# Allocation of Scientific Credit with Altruistic Players

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## ABSTRACT

The process by which a researcher selects the focus of their studies can be impacted by both selfish and selfless motivations. The collective result of these motivations shapes the direction of the body of literature of scientific communities. In this work, we adapt Kleinberg and Oren's 2010 model of the allocation of credit in the scientific community to include the notion of altruism. We redefine each researcher's utility to be a linear combination of their own credit and the credits of their friends, weighted by an altruism parameter. Through this modification, we analyze the impact of altruism on the Price of Anarchy, demonstrating that altruistic behavior can lead to more socially optimal outcomes under certain conditions. Additionally, we note that researchers realistically do not have full information about which other researchers are working on certain projects. We thus explore learning dynamics using multiplicative weights, showing that altruistic researchers are more likely to converge to efficient allocations. These results suggest that altruistic motivations within scientific communities can enhance overall productivity.

## 1 INTRODUCTION

When modeling social decision-making processes, it is often assumed that individuals act solely out of self-interest. Such is the case in Kleinberg and Oren's model of allocation of credit in the scientific community [6]. This framework tries to model the collective productivity of the scientific community by considering the ways in which certain research questions may receive disproportionate shares of research credit. Here, *researchers* must balance the varying importance and difficulty of prospective *projects* in competition with other researchers.

However, we seek to model that scientists aren't motivated by self-interest alone. We modify this algorithm to account for individuals' desire to improve the scientific community at large. Based on Chen and Kempe's model of *altruism* [3], we define each researcher's payoff as a linear combination of their expected credit and a weighted sum of the expected credit of their friends. Based on Yu, Kempe, and Vorobeychik's model of *altruism networks*, we model friendships as an undirected graph with each researcher as a node. [10]

We study the price of anarchy, which helps represent how much social cost there is from players operating on their own rather than coordinated for social gain. This general case proved hard to analyze, so we also examine special cases that restrict this friendship graph, including the case where all researchers are friends, which becomes equivalent to the general altruism framework proposed by [3].

## 2 GAME SETUP

We adapt the model of Kleinberg and Oren [6], which is based on the model proposed by Kitcher [5][4] and Strevens[9], which has roots in the work of Peirce [8], Aaronson [1], and Bourdieu [2].

- There are $n$ *researchers* (players), each with identical problem-solving ability. Each researcher must choose exactly one of $m$ *open projects*.
- Each *project* $j$ has *importance* $w_j$ and *probability of success* $q_j$ (probability of failure $f_j = 1 - q_j$), with each of these values being rational.
- We denote the choices of all players by the *strategy vector* $s$, where each player $i$ works on project $s_i$. Recall that $s_{-i}$ denotes the strategy vector $s$ except for player $i$ and $(j, s_{-i})$ is the strategy vector $s_1, \ldots, s_{i-1}, j, s_{i+1}, \ldots, s_n$.
- The set of players working on project $j$ under strategy vector $s$ is $K_j(s)$, and $k_j(s) = |K_j(s)|$.
- The original model defines each player's credit/original payoff to be $u_i(s) = \frac{w_{s_i}}{(1-f_j^k)k}$, where $k = k_{s_i}(s)$.
- We define $F(i)$ to be the set of all researchers which are friends with $i$.
- We define each player's true payoff (with altruism) as the sum of their personal payoff and $\epsilon$ (the altruism rate) times the sum of their friends' personal payoffs: $u_i'(s) = u_i(s) + \epsilon \sum_{i' \in F(i)} u_{i'}(s)$.
- The social welfare of strategy vector $s$ remains as $u(s) = \sum_{i=1}^{n} u_i(s)$, as in the original model.

## 3 DEVIATIONS FROM ORIGINAL GAME

The base game is shown to be a monotone valid-utility game [6]. In particular, its total utility function satisfies the following properties:

(1) *Submodularity:* The marginal gain from adding a player decreases as the number of other players increases.
(2) *Monotonicity:* Adding more players cannot decrease the total utility.
(3) *Marginal Contribution Lower Bound:* The total utility is at least as large as the sum of the marginal contributions of each player.
(4) *Individual Utility Lower Bound:* The total utility is at least as large as the sum of the individual utilities.

This important result is leveraged to prove an upper bound on the price of anarchy and support other claims throughout the paper. However, the modified version of the game we present with altruism is no longer a monotone valid-utility game. We have already defined $u_i'(s)$ in the game setup, so it remains to define $u'(s)$. The true social welfare that we care about is the sum of everyone's personal payoff $\sum_i u_i(s)$, as this is equal to the expected sum of importance for all solved problems. However, this would violate the fourth condition. Thus, to satisfy the fourth condition, we can define $u'(s) = \sum_i u_i'(s)$.

Now, $u'(s)$ no longer represent the true social benefit from a strategy $s$. However, we note that under the assumption that each researcher has an equal number of friends (let this be $z$), we get $u'(s) = \sum u_i'(s) = (1 + z\epsilon)u(s)$. Thus, this would allow us to analyze the price of anarchy according to $u'(s)$, and conclude that the same price of anarchy applies to the function $u(s)$ that we truly care about, as this $1 + z\epsilon$ scaling factor would appear in the social welfare both at the optimum and at Nash.

Continuing with the assumption that each researcher has an equal number of friends, properties one and two hold for this game for the same reason as they do in the base game since $u'$ is a constant multiple of $u$. However, the third condition

$$u'(\vec{a}) \geqslant u(a_i|a_{-i})$$

no longer holds.

We define the marginal social utility $u(a_i|a_{-i})$ in the natural way, as the difference in social utility between researcher $i$ working on a project $a_i$ and not working on any project. Then, this condition can be expressed as:

$$u_i(s) + \epsilon \sum_{i' \in F(i)} u_{i'}(s) \geqslant r_j(k_j) \cdot (1 + \epsilon|F(i)|).$$

We know that $u_i(s) \geqslant r_j(k_j)$ as in the base game, as the benefit a researcher gets from joining a project is the total benefit of the project averaged among all researchers, while $r_j(k_j)$ is a monotone decreasing function and thus the last researcher to join the project has the least marginal social benefit. However, it is not always true that the total utility of the player's friends exceeds the player's marginal contribution factor scaled by the number of friends; that is, $\sum_{i' \in F(i)} u_{i'}(s) \geqslant r_j(k_j) \cdot |F(i)|$ may not be true. In other words, if a player's utility surpasses the average utility of their friends, the inequality no longer holds, and so the game is no longer a valid monotone-utility game.

## 4 PRICE OF ANARCHY

The main question we would like to answer with the addition of altruism to this game is how much it benefits society. A natural way to quantify this is to look at how the price of anarchy (*PoA*) changes, as this measures how large the gap in social welfare is between Nash equilibria and the social optimum. By modeling players as not entirely selfish, we expect that it may decrease this gap. The base-game had an upper-bound of $2 - \frac{1}{n}$ on the *PoA* for any instance, so this is our point of comparison. We were not able to prove a tighter bound for this game in the general case, but we can for a special case.

To motivate our choice of this special case, we note that it seems like the success probability being equal to 1 often creates the worst case for the price of anarchy. If the success probability is 1, then there is zero marginal social welfare from additional researchers working on the same problem (as opposed to some positive non-zero marginal social welfare otherwise). Thus, since the maximal *PoA* occurs when the Nash is the least socially optimal, it is reasonable to expect that we may be able to produce the worst possible *PoA* when success probabilities are equal to 1. This is especially true since for the base game, there exists an example that realizes the upper bound on the *PoA* using success probabilities of 1. This makes it an interesting special case to study.

### 4.1 Special Case: Projects Guaranteed to Succeed

We consider a special case with two assumptions: (1) that each project is guaranteed to succeed ($q_j = 1$ for all projects $j$) and (2) that the friendship graph is a complete graph (everyone is friends with everyone).

Consider an instance of this game with $n$ researchers and $m$ projects. Without loss of generality, let the problems be numbered such that $w_1 > w_2 > \cdots > w_n$. Consider any Nash equilibrium $S$. We define $\beta$ to be the number of distinct projects that researchers are working on in $S$, and $\psi$ to be the number of projects with at least two researchers working on them at Nash.

- If $\beta = m$, then the Nash obtains credit for every project and thus trivially has the optimal social welfare, so we presume that $\beta < m$ moving forward.
- Similarly, if $\psi = 0$, then there is only one researcher assigned per project at Nash, and these researchers will choose the projects with the highest weights, so again Nash trivially obtains the optimal social welfare. Thus, we presume $\psi > 1$ moving forward.

Since $w_1 > w_2 > \cdots > w_n$, we thus have that there are researchers on projects $w_1, \ldots, w_\beta$ but not on projects $w_{\beta+1}, \ldots, w_m$. Thus, the social welfare at this Nash is $\sum_{j=1}^{\beta} w_j$.

Now, consider a researcher $i$ currently working on project $p$ that has at least 2 researchers on it at Nash (e.g. $k_p(S) \geqslant 2$) that could switch to project $\beta + 1$, the Nash condition tells us that

$$\frac{w_p}{k_p(S)} + \epsilon \left( \sum_{j=1}^{\beta} w_j - \frac{w_p}{k_p(S)} \right) \geqslant w_{\beta+1} + \epsilon \sum_{j=1}^{\beta} w_j$$

since the total social welfare at Nash is $\sum_{j=1}^{\beta} w_j$, and since there are at least two researchers on project $p$, once researcher $i$ switches to project $\beta + 1$ the total welfare of the other $n - 1$ researchers remains as $\sum_{j=1}^{\beta} w_j$. Now, rearranging the above yields

$$w_p \geqslant k_p(S) \frac{w_{\beta+1}}{1-\epsilon}$$

Recall that $\psi$ is the number of projects with at least two researchers working on them at Nash. Since $w_1 > w_2 > \cdots > w_n$, these are clearly the first $\psi$ projects. Summing the previous inequality over these projects $p$, we have:

$$\begin{aligned}
\sum_{p=1}^{\psi} w_p \geqslant \sum_{p=1}^{\psi} k_p(S) \frac{w_{\beta+1}}{1-\epsilon} \\
= \left( \sum_{p=1}^{\beta} k_p(S) - \sum_{p=\psi+1}^{\beta} k_p(S) \right) \frac{w_{\beta+1}}{1-\epsilon} \\
= (n - (\beta - \psi)) \frac{w_{\beta+1}}{1-\epsilon}
\end{aligned}$$

Now, we consider the social optimum. The optimal solution will place at most one researcher per project unless every project has a researcher; since each problem is guaranteed to succeed, there is no increase in social welfare from placing an additional researcher on the problem. If there are less researchers than projects, the optimal solution will prioritize the problems with larger weights. Thus, the optimal solution places the $n$ researchers on the $n$ problems with the largest weights, unless $m < n$ in which case it places at least one researcher on each problem. Thus, the optimal social welfare is $\sum_{j=1}^{\min\{m,n\}} w_j$. We can now bound the price of anarchy:

$$\begin{aligned}
PoA &= \frac{SW(OPT)}{SW(NASH)} \\
&= \frac{\sum_{j=1}^{\min\{m,n\}} w_j}{\sum_{j=1}^{\beta} w_j} \\
&\leqslant \frac{\sum_{j=1}^{\min\{m,n\}} w_j - \sum_{j=\psi+1}^{\beta} w_j}{\sum_{j=1}^{\beta} w_j - \sum_{j=\psi+1}^{\beta} w_j} \\
&= \frac{\sum_{j=1}^{\psi} w_j + \sum_{j=\beta+1}^{\min\{m,n\}} w_j}{\sum_{j=1}^{\psi} w_j} \\
&= 1 + \frac{\sum_{j=\beta+1}^{\min\{m,n\}} w_j}{\sum_{j=1}^{\psi} w_j} \\
&\leqslant 1 + \frac{(n-\beta) w_{\beta+1}}{\sum_{j=1}^{\psi} w_j} \\
&\leqslant 1 + \frac{(n-\beta) w_{\beta+1}}{(n-(\beta-\psi))\frac{w_{\beta+1}}{1-\epsilon}} \\
&= 1 + \frac{n-\beta}{n-(\beta-\psi)}(1-\epsilon) \\
&\leqslant 1 + \frac{n-\psi}{n}(1-\epsilon) \\
&\leqslant 1 + \frac{n-1}{n}(1-\epsilon)
\end{aligned}$$

We use the algebraic fact that if $x > 0$, then
$a/b \geqslant 1 \implies \frac{a}{b} \geqslant \frac{a+x}{b+x}$ and $a/b \leqslant 1 \implies \frac{a}{b} \leqslant \frac{a+x}{b+x}$.

If $\epsilon = 0$, the players have no altruistic tendencies whatsoever, and the game is identical to the base game. We note that substituting $\epsilon = 0$ into this bound gives us $1 + \frac{n-1}{n} = 2 - \frac{1}{n}$, which is known to be the tightest *PoA* bound for the base game. [6]

## 4.2 Attempted Generalizations

*4.2.1 General Friend Graph.* We attempted to generalize the above result by removing the assumption that the friendship graph is complete, however, the proof does not easily extend since we no longer know whether any given pair of researchers are friends. Specifically, the above proof utilizes that when a researcher A is on the same project as another researcher B at Nash, then once professor A leaves it will increase professor's B's utility, which in turn increases professor A's utility since A and B are friends. However, without the assumption that everyone is friends, we cannot know whether A and B are friends for some random A and B (even if we make a moderate assumption, like that each researcher has the same number of friends). Thus, the Nash condition does not give us an inequality any stronger than the one from the base game, and so this proof strategy fails to produce an improved *PoA* bound.

*4.2.2 General Probability of Success.* All of the literature's analysis of the *PoA* of the original model [6] relied on it being a valid monotone-utility game. But, as discussed in Section 3, our game with altruism is not a valid monotone-utility game. Because of this, we cannot analyze our game using any of the same frameworks proposed in the original paper, and had to search for different strategies to produce bounds. Importantly, this limits the extent we can generalize our findings, as analyzing this complex game proved difficult without being able to use any of the established results for monotone utility-games.

## 4.3 Limiting the PoA Upper Bound

While we were not able to prove an improved general upper bound on the *PoA*, we can quantify the maximum amount this *PoA* upper bound can improve by attempting to construct instances with the highest *PoA* possible. These provide a "limit" to how much the upper bound on *PoA* improves with the introduction of altruism to the players.

*4.3.1 General Case.* Consider an instance with $n$ researchers and $n$ open projects. We place no restrictions on the friendships, and let $z = \max_i |F(i)|$ denote the maximum number of friends that any researcher has. We define the projects as follows: $w_1 = 1$, and $w_j = \frac{1}{n} - \frac{\epsilon z}{n(n-1)}$ for all $j \neq 1$; $q_j = 1$ for all $j$.

Since the probability of success is 1 for every project, the social optimum occurs with one researcher assigned to each project, which has social welfare $1 + (n-1)\left(\frac{1}{n} - \frac{\epsilon z}{n(n-1)}\right) = 1 + \frac{n-1}{n} - \frac{\epsilon z}{n} = 1 + \frac{n-1}{n}\left(1 - \epsilon\frac{z}{n-1}\right)$.

Now, we claim that each researcher working on project 1 is a Nash equilibrium. Let this strategy vector be $\vec{a}$, and observe that researcher $i$'s payoff is $u_i(\vec{a}) + \epsilon \sum_{i' \in F(i)} u_{i'}(\vec{a}) = \frac{1}{n} + \epsilon|F(i)|\frac{1}{n} = \frac{1}{n}(1+\epsilon|F(i)|)$. If this researcher $i$ was to switch to a different project $j' \neq 1$ (which are all identical), then their payoff would be

$$\begin{aligned}
u_i(\vec{a}) + \epsilon \sum_{i' \in F(i)} u_{i'}(j', \vec{a}_{-i}) &= \frac{1}{n} - \frac{\epsilon z}{n(n-1)} + \epsilon|F(i)|\frac{1}{n-1} \\
&\leqslant \frac{1}{n} - \frac{\epsilon|F(i)|}{n(n-1)} + \epsilon|F(i)|\frac{1}{n-1} \\
&\qquad (z = \max_i |F(i)|) \\
&= \frac{1}{n} + \frac{\epsilon|F(i)|}{n}\left(\frac{-1}{n-1} + \frac{n}{n-1}\right) \\
&= \frac{1}{n}(1 + \epsilon|F(i)|) \\
&= u_i'(\vec{a})
\end{aligned}$$

Thus, for all $i$ and $j' \neq 1$, we have $u_i'(\vec{a}) \geqslant u_i'(j', \vec{a}_{-i})$, and thus $\vec{a}$ is indeed a Nash. The social welfare at this Nash is 1, and hence the price of anarchy for this example is at least $1 + \frac{n-1}{n}\left(1 - \epsilon\frac{z}{n-1}\right)$.

Observe that if we assume a complete friendship graph, then $z = n-1$, and the price of anarchy for this example becomes $1 + \frac{n-1}{n}(1-\epsilon)$. This shows that the special case upper bound we proved in the previous section is a tight upper bound for any value of $\epsilon$.

#### 4.3.2 *No Friends on the Same Project.*

We observed above that our *PoA* upper bound were difficult to extend to general friend graphs because the "benefit" that altruism provides towards the *PoA* comes from when two friends are working on the same project at Nash. This motivates the exploration of what happens to the *PoA* when no two friends work on the same project – does it still decrease?

Let us consider an example with $n$ researchers and $n$ projects, where we assume that we can break the researchers up into $\gamma$ groups $G_1, \ldots, G_\gamma$ such that no two friends are in the same group. We define the project as follows: $w_j = \frac{|G_j|}{n}$ for $j \in \{1, \ldots, \gamma\}$ and $w_j = \frac{1}{n}$ for $j \in \{\gamma+1, \ldots, n\}$; $q_j = 1$ for all $j$.

Since the probability of success is 1 for every project, the social optimum occurs with one researcher assigned to each project, which has social welfare $\sum_{j=1}^{\gamma} \frac{|G_j|}{n} + (n-\gamma)\frac{1}{n} = 1 + \frac{n-\gamma}{n}$.

Now, we define a strategy vector $\vec{a}$ by assigning each researcher in group $G_j$ to work on project $j$ (for each $j \in \{1, \ldots, \gamma\}$), and we claim $\vec{a}$ is a Nash equilibrium. Note that each researcher $i$'s payoff is $\frac{1}{n} + \epsilon|F(i)|\left(\frac{1}{n}\right) = \frac{1}{n}(1 + \epsilon|F(i)|)$.

Note that if researcher $i$ was to switch to a different project $j$, then none of their friend's utility would increase, meaning $u_{i'}(\vec{a}) \geqslant u_{i'}(j, \vec{a}_{-i})$ for all $i' \in F(i)$ and all $j$. This is because they currently have no friends on their project, so none of their friends experience the utility increase from their leaving. Similarly, researcher $i$'s personal utility $u_i$ would not increase if they leave to project $j$. This is because all researchers on other projects have current utility $\frac{1}{n}$ and all projects with no researchers have weight $\frac{1}{n}$, so $u_{i'}(\vec{a}) = \frac{1}{n} \geqslant u_{i'}(j, \vec{a}_{-i})$ for all $j$. Thus, for all $j$ we have $u_i(\vec{a}) + \epsilon \sum_{i' \in F(i)} u_{i'}(\vec{a}) \geqslant u_i(j, \vec{a}_{-i}) + \epsilon \sum_{i' \in F(i)} u_{i'}(j, \vec{a}_{-i})$, and this is indeed Nash.

At Nash, the social welfare is $\sum_{j=1}^{\gamma} \frac{|G_j|}{n} = 1$. Thus, the price of anarchy for this example is at least $1 + \frac{n-\gamma}{n}$.

We can observe that for the base game where there are no friendships, we are able to place all the researchers in $\gamma = 1$ group without any friends being in the same group. This gives us that the price of anarchy for this example is $1 + \frac{n-1}{n} = 2 - \frac{1}{n}$, which is the maximal price of anarchy for the base game.

In general, it is a bit harder to draw conclusions from this price of anarchy example, as the minimum value of the $\gamma$ parameter for an example is not immediately obvious. The one exception is if we consider a friend graph of "cliques," where everyone in each clique is friends with the others in their clique and nobody else. Then, $\gamma$ is simply equal to the maximum size of a clique. This is generally not the most realistic situation, as the world is very connected; however, one situation where these cliques do arise is if we model researchers as caring about their university's social welfare, so their friends are exactly the other researchers from the same institution.

Additionally, this example is interesting because it produces a limit to the *PoA* bound that does not depend on $\epsilon$, but instead purely on the friendship graph. The previous example did depend on $\epsilon$, and for different values of $\epsilon$ and friendship graphs, we find that there are cases for each examples where its *PoA* is higher. This means that neither value can be an upper bound.

### 4.4 Price of Anarchy Conclusion

We have proven that price of anarchy is bounded above by $1 + \frac{n-1}{n}(1-\epsilon)$ when we assume that all projects are guaranteed to succeed and the friendship graph is complete, and that this bound is tight. We expect that this may hold for all instances where the friendship graph is complete, though we were unable to prove it. Attempts to expand this to different friendship graphs were unsuccessful, though we were able to produce interesting examples that provides limits to what the *PoA* may be for arbitrary friendship graphs. These two examples each provide a bound that is higher than the other in some cases, demonstrating that neither can be a true upper bound, despite the fact that they each equal the upper bound in the case where $\epsilon = 0$ and it collapses to the base game.

## 5 LEARNING THE GAME

### 5.1 Motivation

In realistic scenarios, researchers often do not have complete information about the ongoing projects being conducted by their peers. Additionally, the research process can be more-so nonlinear, as researchers may experiment with different projects, observe the results of their efforts, and gradually refine their choices over time based on their own results and those of others in their community. To capture both the project exploration process by researchers and to investigate whether a fully *altruistic* setting can guide them to a social optimum, we employ a learning-algorithm approach, specifically using multiplicative weights (MW).

Our adaptation of the MW algorithm proceeds as follows: initially, each researcher assigns equal probability to all projects. In each iteration, every researcher samples a project according to their current distribution, after which their payoff is computed via the following: Let $c_i$ denote the credit received by researcher $i$ and $\epsilon \in [0, 1]$ represent the altruism parameter. The utility function for researcher $i$ is

$$u_i(S) = c_i + \epsilon \sum_j c_j,$$

where the sum extends over all researchers $j$ that $i$ considers friends. Each researcher then updates their probability distribution, increasing the weight on projects that yielded higher payoffs and decreasing it on those that performed poorly, both scaled by some the learning rate $\eta$.

### 5.2 Finding Social Optimum

We illustrate the performance of our learning framework using a challenging instance inspired by Kleinberg and Oren [6]. Consider $n$ researchers and $n$ projects, each guaranteed to succeed ($q_j = 1$ for all $j$), with weights $w_1 = 1$ and $w_j = 1/n$ for all $j > 1$. Without altruism ($\epsilon = 0$), it is a Nash equilibrium for every researcher to work on project 1, yielding a social welfare of 1. However, the socially optimal solution assigns each researcher to a distinct project, achieving a social welfare of $2 - 1/n$.

Remarkably, we see that in this instance the algorithm is able to converge to the social optimum. We test the aforementioned example with $N = 12$ and $\epsilon = 1/2$. We keep $N$ small due to computation constraints but the results generalize even with more researchers/projects. Figure 1 shows the convergence of all strategies for all players over time. Most importantly, note that all researchers work on unique projects with probability 1.

Furthermore, for values of $\epsilon > 0.5$, our simulations exhibit similar patterns of convergence toward the social optimum, but with notably accelerated convergence rates. While these results serve as a valuable consistency check for our framework, the underlying assumption of such a high altruism parameter is arguably less representative of typical scientific communities, where individuals are rarely so strongly aligned with collective welfare.

We also observe an intriguing phenomenon in which a particular researcher (e.g., researcher 1) initially collocates with another researcher (e.g., researcher 3) on the same project for a substantial portion of the iterations, but eventually transitions to a different "open" project. We conjecture that this shift occurs at a point where most researchers become uniquely assigned to their own projects prompting the realignment towards the social optimum.

Additional simulations confirm that such convergence patterns are quite common. In the rare cases where the algorithm does not fully reach the optimal assignment, we still observe outcomes that are nearly optimal: all but one project is uniquely assigned, and at most two researchers share a single project.

## 5.3 Social Welfare Improvements

When $\epsilon = 0$, the scenario effectively models a community of researchers each focused solely on their own publication record, naturally converging to the proposed Nash equilibrium where all researchers place probability one in the single highest weight project ($w_1$). As we increase $\epsilon$, the researchers become more inclined to spread out, leading to less sparse solutions with respect to the number of projects worked on. When $\epsilon = 1$, researchers act purely altruistically and simply want to maximize the total welfare, implying that the social optimum should be reached. and the algorithm frequently drives the system toward distributions that approximate the social optimum. This could be seen as analogous to a national research laboratory dedicated to advancing global knowledge. The following plot shows the percent improvement of social welfare for different instances of the game with varying altruism factors. We generate $N$ random weights and $N$ random corresponding probabilities both between $[0, 1)$. The following plot shows the improvement across varying epsilon:

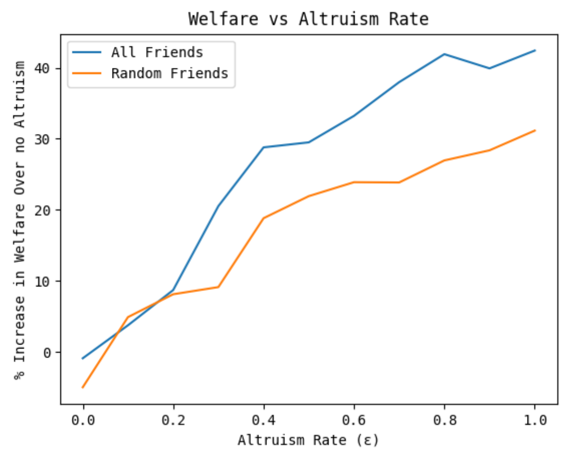


**Figure 2: Average difference in social wellfare for scenarios with complete friend network and random friend network when compared to no altruism**

Figure 2 illustrates how as the altruism paramter increases from 0 to 1, there is a corresponding average increase in the social wellfare for both scenarios with a randomized friend network and complete friend network, reaching approximately 42.4% for complete friendship graphs and 31.1% for random friendship graphs at full altruism. This trend is consistent with the theoretical Price of Anarchy bound of $2 - \frac{1}{n}$, which implies that social welfare improvements are capped at a $100\left(\frac{n-1}{n}\right)\%$ increase. The 42.4% improvement in complete graphs indicates that Nash equilibria in these scenarios are already relatively closer to the social optimum, limiting potential gains from increased altruism.

For a more extensive overview of our approach, including the robustness of our code implementation, please refer to [7]

# 6 CONCLUSION

This work extends the original model of credit allocation in scientific communities by introducing altruistic preferences, guided by connections within a social network of researchers. Despite losing the property of being a monotone valid-utility game, our analysis and simulations show that partial altruism can improve equilibrium outcomes, pushing the system closer to the social optimum. In particular, our results suggest that even moderate levels of altruism can enhance collective welfare, reduce the price of anarchy in special cases, and drive learning-based systems toward more efficient allocations. Though questions remain regarding general network structures and success probabilities, these findings underscore the potential benefits of encouraging collaborative incentives in scientific environments.

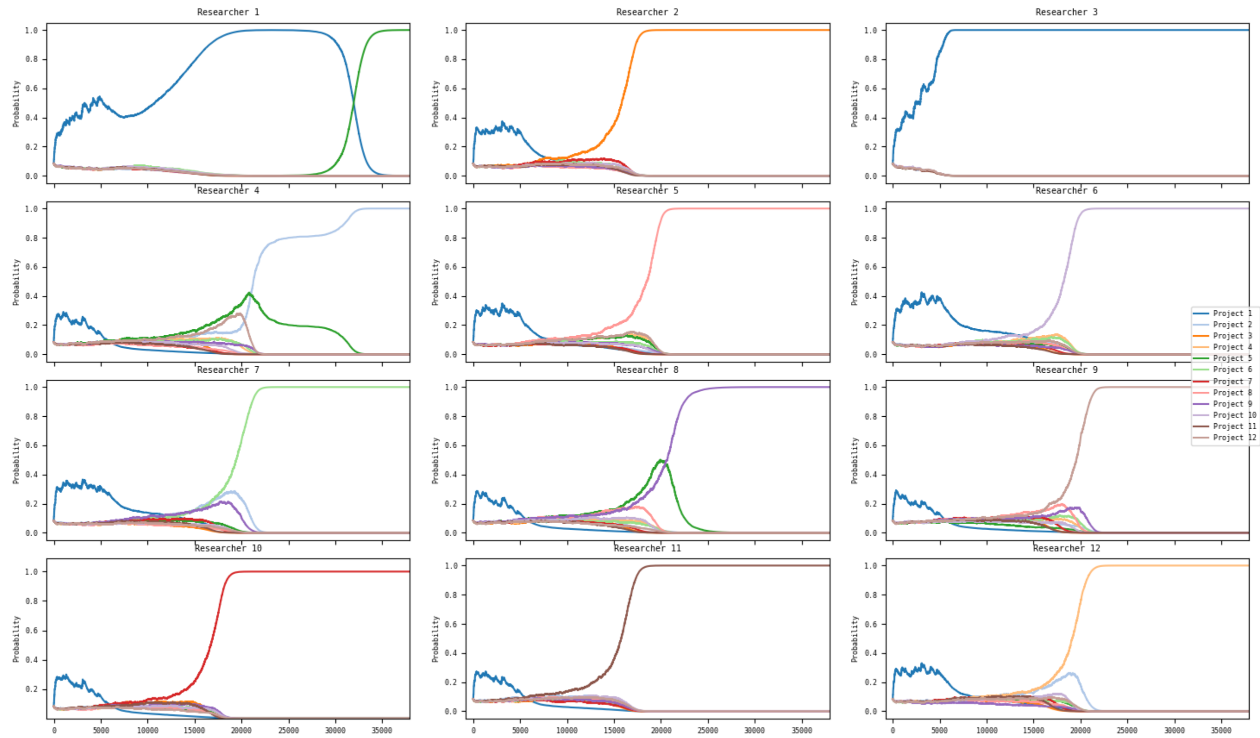


**Figure 1: Change of project selection probabilities for researchers over multiple iterations of the multiplicative weights algorithm.** $N = 12$ **researchers,** $m = 12$ **projects,** $w_1 = 1$**,** $w_j = 1/N$ **for** $j > 1$**, and** $\epsilon = 0.5$**.**

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