

¹ aurel: A Python package for automatic relativistic calculations

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⁵ Summary

⁶ aurel is an open-source Python package designed to automatically calculate *relativistic*
⁷ quantities. It uses an efficient, flexible and user-friendly caching and dependency-tracking
⁸ system, ideal for managing the highly nonlinear nature of general relativity. The package
⁹ supports both symbolic and numerical calculations. The symbolic part extends SymPy with
¹⁰ additional tensorial calculations. The numerical part computes a wide range of tensorial
¹¹ quantities, such as curvature, matter kinematics and much more, directly from any spacetime
¹² and matter data arrays using finite-difference methods. Inputs can be either generated from
¹³ analytical expressions or imported from Numerical Relativity (NR) simulations. For users
¹⁴ of Einstein Toolkit, aurel also provides utilities to load 3D data generated with Carpet.
¹⁵ Given the increasing use of NR, aurel offers a timely post-processing tool to support the
¹⁶ popularisation of this field.

Statement of need

¹⁸ General relativity describes matter as moving according to how distances shrink or expand;
¹⁹ likewise, the intervals of space and time evolve depending on the distribution of matter.
²⁰ Handling this dynamic “mesh” of distances and times requires elaborate tensor algebra that, in
²¹ some cases, can only be managed with symbolic or numerical tools. Naturally, NR has become
²² essential for modern astrophysics, cosmology, and gravitational physics, most notably in the
²³ modelling of gravitational-wave signals.

²⁴ While established computational frameworks focus on solving and evolving Einstein’s field
²⁵ equations, with specific key diagnostics, they leave calculations of the remaining analysis to
²⁶ the discretion of the researchers. Newcomers to the field then face a substantial overhead until
²⁷ they develop their own personal post-processing codes. Established researchers also face the
²⁸ tedious task of handling intermediary variables and indices when calculating new quantities.
²⁹ The field then suffers from this error-prone, time-consuming process and would benefit from
³⁰ an accessible, open-source, standardised framework to automate these steps.

³¹ We therefore present aurel, an open-source Python package designed to streamline relativistic
³² calculations. It is hosted on [GitHub](#) and is available on [PyPI](#). The documentation is available
³³ through [GitHub Pages](#).

³⁴ State of the field

³⁵ When looking for general relativity Python packages, there are a number of tools that provide
³⁶ symbolic calculations ([Bapat et al., 2020](#); [Czaja, n.d.](#); [Della Monica, 2025](#); [Gourgoulhon et al.,](#)
³⁷ [2015](#); [Hackstein & Hackmann, 2025](#); [Martín & Sureda, 2022](#); [Shoshany, 2025](#); [Wittig & Grover,](#)
³⁸ [2017](#)). Or, one may also consider computer algebra systems ([Maplesoft, 2025](#); [Martín-García](#)

& others, 2025; Wolfram Research, 2025). However, when non-linearities become too complex for symbolic packages, NR is used instead.

Einstein Toolkit (Löffler & others, 2012; Rizzo et al., 2025) is a large community-driven software whose tools enable the evolution of Einstein's field equations. Diagnostic and further analysis calculations are typically performed on the fly, during simulations. To study the outputs of such simulations, provided by Carpet, there are Python reading packages available (Bozzola, 2021; Ferguson et al., 2025; Kastaun, n.d.; Radice, n.d.). These extra calculations can slow down the simulation of the spacetime evolution, and if certain relativistic quantities are not available in Einstein Toolkit, or in one of the post-processing packages, then the user needs to code that up themselves.

There are a number of other well-established NR codes (Andrade et al., 2021; Barrera-Hinojosa & Li, 2020; Palenzuela et al., 2025; Wright, 2018; Zhang et al., 2025) that also have their own diagnostic tools. However, these are typically built-in, so going from one code to another, to benchmark or to use their different types of applications, requires learning the ecosystem of each.

To improve the community's versatility and limit the repeated implementation of error-prone calculations, there is a motivation to provide packages for computing relativistic quantities in an NR-code-agnostic way. Especially in the post-processing sense, where all calculations are done from a given NR spacetime and matter solution. A couple of notable packages (Grasso et al., 2021; Pook-Kolb et al., 2019) focus on ray tracing, or apparent-horizon finding, which are currently beyond the scope of aurel. While others have more overlap (Cranganore et al., 2025; Munoz & Bruni, 2023) in calculating curvature terms, they differ in scope and workflow.

Here, aurel innovates in its automatic design, which is easily extendable and provides flexibility and robustness with a large and ever-growing catalogue of relativistic quantities. A precursor to this package was EBWeyl (Munoz & Bruni, 2023), as it provided calculations of gravito-electromagnetic contributions from base spacetime and matter quantities. aurel now has a completely different structure (relying on the automatic dependency resolution), provides calculations of many more terms, over time, and has entirely new features as described in the following section.

Software Design

aurel provides an intuitive interface for the automatic calculation of general relativistic quantities, either symbolically (with AurelCoreSymbolic, built on SymPy (Meurer et al., 2017)) or numerically (with AurelCore, which heavily utilises numpy.einsum (Harris et al., 2020) for efficient operations on array data structures).

Both require base quantities such as the spacetime coordinates or the parameters of the Cartesian numerical grid, as well as the spacetime and matter distributions (the Minkowski vacuum is otherwise assumed). These inputs can either come from analytical expressions, with a couple of built-in solutions available, or from output data from any NR simulations; they just need to be passed as numpy arrays.

Specifically, for simulations run with Carpet in the Einstein Toolkit, the reading module provides helper functions to load and organise the 3D data. These can read the parameter file, summarise available iterations and variables, and handle data separated across restarts, chunks, or refinement levels for normal Carpet data files or checkpoint files. To speed up repeated data reading, read_data can also split the data per iteration, instead of per variable.

Then, once input data is provided, users can directly request a wide range of relativistic quantities, including: spacetime; matter (Eulerian, Lagrangian, or conserved); BSSNOK formulation; constraints; fluid covariant kinematics; null ray expansion; 3- and 4-dimensional curvature; gravito-electromagnetism; Weyl scalars and invariants (including gravitational waves). To see a full list of available quantities, see: [descriptions](#). Tools are also provided for spatial

88 and spacetime covariant derivatives and Lie derivatives. All spatial derivatives are computed by
89 the FiniteDifference class that provides 2nd, 4th, 6th and 8th order schemes, using periodic,
90 symmetric or one-sided boundary conditions.

91 Automatic Computational Pathway

92 The aurel automatic process composes a computational pathway at runtime to evaluate
93 the requested quantities. This is implemented through a lazy-evaluation memoised property
94 pattern, where each quantity is defined as a method of the core class that may depend on
95 other quantities. This design has been chosen for its flexibility and accessibility while remaining
96 robust under future extensions.

97 Quantities are requested via a user-friendly dictionary-style access, e.g. `rel["s_RicciS"]`,
98 which triggers the lazy memoised check to see if this is already cached. If yes, then the result is
99 directly returned. If not, then the corresponding method is called, which recursively triggers the
100 calculation of dependencies (e.g. `rel["s_Ricci_down3"]`). This continues until the requested
101 quantities can be calculated and so returned.

102 To avoid redundant computations, each result is cached, which builds up a cache memory that
103 needs to be efficiently managed. So, inspired by Python's garbage collection, aurel uses an
104 intelligent eviction policy that tracks memory footprint, evaluation counts, and last-access
105 times. When the configurable thresholds are exceeded, the older and heavier cached quantities
106 are removed, while safeguarding protected base quantities. Throughout this process, aurel
107 keeps the user informed on progress by providing verbose updates on the computation and
108 caching workflow.

109 Time dependence

110 All calculations within the AurelCore class are evaluated at a single fixed time, corresponding
111 to one slice in time, so that individual time steps can be treated independently. For multiple
112 time steps, an AurelCore object needs to be created and the requested quantities collected for
113 each.

114 To streamline this process, aurel provides the `over_time` function to do exactly this, and
115 also compute summary statistics over the grid domain (e.g., max/min) at each time step. By
116 design, it is easily extensible, so `over_time` also accepts custom functions of new relativistic
117 quantities and summary statistics. This makes aurel versatile, supporting an infinite number
118 of ways to view a problem and develop diagnostic tools.

119 Research Impact Statement

120 aurel is a specialist tool for general relativity researchers and streamlines numerical relativists'
121 post-processing workflow. Through conference interactions and collaborations involving the
122 authors, this package has gradually been disseminated to individual researchers who appreciate
123 the effortless integration, satisfying dependency resolution and substantial reduction to post-
124 processing overhead. Indeed, in ongoing studies involving NR simulations of primordial
125 black hole formation, aurel has increased capacity and redirected repetitive and error-prone
126 development efforts towards exploring a broader range of simulated scenarios. Additionally, for
127 master students, the straightforward and transparent design has provided an easy gateway for
128 them to analyse NR simulations and so quickly get results within the duration of their projects.
129 Going forward, awareness of this code will build upon publication, reaching a wider audience
130 and supporting the popularisation of NR.

¹³¹ AI usage disclosure

¹³² GitHub Copilot Claude Sonnet 4 was used for the development and documentation of this
¹³³ package. Autocompletion suggestions were accepted via the VSCode Copilot plugin, and upon
¹³⁴ the developer's request, edits and code snippets were generated via the large language model's
¹³⁵ user interface. The most significant AI contributions came in drafting the docstrings and
¹³⁶ scaffolding the test suite, both of which are essential for the accessibility and robustness of this
¹³⁷ package. Each and every suggestion or contribution was meticulously reviewed and adjusted
¹³⁸ before being included by the authors, who made all core design decisions and innovated the
¹³⁹ original structural concept. Finally, this paper was prepared without the use of generative
¹⁴⁰ language models, solely with grammar checkers.

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