Absolutely amazing thesis title

Author’s Full Name

Intitute name

University name

year

The thesis is submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy of the University of Portsmouth.

Whilst registered as a candidate for the above degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award.

Ethical review number: ETHICS CODE.
Approximate word count: MANY NUMBERS

Abstract

I don't want the first paragraph to be indented. Lots of text that all holds in one page. Some times I mention codes I use with the `code font`, sometimes I have a bit of maths $1 + 1 = 2$. But nice overall summary with key messages, one aspect discussed in each paragraph... I do not put references or links or acronyms in the abstract.

Acknowledgements

I am grateful to everybody.

Thank you to all the academic people.

Thank you to all the family and friends.

Dissemination

Papers

- **R. L. Munoz** and M. Bruni. In: *Classical and Quantum Gravity* 40.13 (June 2023), p. 135010.

  "EBWeyl: a Code to Invariantly Characterize Numerical Spacetimes".

  DOI: 10.1088/1361-6382/acd6cf. arXiv: gr-qc/2211.08133

  (Munoz and Bruni, 2023a) see Chapter 5.

- **R. L. Munoz** and M. Bruni. In: *Physical Review D* 107.12 (June 2023), p. 123536.

  "Structure formation and quasispherical collapse from initial curvature perturbations with numerical relativity simulations".

  DOI: 10.1103/PhysRevD.107.123536. arXiv: astro-ph/2302.09033

  (Munoz and Bruni, 2023b) see Chapter 6.

Codes

- `EBWeyl` GitHub

  (Munoz, 2022; Munoz and Bruni, 2023a), see Chapter 5

- `ICPertFLRW` GitHub

  (Munoz, 2023a; Munoz and Bruni, 2023b), see Section **??** and Appendix **??**

- `sphereint` GitHub

  (Munoz, 2023b; Munoz and Bruni, 2023b), see Appendix **??**

Contents

List of Figures

List of Tables

Notation

The speed of light and the gravitational constant are taken to be one: $c = G = 1$.
Greek indices are used for spacetime components $\{0,\ 1,\ 2,\ 3\}$,
while Latin indices are used for space components $\{1,\ 2,\ 3\}$.

General Relativity

Spacetime

τ	Proper time, $\frac{da}{d\tau} = \dot{a}$	Eq. (**??**)
$\delta_{\alpha\beta}$	Kronecker delta	
$g_{\mu\nu}$	Spacetime metric	Eq. (**??**), $\{-,\ +,\ +,\ +\}$
$A_{(\mu\nu)}$	Symmetrisation of the tensor $A_{\mu\nu}$	Eq. (**??**)
$A_{[\mu\nu]}$	Anti-symmetrisation of the tensor $A_{\mu\nu}$	Eq. (**??**)
$\bar{\bar{A}}^{\alpha}{}_{\beta}$ & $\tilde{\tilde{A}}^{\alpha}{}_{\beta}$	Tensor density & pseudo-tensor respectively	Eq. (**??**) & Eq. (**??**)
$\epsilon_{\alpha\beta\mu\nu}$	Levi-Civita tensor	Eq. (**??**)
∂_α	Partial derivative	$\partial_\alpha = \frac{\partial}{\partial x^\alpha}$
∇_α	Covariant derivative w.r.t. $g_{\mu\nu}$	Eq. (**??**)
$\Gamma^{\alpha}{}_{\mu\nu}$	Christoffel symbols	Eq. (**??**)
$R^{\alpha}{}_{\sigma\mu\nu}$	Riemann tensor	Eq. (**??**)
$R_{\alpha\beta}$ & R	Ricci tensor & Ricci scalar (also ${}^{(4)}R$)	Eq. (**??**) & Eq. (**??**)
$C_{\alpha\beta\mu\nu}$	Weyl tensor	Eq. (**??**)

Projection along or orthogonally to a vector

The superscript $\{v\}$ denotes a variable defined in the frame of said timelike vector v^α.
The pre-superscript (3) applies for 3-dimensional quantities built from the spatial metric $P^{\{v\}}_{\alpha\beta}$.
For simplicity, quantities with (3) are not given the $\{v\}$ superscript; instead, the relevant frame should be determined based on context.

$P^{\{v\}}_{\alpha\beta}$	Projection tensor of v^α or Spatial metric	Eq. (**??**), $\{+,\ +,\ +\}$
$A^{\{v\}}_{\langle\mu\nu\rangle}$	Spatially projected, symmetric and tracefree part of $A_{\mu\nu}$	Eq. (**??**)
$\mathcal{L}_v$	Lie derivative along the vector v^α	Eq. (**??**)
$D^{\{v\}}_\alpha$	Spatial covariant derivative w.r.t. $P^{\{v\}}_{\alpha\beta}$	Eq. (**??**)
$D^{\{v\}}\times$	Spatial curl operator	Eq. (**??**)
$\epsilon^{\{v\}}_{\beta\mu\nu}$	Spatial Levi-Civita tensor	Eq. (**??**)
${}^{(3)}\Gamma^{\alpha}{}_{\mu\nu}$	Spatial Christoffel symbols	
${}^{(3)}R^{\alpha}{}_{\sigma\mu\nu}$	Spatial Riemann tensor	Eq. (**??**)
${}^{(3)}R_{\sigma\nu}$ & ${}^{(3)}R$	Spatial Ricci tensor & scalar respectively	

Einstein's field equations

Λ Cosmological constant

$G_{\alpha\beta}$ Einstein tensor .... Eq. (**??**)

κ Einstein's gravitational constant .... $\kappa = 8\pi G c^{-4} = 8\pi$

$T_{\alpha\beta}$ Energy stress tensor .... Eq. (**??**)

T Trace of the energy stress tensor .... Eq. (**??**)

Matter

$\rho^{\{v\}}$ Energy density .... Eq. (**??**)

$q_{\alpha}^{\{v\}}$ Energy flux or momentum density .... Eq. (**??**)

$S_{\alpha\beta}^{\{v\}}$ Stress tensor .... Eq. (**??**)

$S^{\{v\}}$ Trace of the stress tensor .... Eq. (**??**)

$p^{\{v\}}$ Pressure .... Eq. (**??**)

$\pi_{\alpha\beta}^{\{v\}}$ Anisotropic pressure .... Eq. (**??**)

$\varrho^{\{v\}}$ Rest mass energy density .... Eq. (**??**)

$\varepsilon^{\{v\}}$ Specific internal energy .... Eq. (**??**)

$h^{\{v\}}$ Specific enthalpy .... Eq. (**??**)

Kinematical rest-energy frame

All of the following are uniquely defined in the frame of the fluid flow u^{α}.

u^{α} Fluid 4-velocity

$h_{\alpha\beta}$ Projection tensor of u^{α} or Spatial metric .... Eq. (**??**), $\{+,\ +,\ +\}$

$\Theta_{\alpha\beta}$ Expansion tensor .... Eq. (**??**)

$\omega_{\alpha\beta}$ Vorticity tensor .... Eq. (**??**)

a^{α} Acceleration .... Eq. (**??**)

Θ Expansion scalar .... Eq. (**??**)

$\sigma_{\alpha\beta}$ Shear .... Eq. (**??**)

Weyl tensor

$E_{\alpha\beta}^{\{v\}}$ Electric part of the Weyl tensor in the frame v^{α} Eq. (**??**)

$B_{\alpha\beta}^{\{v\}}$ Magnetic part of the Weyl tensor in the frame v^{α} Eq. (**??**)

$Q_{\alpha\beta}^{\{v\}}$ Complex combination of $E_{\alpha\beta}^{\{v\}}$ and $B_{\alpha\beta}^{\{v\}}$ .... Eq. (**??**)

Ψ_0, Ψ_1, Ψ_2, Ψ_3, Ψ_4 Weyl scalars in the frame of the null tetrad $l^{\alpha},\ k^{\alpha},\ m^{\alpha},\ \bar{m}^{\alpha}$ Eq. (**??**)

$I,\ \ J,\ \ L_B,\ M, \mathcal{S}, \mathcal{D}$ Frame independent invariants from $C_{\alpha\beta\mu\nu}$ .... Eq. (**??, ??, ??, ??**)

K, L, N Frame dependent invariants from $C_{\alpha\beta\mu\nu}$ .... Eq. (**??**)

Numerical Relativity

γ_{ij}	Projection tensor of n^α or Spatial metric	Eq. (??), $\{+,\ +,\ +\}$
α	Lapse	
β^i	Shift	Eq. (??)
n^i	Timelike vector normal to γ_{ij}	Eq. (??)
$\mathfrak{a}^i$	Acceleration of n^i	Eq. (??)
t	Coordinate time	Eq. (??)
K_{ij}	Extrinsic curvature	Eq. (??)
K	Trace of extrinsic curvature	Eq. (??)
A_{ij}	Traceless part of the extrinsic curvature	Eq. (??)

Cosmology

Homogeneous & isotropic universe

The overhead bar identifies background quantities.

a	Scale factor	Section ??
η	Conformal time, $\frac{dg}{d\eta} = g'$	Eq. (??)
H	Hubble's scalar	Eq. (??)
$\mathcal{H}$	Conformal Hubble's scalar	Eq. (??)
z	Redshift	Eq. (??)
$\bar{\Theta}$	FLRW expansion scalar	Eq. (??)
$\bar{\rho}$	FLRW energy density in the u^μ frame	Eq. (??)
Ω_k, Ω_m, Ω_Λ	Dimensionless density parameters of curvature, matter and dark energy	Eq. (??)

Perturbation theory & structure formation

The (1) superscript identifies a first-order perturbative term.

ϕ, ω, ψ, χ	Scalar perturbations to the spacetime metric	Eq. (??, ??, ??)
ω_i^S, χ_i^S	Vector perturbations to the spacetime metric	Eq. (??, ??, ??)
χ_{ij}^{TT}	Tensor perturbations to the spacetime metric	Eq. (??, ??)
δ	Density contrast	Eq. (??)
$\delta_C^{(1)}$	Linear density contrast a collapse	Eq. (??)
f_1	Growth factor	Eq. (??)
ϑ_{ij}	Deformation tensor	Eq. (??)
ϑ	Deformation scalar, trace of ϑ_{ij}	Eq. (??)
$\Box_{ij}$	Traceless operator	$\Box_{ij} = \partial_i\partial_j - \frac{1}{3}\delta_{ij}\delta^{kl}\partial_k\partial_l$
∇^2	Laplacian operator	$\nabla^2 = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$
$\mathcal{R}_c$	Comoving curvature perturbation	Eq. (??)
ζ	Uniform-density curvature perturbation	Eq. (??)
Φ, Ψ	Bardeen potentials: Newtonian potential and conformal Newtonian curvature perturbation	Eq. (??)
$\langle\phi\rangle_{\mathcal{D}^{\{v\}}}$	Average of ϕ on the domain $\mathcal{D}^{\{v\}}$ which is defined on the $P_{\alpha\beta}^{\{v\}}$ spatial hypersurface	Eq. (??)
$\mathcal{Q}_{\mathcal{D}^{\{u\}}}$	Backreaction	Eq. (??)

Acronym

ADM	Arnowitt, Deser and Misner
BSSNOK	Baumgarte, Shapiro, Shibata, Nakamura, Ookara and Kojima
CDM	Cold Dark Matter
CMB	Cosmic Microwave Background
EdS	Einstein-de Sitter
FD	Finite Difference
FLRW	Friedmann Lemaître Robertson Walker
GR	General Relativity
LSS	Large-Scale Structure
LTB	Lemaître Tolman Bondi
NP	Newmann-Penrose
NR	Numerical Relativity
OD	Over Density
PDE	Partial Differential Equation
TA	Turn Around
UD	Under Density

1 - Introduction

To site someone I use either: (Newton, 1687) if I just need to include a reference or Newton (1687) if I mention this in the sentence. I create links like this: Eq. (2.1) Fig. (5.1) Table (3.1) Chapter 2 Section 2.1 Appendix B but if I want to reference multiple things at the same time I need to write it out in full: Eq. (2.1, 2.2) or Chapter 2, 3 and 4 and to have footnotes[1]. The first time I use an accronym I give the full name: General Relativity (GR).

1.1 Structure

Here I explain what happens in the chapters: the introduction chapters are: Chapter 2, 3 and 4, research chapters based on papers are Chapter 5 and 6, and in particular I tell you exactly which parts have my original contributions.

[1]This is what I do. And I make sure these are full sentences.

2 - General Relativity

I put punctuation around equations because they are part of a sentence. I shall show you a beautiful equation

$$G_{\alpha\beta} = \kappa T_{\alpha\beta}, \tag{2.1}$$

that is very beautiful. And how are each of these defined?

$$2\nabla_{[\mu}\nabla_{\nu]}u^{\alpha} = R^{\alpha}{}_{\beta\mu\nu}u^{\beta}, \qquad \nabla_{\alpha}T^{\alpha\beta} = 0, \tag{2.2}$$

and with faerie dust.

2.1 Section

2.1.1 Sub Section

2.1.1.1 Sub Sub Section, the & symbol in a title is nice

2.1.1.2 Cosmological constant in a title Λ

To separate things more I use bullet points. Pretty symbol, pifont ding 118, to make clear that these

- ❖ are the points I am making.
- ❖ With long long long long long long long long long long long long long long long long long long long text showing indentation

For a sub bullet I use the ding 70 bullet point and define the leftmargin to give a hierarchal look. I can also define itemindent such that there is no indentation should the text following the bullet be long

✦ **First:** point, with long long long long long long long long long long long long long long long long long long long text showing indentation

✦ **Second:** point

Or if I want a more title like look I can separate them out

❖ **sub sub sub section**

and put some text in between

❖ **sub sub sub section**

3 - Numerical Relativity

This is a table. In caption [Text for table list] Actual caption

Order	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
	Backwards												
1						-1	1						
2					$\frac{1}{2}$	-2	$\frac{3}{2}$						
4			$\frac{1}{4}$	$-\frac{4}{3}$	3	-4	$\frac{25}{12}$						
6	$\frac{1}{6}$	$-\frac{6}{5}$	$\frac{15}{4}$	$-\frac{20}{3}$	$\frac{15}{2}$	-6	$\frac{49}{20}$						
	Centred												
2						$-\frac{1}{2}$	0	$\frac{1}{2}$					
4					$\frac{1}{12}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	$-\frac{1}{12}$				
6				$-\frac{1}{60}$	$\frac{3}{20}$	$-\frac{3}{4}$	0	$\frac{3}{4}$	$-\frac{3}{20}$	$\frac{1}{60}$			
	Forwards												
1							-1	1					
2							$-\frac{3}{2}$	2	$-\frac{1}{2}$				
4							$-\frac{25}{12}$	4	-3	$\frac{4}{3}$	$-\frac{1}{4}$		
6							$-\frac{49}{20}$	6	$-\frac{15}{2}$	$\frac{20}{3}$	$-\frac{15}{4}$	$\frac{6}{5}$	$-\frac{1}{6}$

Table 3.1: Coefficients of backward, centred and forward finite differencing schemes at second, fourth and sixth order (Fornberg, 1988).

4 - Cosmology

5 - EBWeyl

5.1 Test-bed spacetimes

5.1.1 The Λ-Szekeres models of Barrow and Stein-Schabes

This is a figure. I make sure the plot isn't too heavy or the pdf buffers when scrolling through it. In caption [Text for figure list] { Actual caption }

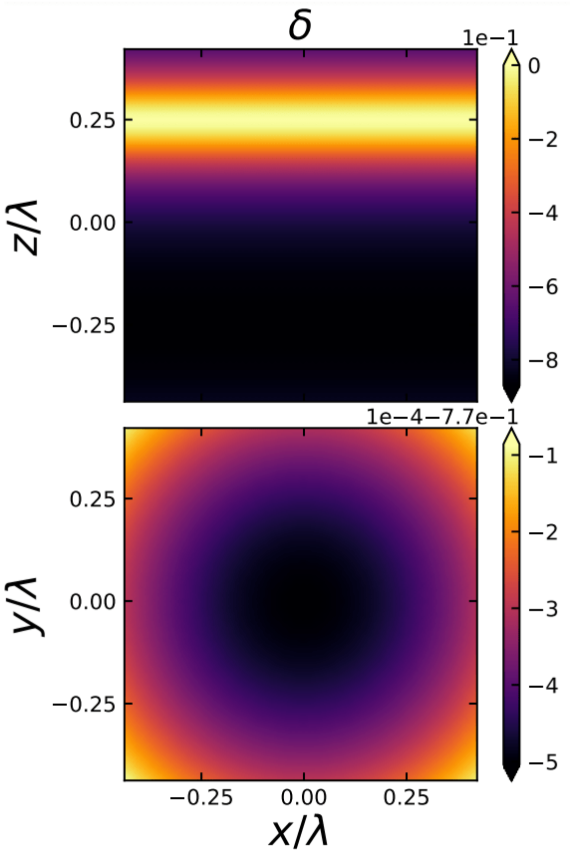

Figure 5.1: Space distribution of the density contrast δ in Eq. (**??**) indicated by colour coding (the top plot ranges from -0.8 to 0 and the bottom plot ranges from -0.7705 to -0.7701) for the Λ-Szekeres spacetime on the z-x and y-x planes (with $y/\lambda = 0$ and $z/\lambda = 0$ respectively) of a data box of size $\lambda = 20$Mpc with 64^3 grid points, at redshift 230.

5.2 Results

5.3 Summary

6 - Quasi-spherical collapse

6.1 Initial conditions

Here's three figures together using minipage

6.2 Simulation results

6.3 Summary

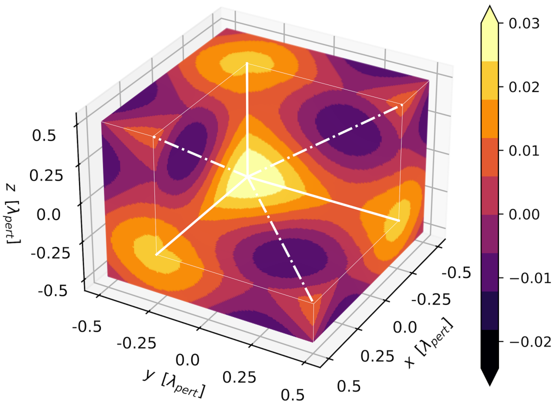

Figure 6.1: Initial distribution at $z_{IN} = 302.5$ of the density contrast δ in the simulation box, for a ΛCDM universe. The x, y, and $z > -0.25\lambda_{pert}$ region is removed exposing the centre of the over-density at $x = y = z = -0.25\lambda_{pert}$, where $\delta_{IN,\,OD} = 0.03$. The full lines go through the vertices and dash-dotted lines through the centre of the edges of an octahedron centred at the over-density.

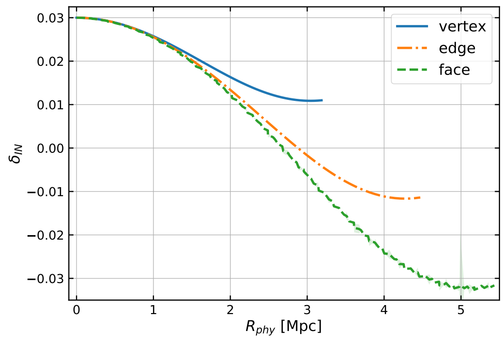

Figure 6.2: Initial radial profile at $z_{IN} = 302.5$ of the initial density contrast δ starting from the centre of the over-density to its minimum in three different directions, towards the vertices, edges, and faces of the octahedral distribution in Eq. (**??**) plotted against the proper radius from the over-dense peak. Error bars, when visible, are indicated as shaded regions.

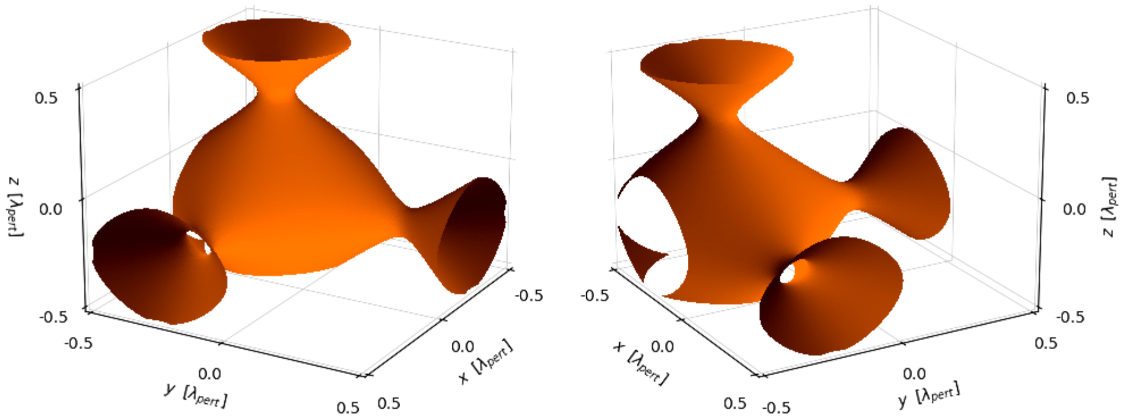

Figure 6.3: Isosurface for $\delta = 0.01$ in the initial distribution of the matter density contrast at $z_{IN} = 302.5$. The two different panels show different points of view. The periodic boundary conditions insure that this distribution is a lattice of over-densities connected by filaments and separated by voids.

7 - Future prospects

One paragraph summary of conclusions, I don't make it long because I already put summaries in the previous chapters. Then I make a list of ideas to pursue and give their relevance in the field.

A - Finite differencing tests

B - Analytic expressions

B.1 The Λ-Szekeres models of Barrow and Stein-Schabes

B.2 A non-diagonal inhomogeneous test metric

B.3 Bianchi II Collins-Stewart

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