t \\ = & \mathbf{g}(\mathbf{s}_t, \mathbf{x}_t, \mathbf{A}_t, \beta_t, \mathbf{1}) - \mathbf{p}_t \end{aligned}$$

- Intuition: Assets with high latent demand are expensive and have low expected returns.

Relation between stock returns and characteristics

Characteristic	All stocks	Excluding microcaps
Expected return	0.18	0.11
	(0.04)	(0.04)
Log market equity	-0.25	-0.15
	(0.08)	(0.08)
Book-to-market equity	0.04	0.06
	(0.04)	(0.05)
Profitability	0.30	0.29
	(0.06)	(0.06)
Investment	-0.38	-0.21
	(0.03)	(0.03)
Market beta	0.08	0.01
	(0.08)	(0.10)
Momentum	0.24	0.37
	(0.08)	(0.10)

Predicting returns by predicting demand

- Modern approaches to return predictability take characteristics of stocks and use it to predict returns directly.
- DSAP provides another approach by first predicting demand and then predict returns via market clearing.
- The conditions under which both approaches are equivalent are quite strong and require a lot of homogeneity across investors.
- Predicting returns by predicting demand can yield new insights by taking a more granular approach. We just provide a simple first example as a "proof of concept."

Additional applications

- In Koijen, Richmond, and Yogo (2021), we provide additional examples of counterfactuals:
  - How important are different investors for pricing characteristics (e.g., governance or environmental characteristics).
  - How did the transition from active to passive investment affect prices, investors' wealth, and price informativeness?
  - Climate stress tests: If there is a shift in demand for green characteristics (e.g., because of growing awareness or because of new regulation for insurers and pension funds), how would this affect prices and investors' wealth.

Lucas critique

- Of course, characteristics-based demand can be used for policy experiments only under the null that it is a structural model of asset demand that is policy invariant.
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- Also, we cannot answer welfare questions without taking an explicit stance on preferences, beliefs, and constraints.
- However, for most asset pricing applications, price (rather than welfare) is the primary object of interest.
- That said, it highlights the importance of developing new micro foundations that can deliver inelastic demand and other key features of the asset demand system.

Conclusions

- We show how to calculate counterfactuals once we have estimated the asset demand system.
- The demand system can be use to connect fluctuations in prices to changes in characteristics and investors' demand.
- This provides a new perspective to start analyzing the "dark matter" in financial markets.
- Moreover, by predicting demand, we provide a new approach to return predictability, where machine learning/AI methods are particularly well suited as holdings data are very high-dimensional.