18.5.5

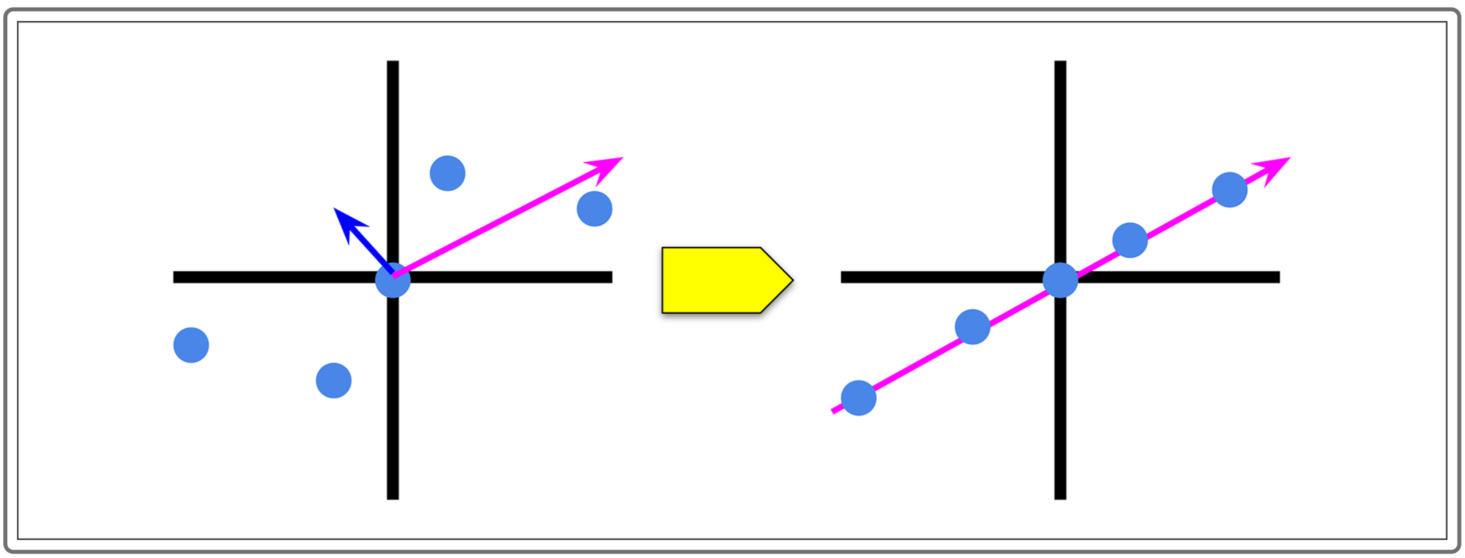
PCA's Underlying Theory

With all the math and fancy words to describe, let's finally see how all this comes together with PCA.

Now it's time to put everything together and show how PCA works. Given our two eigenvalues from before, 7 and 2, take the greater eigenvalue, 7, and eliminate the other since it's less important. The higher eigenvalue is the axis that carries the most amount of information.

We'll also take the corresponding eigenvector, which is (2,1).

Next, extend that eigenvector with the higher value to a line and project all our points onto that line:



Now let's put everything together and show what PCA is doing. We'll up the ante a little bit and expand from two to five columns of data. First, take our data that consists of five columns, or features. Note, the asterisk (*) will represent a number as we'll avoid using numbers to simplify the exercise:

x1	x2	x3	x4	x5
*	*	*	*	*
*	*	*	*	*
*	*	*	*	*

Put all the data points into a 5x5 covariance matrix. The eigenvectors and eigenvalues are calculated for each of those five columns in the matrix. Again, the asterisk (*) represents a number:

$$\begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix}$$

From the matrix we can produce a list of eigenvectors and corresponding eigenvalues:

$$\boxed{\begin{array}{l} V1\ \lambda 1 \\ V2\ \lambda 2 \\ V3\ \lambda 3 \\ V4\ \lambda 4 \\ V5\ \lambda 5 \end{array}}$$

We pick how many eigenvalues we want to keep and which to drop. For this example, we'll keep the top two eigenvalues and drop three:

$V1\ \lambda 1$

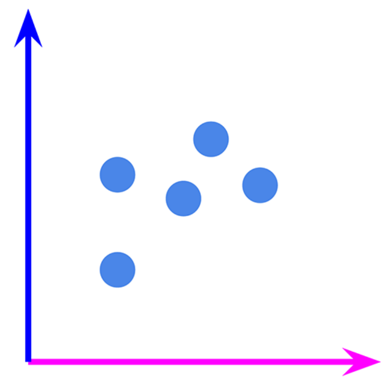
$V2\ \lambda 2$

$\cancel{V3\ \lambda 3}$

$\cancel{V4\ \lambda 4}$

$\cancel{V5\ \lambda 5}$

Taking two will allow us to plot on a 2D plane. The two eigenvalues and eigenvectors will create a plane on which all the points can be plotted:



This now narrows down our five features to two and gives us a good snapshot of what the data should look like because we chose the directions the data spread the most.

Finally, these data points will give us a table of two columns, where the asterisk (*) is a number. Remember, when we coded PCA, the end result was two columns of principal components:

Principal Component 1	Principal Component 2
*	*
*	*
*	*
*	*

IMPORTANT

> The statistics, linear transformations, and eigenvalues and eigenvectors all illustrate how PCA works. As you saw earlier, it is much easier to code than do all of this math. So, don't worry if this is confusing—remember, you've already coded it! It is important to understand, on some level, what PCA is doing in case you're ever asked in an interview.

That wraps up how PCA works. (Wow! That was a lot of fancy jargon!) Thankfully, code has made our work easier and now you have a better understanding of how to reduce dimensions yet still keep the values.

Next, look at another form of clustering.