PROBABILITY ESTIMATES OF EXTREME TEMPERATURE EVENTS: STOCHASTIC MODELLING APPROACH VS. EXTREME VALUE DISTRIBUTIONS

JAN KYSELÝ

*Institute of Atmospheric Physics, Acad. Sci. Czech Republic, Prague, Czech Republic**

ABSTRACT

The paper deals with the probability estimates of temperature extremes (annual temperature maxima and heat waves) in the Czech Republic. Two statistical methods of probability estimations are compared; one based on the stochastic modelling of time series of the daily maximum temperature (TMAX) using the first-order autoregressive (AR(1)) model, the other consisting in fitting the extreme value distribution to the sample of annual temperature peaks.

The AR(1) model is able to reproduce the main characteristics of heat waves, though the estimated probabilities should be treated as upper limits because of deficiencies in simulating the temperature variability inherent to the AR(1) model. Theoretical extreme value distributions do not yield good results when applied to maximum annual lengths of heat waves and periods of tropical days (TMAX ≥ 30°C), but it is the best method for estimating the probability and recurrence time of annual one-day temperature extremes. However, there are some difficulties in the application: the use of the two-parameter Gumbel distribution and the three-parameter generalized extreme value (GEV) distribution may lead to different results, particularly for long return periods. The resulting values also depend on the chosen procedure of parameter estimation. Based on our findings, the shape parameter testing for the GEV distribution and the L moments technique for parameter estimation may be recommended.

The application of the appropriate statistical tools indicates that the heat wave and particularly the long period of consecutive tropical days in 1994 were probably a more rare event than the record-breaking temperatures in July 1983 exceeding 40°C. An improvement of the probability estimate of the 1994 heat wave may be expected from a more sophisticated model of the temperature series.

Keywords: autoregressive model, extreme value distribution, *L* moments, maximum likelihood method, annual temperature maximum, heat wave, 1994 heat wave

* Boční II/1401, 141 31 Prague 4, Czech Republic (honza@ufa.cas.cz)

1. INTRODUCTION

Extreme weather and climate events severely influence ecosystems and human society. High temperatures are among the most frequently investigated extreme events; the domains in which they affect society include agriculture, water resources, energy demand and human mortality (e.g., *Watson et al., 1996*). Many research activities focus on extreme climate phenomena both because of their current impacts and the threat of their possible increases in frequency, duration and severity in a climate perturbed by enhanced concentrations of greenhouse gases in the atmosphere. Impacts of climate change would result from changes in variability and extreme event occurrence rather than from an increase in mean temperature (*Houghton et al., 1996; Watson et al., 1996; Parmesan et al., 2000*) and even relatively small changes in the means and variations of climate variables can induce considerable changes in the severity of extreme events (*Katz and Brown, 1992; Hennessy and Pittock, 1995; Colombo et al., 1999*).

Impacts of extreme events are more serious when extreme weather conditions prevail over extended periods. That is why prolonged extreme temperature events (usually referred to as heat waves and cold spells) are frequently investigated (e.g., *Sartor et al., 1995; Rooney et al., 1998; Colombo et al., 1999; Huth et al., 2000; Kyselý, 2000*).

This study focuses on the probability of recurrence of extreme high daily maximum temperatures and extreme heat waves in the Czech Republic. For more details concerning the definition and characteristics of heat waves see, e.g., *Huth et al. (2000), Kyselý et al. (2000)* or *Kyselý (2000)*. Two statistical methods of the probability estimations are compared; one is based on the stochastic modelling of time series of the daily maximum temperature (*TMAX*) using the first-order autoregressive model, the other consists in fitting an extreme value distribution to the sample of annual temperature extremes.

The paper is organized as follows: In Section 2, basic facts about extreme high daily air temperatures and extreme heat waves in the Czech Republic are summarized. A description of the data used in this study is given in Section 3. The first-order autoregressive model is described and validated in Section 4 where it is also used to estimate the return periods of extreme heat waves. Extreme value distributions are described (with particular reference to the method of *L* moments) and applied to annual temperature extremes and extreme heat waves in Section 5. Discussion and conclusions follow in Sections 6 and 7.

2. EXTREME HIGH TEMPERATURES AND EXTREME HEAT WAVES IN THE CZECH REPUBLIC

Unusually high summer temperatures occurred in several continental areas of the Northern Hemisphere in the 1980s and mainly in the 1990s; Central Europe was one of the most affected regions. Especially the 1994 and 1995 summer seasons brought a lot of record-breaking temperatures at a large number of locations across Europe. In 1994 the most unusual temperatures occurred in Central and Northeastern Europe, encompassing Sweden, Denmark, Belgium, the Netherlands, Germany and Poland, while in 1995, the main warm areas extended further west from the British Isles to Spain (*Nicholls, 1998*). In addition to record-high daily temperatures (e.g., 36.7°C at Lycksele, northern Sweden, 1994; 38.9°C at Virton, Belgium, 1994; 46.6°C at Cordoba, Spain, 1995), long periods

Table 1. The most severe heat waves (as measured by cumulative *TMAX* excess above 30°C, *TS30*) and the longest continuous periods of days with *TMAX* ≥ 30.0°C at Prague-Klementinum, 1901−1997. From *Kyselý (2002)*.

a) The most severe heat waves

Year	Beginning	End	Duration [days]	Peak temperature [°C]	*TS30* [°C]
1994	Jul 21	Aug 11	22	36.0	47.6
1957	Jun 28	Jul 10	13	37.6	34.2
1992	Jul 16	Aug 10	26	35.8	33.3
1921	Jul 23	Aug 12	21	34.7	31.4
1952	Jul 31	Aug 16	17	35.5	21.9

b) The longest continuous periods of days with TMAX ≥ 30.0°C

Year	Duration [days]	Peak temperature [°C]
1994	16	36.0
1911	9	33.5
1921	7	34.7
1929	7	34.0
1942	7	31.5

(heat waves) with gradual development of extreme heat were particularly interesting in 1994 and 1995. Several studies have dealt with increases in the daily mortality rates during these heat waves, e.g., *Sartor et al. (1995)* for the 1994 heat wave in Belgium and *Rooney et al. (1998*) for the 1995 heat wave in Great Britain. The economic impacts of the hot summer of 1995 in Great Britain were analysed by *Agnew and Palutikof (1999)*.

In the Czech Republic, particularly the July and August 1994 heat wave must be considered exceptional. It was the most severe heat wave (as measured in terms of the cumulative *TMAX* excess above 30.0°C) at Prague-Klementinum during the 20th century, and very likely from the beginning of continual temperature measurements in 1775 (*Kyselý and Kalvová, 2000; Kyselý, 2002*). The period of 16 successive tropical days (days with *TMAX* ≥ 30.0°C), recorded at Klementinum between July 22 and August 6, 1994, is also unprecedented. The second longest period of tropical days (in 1892) lasted 11 days, and the second longest period during the 20th century (in 1911) only 9 days (see Tab. 1). The difference between the 1994 heat wave and the others cannot be attributed to the increased urban heat island, because its intensification was found to be insignificant in summer seasons during the 20th century (*Brázdil and Budíková, 1999*).

Towards the east, the duration of the period of consecutive tropical days in 1994 was even higher. It reached 19 days at a few Moravian stations and exceeded 20 days in eastern Slovakia (*Krška and Racko, 1996*). The period lasted 18 days even in places located above 450 m a.s.l. (*Kyselý, 2000*). At some stations, e.g., Svratouch (737 m a.s.l.), the number of tropical days recorded within the 1994 heat wave was nearly the same as during the whole 30-year period 1961-1990 (*Kyselý, 2000*). However, absolute record-breaking daytime and nighttime temperatures were not reached in 1994.

The highest temperatures ever recorded in the Czech Republic, reaching 40°C in south and central Bohemia (*Krška and Munzar, 1984*), were observed in 1983. Extreme temperatures were then confined to relatively short periods, most of the heat wave characteristics not reaching a severity comparable to those of 1994. The absolute highest temperature, 40.2°C, was observed at Prague-Uhříněves on July 27, but 40°C was also recorded at a few other places on the same day, e.g., at Sedlčany (360 m a.s.l.), Klatovy (430 m a.s.l.) and Husinec (536 m a.s.l.) (*Krška and Munzar, 1984; Kyselý, 2000*). Despite the record-breaking temperatures, the duration of heat waves was much shorter in 1983 than in 1994. This reflects the relatively frequent cold front passages in 1983 (*Krška and Munzar, 1984*); although the cold fronts were weak, they interrupted the hot period several times. In 1994, long periods with high air temperature and low interdiurnal temperature variability were related to persistent circulation patterns over Europe with high-pressure systems influencing central Europe. It is worth noting that in Moravia, higher temperatures were recorded in 1994 than in 1983. The above-mentioned differences between the 1983 and 1994 summer weather are evident from Fig. 1 which compares the course of *TMAX* of June to August at two stations, Klatovy (southwest Bohemia) and Strážnice (southeast Moravia).

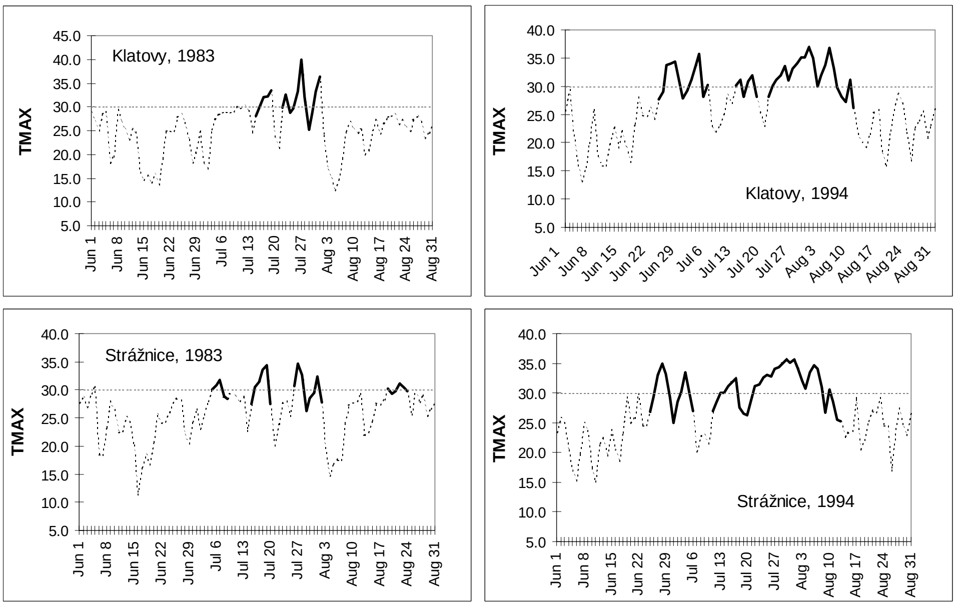

Fig. 1. *TMAX* course in summer of 1983 and 1994 at stations Klatovy and Strážnice. Heat waves are plotted bold, the horizontal line shows the threshold for a tropical day (30°C).

It appears to be clear that both the 1983 record-high temperatures and the 1994 heat wave were exceptional, at least in the 20th century. The aim of this contribution is to evaluate the return periods of these events, and compare the methods used in their probability estimations.

As for the 1994 period of consecutive tropical days, *Krška and Racko (1996)* estimated the probability of its recurrence to be in the order of 100–200 years. This estimate is based on a previous study by *Racko (1987)* who evaluated the probability of occurrence of long tropical day periods in Slovakia using Gumbel statistics; the parameters of the Gumbel distribution were derived from the data covering the period 1951–1975. It is demonstrated further that this method leads to very different values if the individual years with the most severe heat waves are included or omitted, and is, therefore, inappropriate.

3. DATA

The analysis was performed at three stations located in different parts of the Czech Republic and various climatological settings, namely Klatovy (430 m a.s.l., southwestern Bohemia, extremely high temperature in 1983), Strážnice (187 m a.s.l., southeastern Moravia, extremely long heat wave and tropical day period in 1994), and Prague-Ruzyně (364 m a.s.l., central Bohemia). Available observations cover the period 1961–1998.

4. THE FIRST-ORDER AUTOREGRESSIVE MODEL

4.1. Basic description

One way of estimating the probabilities of occurrence of extreme events is to employ stochastic time series modelling. First-order autoregressive (AR(1)) models are frequently used to simulate time series of *TMAX* and provide characteristics of heat waves and temperature threshold excesses that are in good agreement with observations (*Mearns et al., 1984; Macchiato et al., 1993; Hennessy and Pittock, 1995; Colombo et al., 1999; Kyselý, 2000*). In a recent study, *Kalvová et al. (2000)* estimated the optimum order of autoregressive models for *TMAX* in observed (south Moravia) and simulated (ECHAM3 GCM) climates using a non-parametric method. While AR(2) was identified to be the closest to the simulated series, AR(1) is the best choice for the ones observed.

The appropriateness of the AR(1) model is discussed in more details, e.g., in *Colombo et al. (1999)* for Canadian stations. Using the Bayesian information criterion (*Katz and Skaggs, 1981; von Storch and Zwiers, 1999*) they arrived at the conclusion that higher-order models (e.g., AR(4) with the second-order parameter constrained to zero) may yield slightly better results at some sites, but the improvement in accuracy of the fit cannot compensate the parsimony of the AR(1) model.

The AR(1) model was applied at each of the three sites (Klatovy, Strážnice and Prague) to generate long series of *TMAX* from which the probabilities of recurrence of extreme temperature events can be estimated. The model is based on three parameters of *TMAX* series, namely the mean ($\mu(t)$), the variance ($\sigma^2(t)$) and the first-order autocorrelation coefficient ($\Phi(t)$); t denotes the time parameter because the seasonal cycle is considered here (see below). Synthetic time series of *TMAX* were created by employing the algorithm described, e.g., in *Mearns et al. (1984)* and *Macchiato et al. (1993)*.

Generally, there are two approaches of simulating temperature series when only a few months (typically one to three) of the year are examined; first, the seasonal cycle is considered explicitly as the deterministic part, and only deviations from this cycle are simulated by the AR(1) model (as in *Macchiato et al., 1993*), or the deterministic part is not considered at all and the whole series (one year consecutive to another) is treated as if it were a mere realization of the AR(1) process (as in *Mearns et al., 1984*). Here, the former method (physically more reasonable) was adopted since the seasonal cycle may play an important role both in supporting the heat wave development (smaller deviations from the long-term mean, say, in July than in June are necessary to make up a heat wave) and in imposing some limitations on the length of the heat wave, and because the whole five-month period (of May to September) when heat waves may occur should be considered.

The values of *TMAX* were then determined according to the following recursion formula:

$$TMAX(t) = \mu(t) + \phi(t)\left(TMAX(t-1) - \mu(t-1)\right) + \varepsilon(t)\,.$$

First, an initial value for the series is generated from a normal distribution $N\left(\mu(1), \sigma^2(1)\right)$ for the first day considered in each year, and random variable $\varepsilon(t)$ is then generated for each day from the $N\left(\mu(t), \sigma_\varepsilon^2(t)\right)$ distribution where the variance of $\varepsilon(t)$ is $\sigma_\varepsilon^2(t) = \left(1 - \phi^2(t)\right)\sigma^2(t)$. The estimate of $\Phi(t)$ was computed according to *Kendall and Stuart (1976)*

$$\Phi(t) = \frac{\sum\limits_{i=t-L}^{t+L-1} u(i+1)u(i)}{\sqrt{\sum\limits_{i=t-L}^{t+L-1} u^2(i) \sum\limits_{i=t-L}^{t+L-1} u^2(i+1)}}\,,$$

where $u(i)$ stands for the standardized variable $\dfrac{TMAX(i) - \mu(i)}{\sigma(i)}$ and L is the half-width of the moving window (here 30 days). The Box-Muller random number generator (*Press et al., 1992*) was used to generate the independent random variable with normal distribution.

Time series of *TMAX* were generated for the months of May to September (153 days in each year). Seasonal changes in the mean temperature, the standard deviation and the first-order autocorrelation coefficient were considered; the seasonal cycle of $\mu(t)$ was smoothed using 15-day running means, and the standard deviation and first-order autocorrelation coefficient were estimated for moving 61-day windows. Sometimes models of this type are called dynamic-stochastic models as they contain a deterministic (the seasonal or annual cycle) and a stochastic component (which models the deviations from the deterministic part).

4.2. Validation

The AR(1) model reproduces all three characteristics of *TMAX* (mean, variance and first-order autocorrelation coefficient) in good agreement with observations. As to the variance, the model reflects its overall value well, but it tends to underestimate the interannual variability and overestimate the intraseasonal variability. That is why the day-to-day changes in *TMAX* are slightly overestimated (see the last column in Tab. 2) and the year-to-year changes in seasonal means slightly underestimated; this is a typical feature of the temperature series obtained using an AR(1) model (e.g., *Madden and Shea, 1978*).

For a basic evaluation of the ability of the AR(1) model to reproduce heat wave characteristics, twenty daily data samples of the same length as the observations (i.e. 38 years) were generated for each of the three stations, followed by a heat wave analysis. The comparison of heat wave characteristics from observed and simulated *TMAX* is shown in Tab. 2 for the Klatovy station. The AR(1) model is successful in reproducing most heat wave properties (their frequency, annual duration, cumulative *TMAX* excess above 30°C, location within a year, individual length) since the observed values are within the range of the simulated characteristics and close to their mean. Also the mean annual frequency of tropical days and days with *TMAX* ≥ 32.0°C are reflected reasonably well. Only the temporal evolution of a heat wave (which is a property of marginal importance in this analysis) with the temperature peak typically shifted towards the end of a hot period is not reproduced by the AR(1) model, simulated heat waves tend to peak at too high temperatures, and the inclusion of tropical days into prolonged periods is too low.

Table 2. Comparison of the mean heat wave characteristics at the Klatovy station in 1961–1998 and in twenty simulations of the same length with the AR(1) model. *MIN* (*AVG*, *MAX*) denotes the minimum (average, maximum) value from the sample of twenty generated series. Mean annual characteristics (frequency of heat waves *f**, number of tropical days *T**, number of days with *TMAX* ≥ 32.0°C *T32**, duration of heat waves *D**, cumulative temperature excess *TS30**) together with mean characteristics of the individual heat waves (length *d*, mean relative position of the peak within a heat wave *p*, elevation of 1-day and 3-day temperature peaks above 30.0°C *TX1* and *TX3*) are shown. *W*/*T* denotes the ratio of the number of tropical days occurring within heat waves to the number of all tropical days. The mean interdiurnal change of *TMAX* in July–August is given in the last column (*TX78i*). Further details concerning heat wave characteristics can be found in *Kyselý (2000)*.

	Mean annual characteristics					Characteristics of individual heat waves					
	*f**	*T** [days]	*T32** [days]	*D** [days]	*TS30** [°C]	*d* [days]	*p*	*TX1* [°C]	*TX3* [°C]	*W*/*T* [%]	*TX78i* [°C]
Klatovy	1.05	8.6	3.4	7.9	11.6	7.5	0.67	3.8	2.4	61.0	2.97
MIN	0.79	6.9	2.7	5.6	8.9	7.0	0.43	4.1	2.5	46.6	3.04
AVG	0.96	8.2	3.6	7.4	12.4	7.7	0.51	5.0	3.1	54.0	3.10
MAX	1.34	10.1	4.7	10.8	16.5	8.8	0.56	5.7	3.7	65.8	3.18

Similar, favourable results were also obtained for the two other stations. Moreover, the model is able to reproduce some differences between the characteristics from different stations that are not directly determined by the parameters of the AR(1) model; e.g., a growing inclusion of tropical days in hot periods with increasing heat wave frequency and higher temperature peaks at Klatovy compared to other stations are reflected.

Most of the model's advantages and drawbacks are the same as that of the four-variable stochastic weather generator Met&Roll (*Dubrovský, 1997*) which is currently used in impact studies of climate change. The main improvement due to the use of the AR(1) model is the correct reproduction of the mean length of heat waves which is not underestimated, in contrast with the results of Met&Roll (*Kyselý et al., 2001*).

The AR(1) model also reproduces successfully the distribution of lengths of heat waves. A comparison of simulated and observed frequencies of heat waves according to their lengths is shown in Fig. 2 for the Strážnice station. The model's curve was determined from a 100 000-year long series and is, therefore, relatively smooth compared to observation. The AR(1) model slightly underestimates the percentage of short heat waves (lasting 3 to 7 days) and overestimates frequencies of medium-length heat waves (8−11 days). The percentage of long heat waves (12 days or longer) appears to be simulated well; averaged over the three stations, they constitute 13.3% (14.5%) of all heat waves in the observed (simulated) data. The ability of the AR(1) model to reflect even very long heat waves is demonstrated by the fact that, in long simulations based on the current climate, the model's most extreme heat waves considerably exceed in lengths all the observed heat waves at all three stations. Therefore, the probability of recurrence of long heat waves can be estimated from a long series of *TMAX* generated by the AR(1) model.

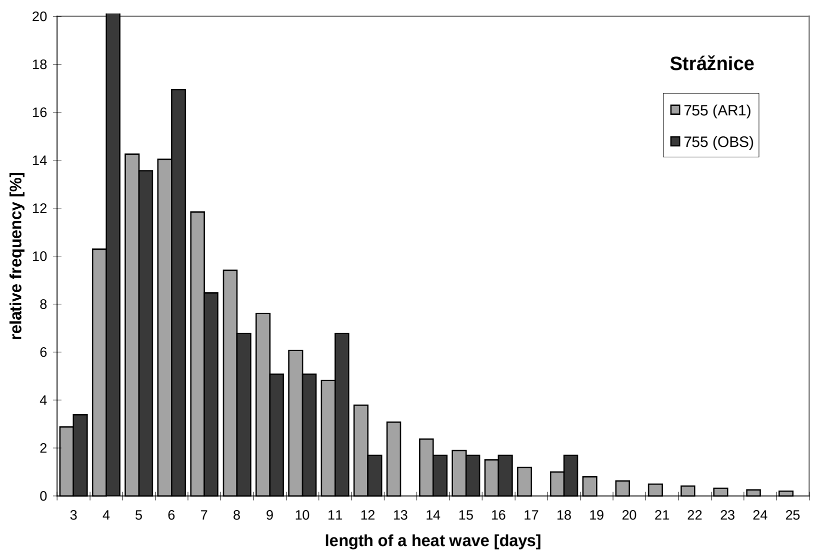

Fig. 2. Distribution of lengths of simulated and observed heat waves at the Strážnice station.

Unlike the probabilities of long heat waves, return periods of extreme high daily maxima cannot be estimated using the AR(1) model simulations. The reason is that for threshold values of *TMAX* higher than approximately 32.0°C, the model overestimates frequencies of the threshold crossings, and the overestimation increases with increasing threshold value. For instance, the frequencies of days with $TMAX \geq 34.0$°C are overestimated by a factor of 1.2 in the simulated series at the Klatovy station, while the occurrence of days with $TMAX \geq 36.0$°C is higher by a factor of 1.7. This property of the model's series is reflected also in the overestimated temperature peaks in heat waves (Table 2).

4.3. Return periods of extreme heat waves

For all three stations (Klatovy, Strážnice and Prague-Ruzyně), a 100 000-year long series of *TMAX* (i.e. 1.53×10^7 daily values of *TMAX* in the period of May to September) was generated by the AR(1) model. Probabilities of the occurrence of heat waves lasting at least 15, 20, 25, 30 and 35 days were evaluated and are summarized in Tab. 3 in terms of return periods. It is evident that long heat waves observed at all three stations in 1994 must be considered exceptional and the probabilities of their recurrence under unchanged climatic conditions are low. For example, at Prague-Ruzyně the return period of a heat wave lasting at least 17 days (July 21 − August 6, 1994) is around 100 years according to the AR(1) model simulations, whereas at Strážnice, the return period of a 34-day heat wave (July 11 − August 13, 1994) is ca 700 years. These values must be treated as first estimations and they probably represent upper limits for the real return periods. This is due to the deficiencies in simulating the heat wave characteristics and temperature variability inherent to the AR(1) model; the observed series of *TMAX* are not exact realizations of the AR(1) process (they may reflect, e.g., persistence with longer lags, long-term variations and trends, etc. which are not described in the AR(1) model). It is worth noting that also in generated series, extremely long heat waves are not characterized by breaking long-term temperature records on individual days, but usually by long periods without considerable cooling and with damped interdiurnal temperature variability.

The AR(1) model has a limited efficiency of reproducing periods of successive tropical days. For example, the return period of at least 12 consecutive tropical days was estimated to be 200 years at Strážnice. On the other hand, extreme events analogous to those recorded in 1994 occur rather exceptionally in the simulated series; e.g., an

Table 3. Return periods (in years) of heat waves lasting at least 15, 20, 25, 30 and 35 days at three observing stations. The values are derived from the 100 000-year long series simulated by the AR(1) model.

	15 days	20 days	25 days	30 days	35 days
Klatovy	19	78	310	1320	---
Prague-Ruzyně	55	330	2000	---	---
Strážnice	8	25	79	270	910

uninterrupted period of tropical days lasting at least 18 (16) days (which was the duration in 1994) occurs once in 7 000 (6 000) years at Strážnice (Klatovy). To obtain a more realistic estimate of the return period of a continuous tropical day period observed in 1994, a higher-order AR model, probably also with a better simulation of the interannual variability would have to be considered. Nevertheless, it appears to be clear that the conditions during the 1994 heat wave were quite exceptional, that the probability of their recurrence is very low under present climate conditions, and that a consecutive period of up to 19 tropical days is an even more sporadic event than the heat wave lasting more than a month.

Despite the limitations that are inherent to the model, interesting results can be achieved when the return periods of consecutive tropical days are evaluated under changed mean temperature conditions. According to the AR(1) model simulations, an 1°C (3°C) increase in the mean *TMAX* at Strážnice in summer would result in a substantial decline of the return periods. For instance, the recurrence time for an 18-day uninterrupted tropical period would decline from 7000 years under current (1961–1998) climate to 1100 years (65 years). In a climate warmer by 3°C, tropical periods lasting more than 30 days would be of the same frequency as periods of at least 18 tropical days in the current climate. Changes in the variance and autocorrelation structure of *TMAX* (that were not considered here) may lead to even more pronounced intensification of the extremes.

5. EXTREME VALUE DISTRIBUTIONS

5.1. Basic description

Extreme-value data can be characterized by theoretical probability distributions. A number of extreme value distributions exist; probably the Gumbel distribution is the best-known (e.g., *Wilks, 1995*). For the probability estimates of annual temperature extremes, the two-parameter (ξ, β) Gumbel distribution and the three-parameter (ξ, β, k) generalized extreme value (GEV) distribution are commonly applied (*Faragó and Katz, 1990; Brown and Katz, 1995; Zwiers and Kharin, 1998; Kharin and Zwiers, 2000*). The probability density functions of these distributions are analytically integrable and lead to distribution functions

$$F(x) = \exp\left\{-\exp\left(-\frac{x-\xi}{\beta}\right)\right\}$$

for the Gumbel distribution, and

$$F(x) = \exp\left\{-\left(1-k\frac{x-\xi}{\beta}\right)^{\frac{1}{k}}\right\}, \quad x < \xi + \frac{\beta}{k}, \quad k > 0$$

$$F(x) = \exp\left\{-\exp\left(-\frac{x-\xi}{\beta}\right)\right\}, \quad k = 0$$

$$F(x) = \exp\left\{-\left(1 - k\frac{x-\xi}{\beta}\right)^{\frac{1}{k}}\right\}, \quad x > \xi + \frac{\beta}{k}, \quad k < 0$$

for the GEV distribution. One can see that the Gumbel distribution is a special case ($k = 0$) of the GEV distribution. The introduction of the third parameter k (shape parameter) into the GEV distribution improves the fit to the upper tail when the extremes are not Gumbel distributed; for $k < 0$ ($k > 0$) the probability density function of the GEV distribution converges more slowly (more rapidly) to zero. Various methods (e.g., method of moments, method of probability-weighted moments, maximum likelihood method) are used to estimate the parameters of the distributions; their description can be found, e.g., in *Faragó and Katz (1990)*, *Dufková (1997)* and *von Storch and Zwiers (1999)*. The simplest way of estimating parameters ξ and β of the Gumbel distribution is based on the sample mean μ and the sample standard deviation σ; the parameter estimates then being $\beta = \sigma\sqrt{6}/\pi$ and $\xi = \mu - \gamma\beta$ where γ stands for the Euler constant (approximately $\gamma = 0.57721$; *Wilks, 1995*). Since the application of the three-parameter distribution may increase the uncertainty of the results when extreme data follow the Gumbel distribution, the two-parameter distribution is more frequently used in climatological studies (e.g., *Šamaj et al., 1982; Racko, 1987*) than the three-parameter one. Various statistics have been proposed for testing hypothesis H: $k = 0$ (which helps to decide whether to use the Gumbel or GEV distribution); brief description of these tests can be found in *Faragó and Katz (1990)* or *Dufková (1997)*.

The method of the maximum likelihood parameter estimation is asymptotically optimal but it is not necessarily the best for finite sample sizes. Recently, a new parameter estimation method for extreme value distributions has emerged which is based on L moments (*Hosking, 1990*). The estimators obtained using the method of L moments have better sampling properties than those for the method of maximum likelihood and/or the method of conventional moments since L moments are simple linear combinations of the data.

The method of L moments was applied to estimating the parameters of the GEV distribution, among others, by *Angel and Huff (1992)*, *Zwiers and Kharin (1998), Kharin and Zwiers (2000)* and *Gellens and Demarée (2001)*. Since it has not been commonly used in climatological practice yet, and since it yields results that are in some respects superior to the results of other methods, its description is presented here.

5.2. Estimation of parameters using L moments

L moments represent an alternative set of scale and shape statistics of a data sample, or a probability distribution. They play a role similar to that of conventional moments and any distribution can be completely specified by either L moments, or conventional moments (*von Storch and Zwiers, 1999*). The advantage of L moments over conventional moments is that the higher L moments can be estimated more reliably and are less sensitive to outlying data values. This is because ordinary moments (unlike L moments) require involution of the data which causes greater weight to be given to the outlying values. Robust estimators of higher moments are needed to identify and fit distributions used in extreme value analysis.

The method of L moments is computationally simpler than the method of maximum likelihood which is frequently applied to fit extreme value distributions. The derivation of L moments is based on order statistics which are obtained simply by sorting the sample $\{X_1, X_2, ..., X_n\}$ of n independent realizations of variable X in ascending order $\{X_{1:n}, X_{2:n}, ..., X_{n:n}\}$; the subscript $k{:}n$ denotes the k-th smallest number in the sample of length n. L moments are defined as expectations of linear combinations of these order statistics,

$$\lambda_1 = E(X_{1:1}),$$

$$\lambda_2 = \frac{1}{2}E(X_{2:2} - X_{1:2}),$$

$$\lambda_3 = \frac{1}{3}E(X_{3:3} - 2X_{2:3} + X_{1:3}),$$

and generally for the k-th L moment

$$\lambda_k = \frac{1}{k}\sum_{j=0}^{k-1}(-1)^j \binom{k-1}{j} E(X_{k-j:k}),$$

where E stands for the expectation operator (*Hosking, 1990; von Storch and Zwiers, 1999*). The first L moment is the expected smallest value in a sample of one, i.e. the conventional first moment. The second L moment is the expected absolute difference between any two realizations, multiplied by 1/2 (i.e. the analogue to the conventional second moment). The third and fourth L moments are shape parameters. Standardized L moments are the L-coefficient of variation λ_2/λ_1, the L-skewness λ_3/λ_2 and the L-kurtosis λ_4/λ_2; they take values between −1 and +1 (except for some special cases of small samples).

Hosking (1990) proved that the k-th L moment λ_k ($k \le n$) can be estimated as

$$l_k = \sum_{l=0}^{k-1}(-1)^{k-l-1}\binom{k-1}{l}\binom{k+l-1}{l} b_l ,$$

where

$$b_l = \frac{1}{n}\sum_{i=1}^{n}\frac{(i-1)(i-2)...(i-l)}{(n-1)(n-2)...(n-l)} X_{i:n} , \; l \ge 1, \text{ and}$$

$$b_0 = \frac{1}{n}\sum_{i=1}^{n} X_{i:n}$$

For the first three L moments, estimators can be expressed in much simpler form as

$$l_1 = \frac{\sum_i X_i}{n}, \; l_2 = \frac{\sum_{i>j}\left(X_{i:n} - X_{j:n}\right)}{2\binom{n}{2}}, \text{ and } l_3 = \frac{\sum_{i>j>k}\left(X_{i:n} - 2X_{j:n} + X_{k:n}\right)}{3\binom{n}{3}}.$$

Details concerning L moments for probability distributions can be found in *Hosking (1990)*. If X has the GEV distribution ($k \neq 0$), the first three L-moments λ_1, λ_2, λ_3 are given by (note that in *Zwiers and Kharin, 1998* there is a misprint in the expression for λ_3)

$$\lambda_1 = \xi + \beta \frac{1 - \Gamma(1+k)}{k}, \lambda_2 = \beta \frac{\left(1 - 2^{-k}\right)\Gamma(1+k)}{k}, \text{ and } \lambda_3 = \beta \frac{\Gamma(1+k)\left(-1 + 3.2^{-k} - 2.3^{-k}\right)}{k},$$

where Γ stands for the gamma function and (ξ,β,k) are parameters of the GEV distribution (e.g., *Kharin and Zwiers, 2000*).

Finally, the method of L moments fits the GEV distribution by choosing its parameters so that the first three L moments λ_1, λ_2, λ_3 match the corresponding estimates l_1, l_2, l_3. The resulting L moment estimators of ξ, β and k are given by (note that in *Zwiers and Kharin, 1998* there is again a misprint in the expression for β)

$$k = 7.8590z + 2.9554z^2, \; \beta = \frac{l_2 k}{\left(1 - 2^{-k}\right)\Gamma(1+k)}, \text{ and } \xi = l_1 + \beta \frac{\Gamma(1+k) - 1}{k},$$

where $z = \left(\frac{2}{3 + \frac{l_3}{l_2}} - \frac{\ln 2}{\ln 3} \right)$.

For k approaching zero, one should confine oneself to the two-parameter Gumbel distribution for which the L moments estimators are (see, e.g., *von Storch and Zwiers, 1999*) $\beta = l_2/\ln 2$ and $\xi = l_1 - \left(\gamma l_2/\ln 2\right)$ where γ stands for the Euler constant.

5.3. Return periods of annual temperature extremes

The Gumbel and the GEV distributions were fitted to the samples of the annual highest *TMAX* at the three stations using various methods of estimating their parameters (*Faragó and Katz, 1990; von Storch and Zwiers, 1999*). The null-hypothesis H: $k = 0$ was tested against the two-sided alternative ($k \neq 0$) using the three tests which are known as the median test (*Gumbel, 1965*), the maximum likelihood test (*Otten and Van Montfort, 1980*) and the probability-weighted moments test (*Hosking et al., 1985*). The null-hypothesis is not rejected at any station at the significance level of 0.05, which justifies the application of the Gumbel distribution to the annual maxima of *TMAX* (this finding is in accordance with the results of *Dufková, 1997*). The extreme annual maxima of *TMAX* relevant to various return periods at the Klatovy station are shown in Tab. 4.

Table 4. The highest annual maxima of *TMAX* with return periods of 10, 20, 50, 100 and 200 years at Klatovy. The estimates are based on various methods (left column). Description of the *L* moments is in Section 5.2., the other methods are in *Faragó and Katz (1990)* and *Dufková (1997)*. Temperatures are in °C.

Method / Return period (years)	10	20	50	100	200
Two-parameter methods					
Method of theoretical moments	36.52	37.74	39.31	40.50	41.68
Method of empirical moments	36.72	38.01	39.68	40.94	42.19
Method of quantiles	37.08	38.44	40.21	41.54	42.86
Lieblein method	36.78	38.11	39.82	41.10	42.38
Method of probability-weighted moments	36.53	37.76	39.34	40.53	41.72
Maximum likelihood method	36.74	38.04	39.72	40.99	42.24
Standard deviation	0.68	0.83	1.03	1.18	1.34
Method of *L* moments	36.64	37.91	39.56	40.79	42.02
Three-parameter methods					
Method of sextiles	36.41	37.49	38.82	39.76	40.65
Method of probability-weighted moments	36.55	37.57	38.80	39.65	40.44
Maximum likelihood method	36.53	37.58	38.85	39.72	40.54
Standard deviation	0.59	0.69	0.87	1.02	1.21
Method of *L* moments	36.63	37.72	39.03	39.95	40.81

The return period of the annual maximum temperature ≥ 40.0°C at Klatovy (the value reached in 1983) was estimated to be within 50–100 years by all the two-parameter methods except for the method of quantiles, while the application of the three-parameter methods leads to longer return periods of 100–200 years. For example, the method of empirical moments which is the simplest tool of parameter estimations, leads to 75 years for the Gumbel distribution, and the application of *L* moments yields the value of 65 years. Using three-parameter methods, *L* moments lead to a return period of 105 years while the maximum likelihood method yields a return period of 130 years. The differences between the two- and three-parameter methods are similar at the other two stations. *TMAX* exceeded 36.0°C in one year only, both at Prague-Ruzyně and Strážnice, and the return periods of these extremes are around 20–30 (30–50) years according to the Gumbel (GEV) distribution.

Among the three-parameter methods, the method of *L* moments tends to provide lower return periods of the extremes than the other three-parameter methods (particularly compared to the maximum likelihood method), and its results are, therefore, closer to the estimates obtained by the two-parameter methods. Among the two-parameter methods, on

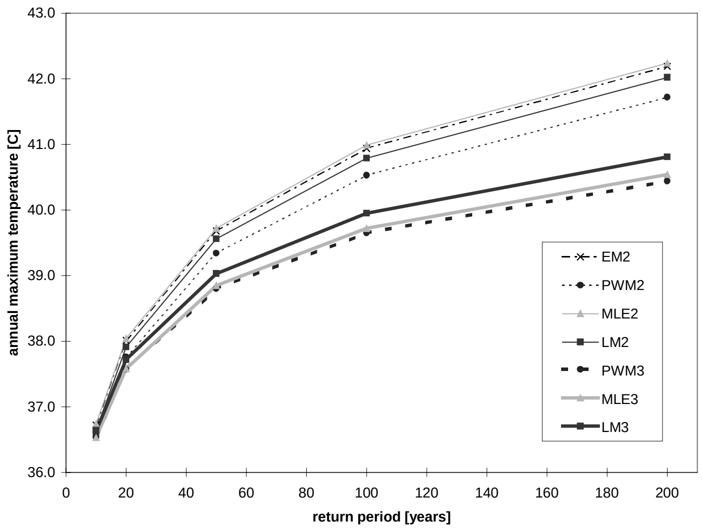

Fig. 3. Annual maxima of *TMAX* with return periods of 10, 20, 50, 100 and 200 years at the Klatovy station. Values based on various parameter estimation methods of the Gumbel distribution (EM2 - method of empirical moments, PWM2 - method of probability-weighted moments, MLE2 - maximum likelihood method, LM2 - method of *L* moments) and the GEV distribution (PWM3 - method of probability-weighted moments, MLE3 - maximum likelihood method, LM3 - method of *L* moments) are depicted.

the other hand, the return periods of temperature extremes are higher for the method of *L* moments than for the maximum likelihood method (Fig. 3). This means that the estimates of fixed quantiles of the GEV distribution are considerably closer to those of the Gumbel distribution if *L* moments are applied as compared to the maximum likelihood method. Since we are dealing with samples of an unknown population, it is difficult to ascertain which values are the most reliable. Relatively good agreement between the results of the two- and the three-parameter methods of *L* moments seems, however (if the hypothesis $k = 0$ cannot be rejected, which is the case of the data analysed), to be an advantage of the parameter estimators obtained from the *L* moments. These findings should be tested at other stations in the near future.

5.4. Return periods of extreme heat waves

The application of both the two-parameter Gumbel (as in *Racko, 1987*) and the three-parameter GEV distribution to estimating the probability of extreme heat waves and/or extreme periods of tropical days leads to confusing results. This is demonstrated here for the Strážnice station, where at least one tropical day was observed in each year between 1961−1998. The duration of the longest continuous period of tropical days was set in each year; the sample of annual extremes was then tested for the hypothesis H: $k = 0$, and the extreme value analysis was performed with (i) inclusion and (ii) omission of the extreme year of 1994. Whereas in (i) hypothesis $k = 0$ is rejected and the application of the GEV distribution leads to return values of a 16-day tropical period around 150–200 years, in (ii) the null hypothesis cannot be rejected at $\alpha = 0.05$ and the application of both the Gumbel and GEV distributions yields return periods of a 16-day period of tropical days to be in the order of ten thousand to hundred thousand years (i.e. values higher by two to three orders).

An uncritical application of either the two-parameter (Gumbel) or the three-parameter (GEV) distributions may lead to unrealistic values particularly for long return periods, and the problem becomes even more serious when the parameters of the distribution are estimated from a relatively small sample. These problems rather frequently corrupt the estimated characteristics of extreme events. For example, some results of *Racko* (*1987)* (application of the two-parameter distribution to estimating the probabilities of the occurrence of periods of tropical days in Slovakia) and *Zwiers and Kharin* (*1998)* (three-parameter distribution applied in a comparative study between observed and GCM-simulated extremes) should be accepted with reservations, mainly because of sizes of the samples being too small.

6. DISCUSSION

The AR(1) model of *TMAX* is able to reproduce the main characteristics of heat waves, and the probabilities of extreme heat waves may be estimated from large samples simulated by the model. On the other hand, theoretical extreme value distributions do not yield good results when applied to the maximum annual lengths of heat waves and periods of tropical days. The omission of the highest value from the sample may cause changes even at the method-selection stage of the procedure (the decision between the GEV and Gumbel distribution), so that the estimated return periods differ substantially, often also in their order. It is likely that the distributions of these extremes follow neither the Gumbel, nor the GEV distribution, and another extreme value distribution would be more appropriate (see, e.g., *von Storch and Zwiers, 1999*). Moreover, the application of the GEV or the Gumbel distribution is reasonable only when there are many heat waves/tropical day periods in a year, which is not satisfied in most years (due to the annual cycle which strongly reduces the effective sample size). That is why the previous findings on return periods of consecutive tropical days, presented in *Racko (1987)* and cited in *Krška and Racko (1996)* should be considered uncertain. AR(1) model simulations proved that the recurrence probability of such a long continuous period of tropical days as in 1994 is very small under present temperature conditions, but an increase of 3°C in the mean summer *TMAX* (which is a change that lies within the range indicated for central Europe by most current general circulation models under doubled effective CO_2 concentrations) would result in an increase by two orders in the probability.

A great problem of applying the theoretical extreme value distribution is that the use of the two-parameter Gumbel distribution and the three-parameter GEV distribution often lead to quite different values particularly for large return periods. Probably the most appropriate way of overcoming this difficulty is to test the shape parameter (k) in the GEV distribution, and then applying either the GEV distribution (if k differs significantly from zero), or both the Gumbel and GEV distribution (in the reverse case). A better insight may be achieved by employing a regional analysis with more stations considered together and k estimated, e.g., as the median of station estimates within the region (*Buishand, 1984*).

A relatively new alternative method of estimating distribution parameters, which has some advantages over more traditional methods, is the method of *L* moments (*Hosking, 1990*). The comparison with standard methods shows that quantiles of the GEV and Gumbel distribution estimated with *L* moments may differ slightly from the results of other methods. Nevertheless, this difference is substantially lower than the difference resulting from the application of the two-parameter (Gumbel) distribution, on the one hand, versus the three-parameter (GEV) distribution, on the other, and this holds even in cases in which the shape parameter test justifies the use of the Gumbel distribution.

It appears that the 1994 heat wave and particularly the period of consecutive tropical days was a rarer event than the 40°C heat in July 1983; this result should be verified in the future, if high-quality data from other stations, where *TMAX* reached 40°C in 1983, are available. An improvement in the reliability of the estimated probability for the 1994 heat wave may be expected from a more sophisticated model of the temperature series, the identification and application of which is a topic for a further study.

The application of both the stochastic model and the extreme value distributions assumes that the observed time series (apart from seasonal cycles) are stationary. Recently more evidence has been forthcoming that the current climate is non-stationary, the globally averaged mean annual surface temperature being 0.5°C higher as compared to the end of the 19th century. This global change may also be reflected in the current and future central-European temperature extremes; if so, the return periods of extreme heat waves and extreme high one-day temperatures would be lower than the estimates presented here.

Moreover, great uncertainty exists in how climate change should affect the other two parameters of the AR(1) model, namely the variance and persistence of daily temperatures; they may influence the results in both directions. If modifications of the mean and variance of *TMAX* were confined to the difference between $2\times CO_2$ and $1\times CO_2$ climates of the ECHAM3 general circulation model (*Nemešová et al., 1999*), the recurrence probability of a continuous period of tropical days lasting at least 18 days would be as high as one in three years under $2\times CO_2$ conditions at the Strážnice station (*Kyselý, 2000*). If a climate change is occurring, these temperature extremes are expected to impose increasing stress on the biosphere as well as on a large number of human activities.

7. CONCLUSIONS

The highest daily temperatures and the most severe heat waves ever observed in the Czech Republic appeared in the last two decades, and probabilities of their recurrence were addressed in this contribution. The application of two statistical methods for probability estimations was evaluated; the first one is based on the stochastic modelling of time series of the daily maximum temperature (*TMAX*) using the first-order autoregressive (AR(1)) model, the second one consists in fitting the extreme value distribution to the sample of annual temperature extremes. The main results are as follows:

- the AR(1) model is able to reproduce the main characteristics of heat waves, although the highest one-day temperatures are overestimated by the model,
- the return period of a heat wave lasting at least 34 days (as in 1994) was estimated by the AR(1) model to be 700 years in south Moravia, and this value should be treated as

an upper limit because of the deficiencies in simulating temperature variability, inherent to the model,

- estimations relying on AR(1)-simulated data of a warmer climate indicate that the relatively small increase in the mean temperature (of 3°C) would lead to substantial alterations (by two orders) in the return periods of extreme tropical day periods,
- extreme value distributions should not be applied to maximum annual lengths of heat waves and periods of tropical days, but may be used to estimate the recurrence probabilities of one-day temperature extremes, after the GEV distribution has been shape-parameter tested,
- the heat wave and particularly the long period of consecutive tropical days in 1994 were probably a rarer event than the record-breaking temperatures in July 1983 exceeding 40°C.

Acknowledgements: This research was supported by the Grant Agency of the Czech Republic and Grant Agency of the Academy of Sciences of the Czech Republic under Grant Projects No. 205/99/1561 and A3042903, and by the Ministry of Environment of the Czech Republic within the National Climate Programme. Thanks are due to R. Huth, Institute of Atmospheric Physics, Prague, and J. Kalvová, Charles University, Prague, for helpful comments on various versions of the manuscript; to K. Dejmal, Institute of Atmospheric Physics, Prague, for assistance in computing the *L* moments; to the staff of the Czech Hydrometeorological Institute, Prague, for providing the data sets; and to the two anonymous reviewers whose suggestions resulted in a considerable improvement of the text.

Received: 28 May 2001; Accepted: 30 October 2001

References

Agnew M.D. and Palutikof J.P., 1999. The impacts of climate on retailing in the UK with particular reference to the anomalously hot summer of 1995. *Int. J. Climatol.*, **19**, 1493–1507.

Angel J.R. and Huff F.A., 1992. Comparing three methods for fitting extreme rainfall distributions: *L*-moments, maximum likelihood, and graphical fit. In: *Proc. 12th Conference on Probability and Statistics in the Atmospheric Sciences,* Toronto, Canada, 255–260.

Brázdil R. and Budíková M., 1999. An urban bias in air temperature fluctuations at the Klementinum, Prague, the Czech Republic. *Atm. Environ.,* **33**, 4211–4217.

Brown B.G. and Katz R.W., 1995. Regional analysis of temperature extremes: Spatial analog for climate change? *J. Climate,* **8**, 108–119.

Buishand A., 1984. Bivariate extreme-value data and the station-year method. *J. Hydrol.,* **69**, 77–95.

Colombo A.F., Etkin D. and Karney B.W., 1999. Climate variability and the frequency of extreme temperature events for nine sites across Canada: Implications for power usage. *J. Climate,* **12**, 2490–2502.

Dubrovský M., 1997. Creating daily weather series with use of the weather generator. *Environmetrics,* **8**, 409–424.

Dufková D., 1997. *Modelling Extreme Air Temperatures and Atmospheric Precipitation.* Diploma thesis. Faculty of Mathematics and Physics, Charles University, Prague, 61 pp. (in Czech).

Faragó T. and Katz R.W., 1990. *Extremes and Design Values in Climatology.* WMO/TD-No 386.

Gellens D. and Demarée G., 2001. Regional GEV-growth curves of yearly precipitation depths in Belgium for durations ranging from 10 minutes to 30 days. In: *Proc. 8th International Meeting on Statistical Climatology*, Lüneburg, Germany.

Gumbel E.J., 1965. A quick estimation of the parameters in Fréchet's distribution. *Rev. Int. Stat. Inst.,* **33**, 349−363.

Hennessy K.J. and Pittock A.B., 1995. Greenhouse warming and threshold temperature events in Victoria, Australia. *Int. J. Climatol.* **15**, 591−612.

Hosking J.R.M., Wallis J.R. and Wood E.F., 1985. Estimation of the generalized extreme-value distribution by the method of probability-weighted moments. *Technometrics,* **27**, 251−261.

Hosking J.R.M., 1990. *L*-moments: Analysis and estimation of distributions using linear combinations of order statistics. *J. Roy. Stat. Soc.,* **52B**, 105−124.

Houghton J.T., Meira Filho L.G., Callander B.A., Harris N., Kattenberg A. and Maskell K. (eds.), 1996. *Climate Change 1995. The Science of Climate Change.* Published for the Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, 572 pp.

Huth R., Kyselý J. and Pokorná L., 2000. A GCM simulation of heat waves, dry spells, and their relationships to circulation. *Clim. Change,* **46**, 29−60.

Kalvová J., Jurečková J., Picek J. and Nemešová I., 2000. On the order of autoregressive (AR) model in temperature series. *Meteorologický časopis,* **3**, 19−23.

Katz R.W. and Skaggs R.H., 1981. On the use of auto-regressive moving average processes to model meteorological time series. *Mon. Wea. Rev.,* **109**, 479−484.

Katz R.W. and Brown B.G., 1992. Extreme events in a changing climate: Variability is more important than averages. *Clim. Change,* **21**, 289−302.

Kendall M. and Stuart A., 1976. *The Advanced Theory of Statistics.* **Vol. 3**. Charles Griffin & Co., London, 585 pp.

Kharin V.V. and Zwiers F.W., 2000. Changes in the extremes in an ensemble of transient climate simulations with a coupled atmosphere-ocean GCM. *J. Climate*, **13**, 3760−3788.

Krška K. and Munzar J., 1984. Temperature peculiarities of the tropic summer 1983 in Czechoslovakia and in Europe. *Meteorol. Zpr.*, **37**, 33−40 (in Czech).

Krška K. and Racko S., 1996. Extra-ordinary hot summer 1994 in the Czech and the Slovak Republic. *Meteorol. Zpr.*, **49**, 12−21 (in Slovak).

Kyselý J., 2000. *Changes in the Occurrence of Extreme Temperature Events.* PhD thesis. Faculty of Mathematics and Physics, Charles University, Prague, 97 pp. (in Czech).

Kyselý J. and Kalvová J., 2000. Heat waves in long-term temperature series in the Czech Republic. In: *Proc. 3rd European Conference on Applied Climatology*, Pisa, Italy.

Kyselý J., Kalvová J. and Květoň V., 2000. Heat waves in the south Moravian region during the period 1961−1995. *Stud. geophys. geod.,* **44**, 57−72.

Kyselý J., Huth R. and Dubrovský M., 2001. Simulation of extreme temperature events using general circulation models, statistical downscaling and stochastic generator. *Meteorol. Zpr.,* **54**, 73-82 (in Czech).

Kyselý J., 2002. Temporal fluctuations in heat waves at Prague-Klementinum, the Czech Republic, in 1901−1997, and their relationships to atmospheric circulation. *Int. J. Climatol.* (in print).

Macchiato M., Serio C., Lapenna V. and LaRotonda L., 1993. Parametric time series analysis of cold and hot spells in daily temperature: An application in southern Italy. *J. Appl. Meteorol.,* **32**, 1270−1281.

Madden R.A. and Shea D.J., 1978. Estimates of the natural variability of time-averaged temperatures over the United States. *Mon. Wea. Rev.,* **106**, 1695−1703.

Mearns L.O., Katz R.W. and Schneider S.H., 1984. Extreme high temperature events: changes in their probabilities with changes in mean temperature. *J. Climate Appl. Meteor.,* **23**, 1601−1608.

Nemešová I., Kalvová J. and Dubrovský M., 1999. Climate change projections based on GCM-simulated daily data. *Stud. geophys. geod.*, **43**, 201−222.

Nicholls J.M. (ed.), 1998. *The Global Climate System Review. December 1993 - May 1996.* WMO No. 856.

Otten A. and Van Montfort M.A.J., 1980. Maximum likelihood estimation of the general extreme value distribution parameters. *J. Hydrol.*, **47**, 187−192.

Parmesan C., Root T.L. and Willig M.R., 2000. Impacts of extreme weather and climate on terrestrial biota. *Bull. Amer. Meteorol. Soc.*, **81**, 443−450.

Press W.H., Teukolsky S.A., Vetterling W.T. and Flannery B.P., 1992. *Numerical Recipes in Fortran. The Art of Scientific Computing.* Second Edition, Cambridge University Press, Cambridge, 963 pp.

Racko S., 1987. Periods of summer and tropical days and tropical nights in Slovakia during 1951−1975. *Meteorol. zpr.,* **40**, 135−137 (in Slovak).

Rooney C., McMichael A.J., Kovats R.S. and Coleman M.P., 1998. Excess mortality in England and Wales, and in Greater London, during the 1995 heatwave. *J. Epidemiol. Community Health,* **52**, 482−486.

Sartor F., Snacken R., Demuth C. and Walckiers D., 1995. Temperature, ambient ozone levels, and mortality during summer 1994, in Belgium. *Environ. Res.*, **70**, 105−113.

Šamaj F., Valovič Š., Brázdil R. and Gulčíková V., 1982. Maximum daily precipitation amounts in Czechoslovakia. *Meteorol. zpr.,* **35**, 129−135 (in Slovak).

von Storch H. and Zwiers F., 1999. *Statistical Analysis in Climate Research.* Cambridge University Press, Cambridge, 484 pp.

Watson R.T., Zinyowera M.C., Moss R.H. and Dokken D.J. (eds.), 1996. *Climate Change 1995: Impacts, Adaptations, and Mitigation of Climate Change: Scientific-Technical Analyses.* Published for Intergovernmental Panel on Climate Change. Cambridge University Press, Cambridge, 878 pp.

Wilks D. S., 1995. *Statistical Methods in the Atmospheric Sciences: An Introduction.* Academic Press, New York, 467 pp.

Zwiers F.W. and Kharin V.V., 1998. Changes in the extremes of the climate simulated by CCC GCM2 under CO_2 doubling. *J. Climate,* **11**, 2200−2222.