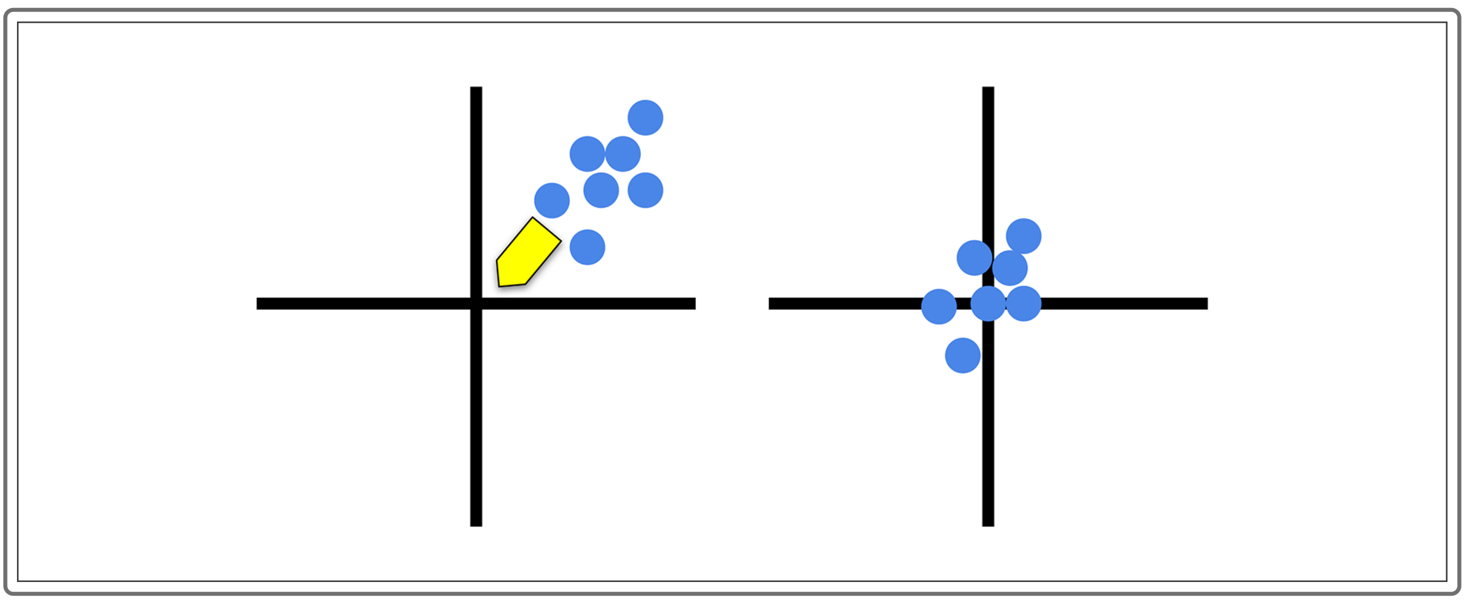
18.5.4

Linear Transformations

Martha appreciates the refreshers on stats but is wondering where this is going. Well, patience is a virtue, and trust that all of this forms the building blocks to really start to understand how PCA works. Next up is **linear transformations**.

Say we have a set of points on a graph. We want to center these points by taking the average of the coordinate, both X and Y. Find the balance point and move that to zero:



Once the points are centered, we're going to create a 2x2 matrix that consists of the variance and covariances that we found in the previous step:

$$\begin{bmatrix} \text{Variance (X)} & \text{Covariance (X, Y)} \\ \text{Covariance (X, Y)} & \text{Variance (Y)} \end{bmatrix}$$

So, let's say the matrix above contains the following:

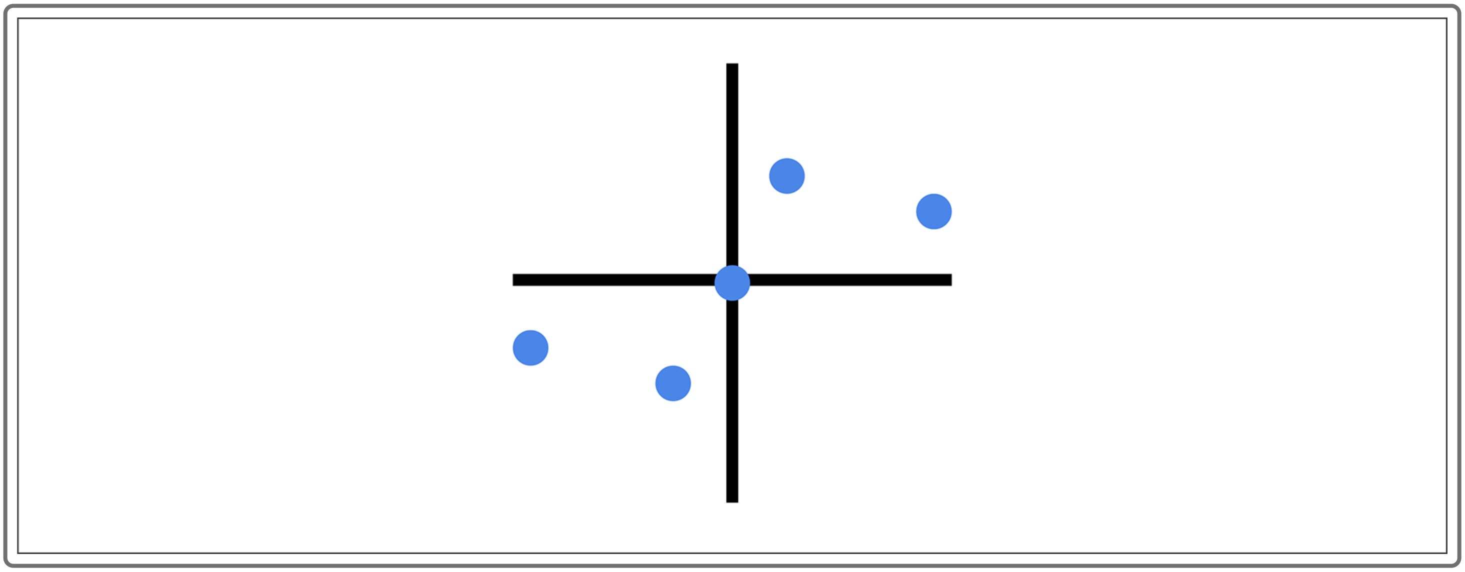
$$\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

This matrix will be used to transform the points from one graph to another by using the numbers to create a formula for our transformation. The top two values of the matrix will correspond to one point and the bottom two values to another.

In our example, the formula for the points becomes (6x + 2y, 2x + 3y). Let's plug some coordinates into the formula:

(x, y)	(6x + 2y, 2x + 3y)
(0,0)	(0,0)
(1,0)	(6,2)
(0,1)	(2,3)
(-1,0)	(-6,-2)
(0,-1)	(-2,-3)

Now, let's plot the new points from the right side of the matrix to create a linear transformation:

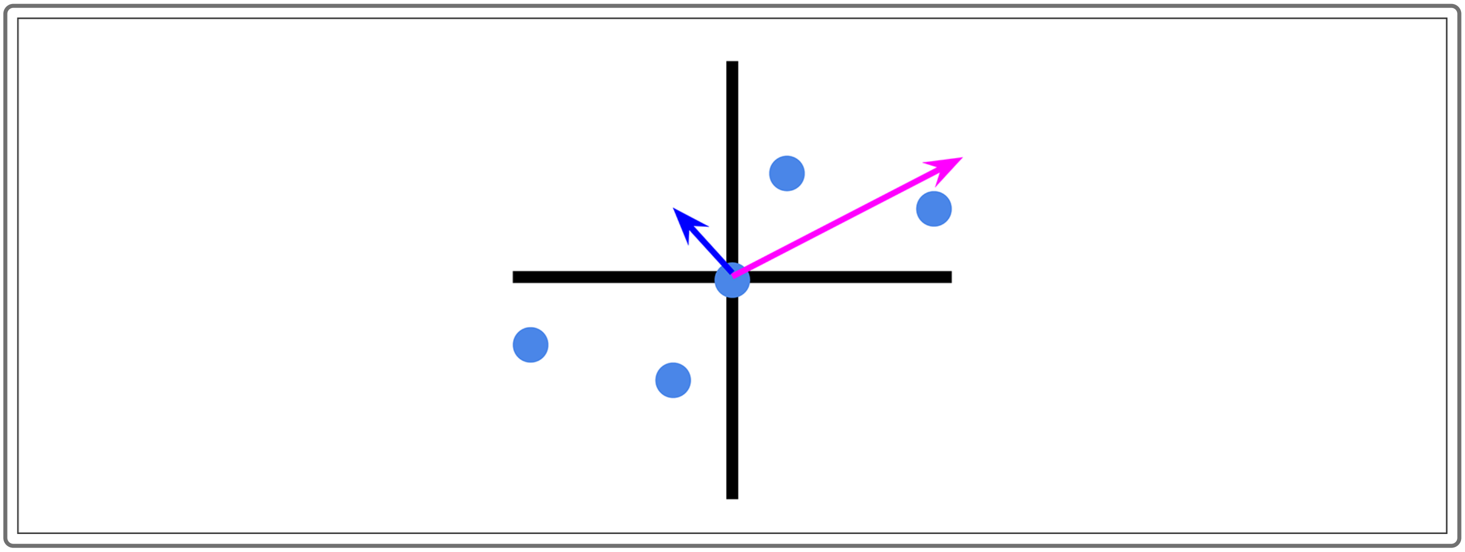


Eigenvectors and Eigenvalues

NOTE

Eigenvectors and eigenvalues can be complicated subjects rooted in linear algebra. We cover these at a very high level, but if you wish to explore more on your own, you can read more about **Eigenvalues and eigenvectors (https://en.wikipedia.org/wiki/Eigenvalues_and_eigenvectors)** and watch this **video (https://www.youtube.com/watch?v=PFDu9oVAE-g)**

As you can see, the points stretch out in our graph in two directions. One direction moves from southwest to northeast direction while another direction moves from southeast to northwest. These are called **eigenvectors**, as indicated by the arrows in the graph below:



There is a way to figure out the vectors and values with algebra, but we use the calculator on **[WolframAlpha (https://www.wolframalpha.com/input/?i=eigenvalues)](https://www.wolframalpha.com/input/?i=eigenvalues)** to simplify the process. Plug in our matrix of {{6,2},{2,3}}, then click calculate.

From the results website, you can see in one direction the shape stretched to a value of 7 and another to a value of 2. The magnitude that each of these stretches is called the **eigenvalue**:

Results:
$\lambda_1=7$
$\lambda_2=2$

We also see the direction that stretched with the eigenvectors of (2, 1) and (-1, 2):

Corresponding eigenvectors:
$v_1 = (2, 1)$
$v_2 = (-1, 2)$

The big takeaway from eigenvectors and eigenvalues is that they show us the spread of the dataset and by how much.