Dynamics in the dynamic walk of a quadruped robot

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Abstract

Walk can be classified as "static walk" and "dynamic walk". It is said that dynamic walk is superior in both speed and energy consumption. This paper describes how a quadruped robot should walk dynamically to realize these advantages. Such consideration is lacking in past research.

In this paper, three criteria are introduced to evaluate the walk – '*stability*', '*maximum speed*' and '*energy consumption*'. The relations between these three criteria and the parameters (gait, speed, period, stride, length of the leg, joint angles, etc.) are formulated accordingly to the dynamics. The conclusions are as follows:

(1) The shorter a period is, the more stably the quadruped can walk.

(2) It is desirable to walk with a longer period and wider stride in order to increase the maximum speed.

(3) There is a period which maximizes the speed.

(4) There is a period which minimizes the energy consumption for a given speed.

(5) Trot gait is desirable when the priority is placed on energy consumption. Pace gait is recommended when the priority is place on maximum speed.

From experiments using the quadruped robot Collie-2, the validity and usefulness of these relations are verified.

Key Words : Quadruped Robot, Dynamic Walk, Stability, Maximum Speed, Energy Consumption, Collie-2

1 INTRODUCTION

The legged locomotion of robots has been studied actively. The researches in this field can be divided into two areas.

a) static walk – the centre of gravity is projected inside the polygon formed by the supporting legs.

b) dynamic walk – the centre of gravity is not necessarily projected within the polygon formed by the supporting legs. However, dynamic balance is to be maintained.)

Control in static walk is quite simple and this makes it easy for a robot to walk over irregular terrain. Dynamic walk is superior in speed and energy consumption.

We have studied the dynamic walk of a quadruped robot with the intention of realizing a robot which can choose either walk according to the environment and realized 'pace gait" and 'trot gait' [1] [2]. Including our studies, researchers in the area of dynamic walk had been concentrating solely on realizing the walk. They had realized various dynamic walks of a one [3], two [4] [5], four [6]- [9], and six [10] legged robot. Now we are going to find out the conditions which make dynamic walk better. In this paper in order to evaluate dynamic walk, three criteria for the quadruped are proposed: '*stability*', '*maximum speed*' and '*energy consumption*'. The relations between these three criteria and the parameters (gait, speed, period, stride, length of the leg, joint angles, etc.) are formulated accordingly to the dynamics. These relations are useful for designing the robot and planning the dynamic walk.

2 EQUATIONS OF MOTION

2.1 Modeling of the quadruped

When we analyse the dynamics of the quadruped, it is necessary to determine the mechanism and the types of actuators (i.e. hydraulic or electric, the gear ratio, etc.). In this paper, the quadruped Collie-2, which can walk dynamically is considered. The following are assumed for the quadruped:

Assumption 1 The coordinates of the quadruped are as shown in Fig. **??**.

Assumption 2 The actuators are DC servo motors with a small gear ratio (about $10 \sim 15$).

Assumption 3 Friction at the joints can be ignored as it is small.

In addition, the following are assumed to simplify the equations of motion:

Assumption 4 The centripetal force and Coriolis' force can be ignored.

Assumption 5 There is no interference between the motion around each axis (pitch, roll and yaw) except that caused by the reaction forces from the floor.

The origin of the coordinate is located at the centre of the body. We choose the state variables as follows:

$$x = \begin{bmatrix} x_p \\ x_{FR} \\ x_{HR} \\ x_{FL} \\ x_{HL} \end{bmatrix} \qquad x_p = \begin{bmatrix} x \\ y \\ z \\ \phi_1 \\ \phi_2 \end{bmatrix} \qquad x_{Leg} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

Leg = FR(Fore Right), HR(Hind Right), FL(Fore Left), HL(Hind Left).

The meanings of the symbols used in this paper are given in Table **??**.

2.2 Dynamics and problems in inverse dynamics

The constraints between the robot and the floor are expressed as

$$e(x) = 0, \tag{1}$$

and the equations of motion with constraint (1) are as (2) [11]

$$A(x)\ \ddot{x} = Cu + g(x) + E^t f, \tag{2}$$

where

$x(t)$: state variables $\in \mathbf{R}^n$,
$u(t)$: joint torques $\in \mathbf{R}^k$,
$f(t)$: reaction forces $\in \mathbf{R}^m$,
$E = \partial e/\partial x \in \mathbf{R}^{m\times n} = [e_{n-m+1}, \cdots, e_n]^t$,
n=25, k=20, and m=5 $\times$ j (j is the number of legs on the floor).

For a given trajectory $x(t)$, the feedforward torque $u(t)$ must satisfy (2). However, the following problems arise:

a) Torque $u(t)$ which satisfies (2) for given arbitrary x and f does not exist because $k < n$ in general;

b) f must satisfy the following conditions:

1) *(vertical floor reaction force)* ≥ 0;

2) *(horizontal floor reaction force)* $\leq$ *(vertical floor reaction force)* $\times$ *(friction coefficient).*

2.3 Reduction of the equations of motion

As long as constraint (1) is satisfied, the precise value of f is not important for realizing the walk. Eliminating the term $E^t f$ in (2), the dimension of the equations is reduced to $n - m$ as shown in (3) (Appendix A):

$$QCu = Q(A\ddot{x} - g), \tag{3}$$

where

$E_p = [e_1, \ldots, e_{n-m}]^t$,

$e_1, \ldots, e_{n-m}$ are the vectors which are independent of $e_{n-m+1}, \ldots, e_n$ and of each other,

$P = A^{-1} - A^{-1}E^t(EA^{-1}E^t)^{-1}EA^{-1}$,

$Q = E_p P$.

Note that (3) is redundant on u contrary to (2), and other conditions are necessary to determine u.

3 BASIC SYMMETRICAL GAITS

3.1 Gaits considered in this paper

The following is assumed about the gait to simplify the analysis:

Assumption 6 The walk is steady.

Assumption 7 The motion of each leg is similar except for the phase difference.

Three basic gaits that use pairs of legs in union are noticed in animals [8]. They are:

trot : diagonal pairs of legs move at the same time;

pace : lateral pairs of legs move at the same time;

bound : the front legs move at the same time, as do the rear.

These gaits are shown in Fig. **??**. As bound needs large power actuators at the pitch hip joints, it is not suitable for the walk.

The duty factor, α ($0 < \alpha < 1$), is defined as the ratio of the time in which the leg contacts the floor over one cycle of the walk. When $\alpha < 0.5$, all legs are off the ground at some particular moment. In this paper, this case is not considered.

Assumption 8 Only trot and pace with $\alpha \geq 0.5$ are considered.

3.2 Realizable trajectory and the model as the inverted pendulum

When we calculate the torques u from (3) in the two-leg supporting phases which appear in the basic symmetrical gaits, condition (b.1) in Section 2.2 is generally not satisfied. This means that the walk is statically unstable and such trajectory is unrealizable. Therefore we must plan the trajectory $x(t)$ which satisfies condition (b.1).

In this paper, we plan this trajectory using the model of an inverted pendulum in which no actuator exsits at the ankle joints of the supporting legs. This model was introduced in the studies of the biped [4]. We let the inverted pendulum shown in Fig. **??**-(b) represent the surface constructed by diagonal pairs of supporting legs in trot. We let the inverted pendulum shown in Fig. **??**-(b) represent the surface constructed by lateral pairs of supporting legs in pace.

4 STABILITY

4.1 Stability and maximum period in dynamic walk

The dynamic walk is realized by a series of unstable inverted pendulums alternatively changing the supporting legs [4]. As described in Section 3.2, we let the inverted pendulums shown in Fig. ??(b)-??(b) represent simple models of the quadruped robot in each gait.

In this paper, the steady walk which satisfies the following equations is considered.

$$\alpha = 0.5 \tag{4}$$

$$\xi_1(0) = -\xi_1(T_0), \tag{5}$$

$$\dot{\xi}_1(0) = -\dot{\xi}_1(T_0), \tag{6}$$

$$T_0 = (1-\alpha)T = T/2. \tag{7}$$

In the steady walk, when the period T is increased , the amplitude of the rolling motion becomes large. If the amplitude is too large, the robot falls over. Therefore, there exists a period T_{max} in each gait such that the robot does not fall.

4.2 Maximum period in trot gait

The motion of the inverted pendulum shown in Fig. ??b is greatly influenced by the initial angle. By carrying out experiments, we can determine the maximum period T_{max} in which the growth of the angle is tolerable. Here increase of the stride S causes an increase in the initial angle of the inverted pendulum (see Fig. ??) and a decrease of T_{max}.

In Collie-2, we obained the following results from experiments in which the roll joints at hips were mechanically constrained:

$$T_{max} = 0.8\,s \quad for\ S = 0\,cm,$$

$$T_{max} = 0.6\,s \quad for\ S = 6\,cm.$$

4.3 Maximum period in pace gait

The roll motion in pace is represented by the double inverted pendulum shown in Fig. ??b. Similar to the biped [4], this inverted pendulum should steadily reciprocate by changing the supporting legs. The equation of motion of the inverted pendulum is written as (Appendix B) :

$$\ddot{\xi}_1 = a\xi_1 - b. \tag{8}$$

The phase plot of the supporting leg's angle ξ_1 from (8) is shown in Fig. ??.

When ξ_1 is small throughout the motion, the second term in (8), b, is dominant. In this case, the motion of the supporting leg is simple and easily satisfies the conditions of steady walk (line a in Fig. ??). When ξ_1 becomes large, the influence of the first term of (8) makes the motion unstable and finally the supporting leg falls down (line b in Fig. ??). Therefore $\xi_{1max}\,(= \xi_1(T_0/2))$ should satisfy the following:

$$\xi_{1max} < \xi_{1limit} < \frac{b}{a}, \tag{9}$$

where ξ_{1limit} is the limit angle obtained from the experiments.

We ignore the first term of (9) as ξ_1 is small. Then from (5)$\sim$(9), we obtain

$$T < \sqrt{32\xi_{1limit} \ / \ b} \tag{10}$$

From the experiments, we find that $\xi_{1limit} = 10°$ and $T_{max} = 0.85\ s$.

5 MAXIMUM SPEED

Let α be 0.5, then the speed V_G can be written as

$$V_G = \frac{2S}{T}, \tag{11}$$

where S and T are the relative stride to the body and the period, respectively.

In order to increase the speed V_G, we need to increase the stride S to decrease the period T. Then the maximum speed V_{Gmax} is determined by U_{limit} (the limitation of the actuator torque which activates the swinging leg.) The relation between V_{Gmax}, S and T is formulated in the following.

To swing a leg by the stride S, it is necessary to accelerate and decelerate a leg. The maximum inertial torque U_{max} is obtained by using the simple model of a swinging leg shown in Fig. **??** (Appendix C) as follows:

$$U_{max} = \frac{48JS}{lT^2}, \tag{12}$$

where S, T, l, and J are the stride, the period, the length of a swinging leg, and the moment of inertia of a swinging leg, respectively.

As U_{max} should be smaller than U_{limit}, we can get the following by substituting (12) into $U_{max} < U_{limit}$:

$$S \leq \frac{U_{limit}}{48J} lT^2. \tag{13}$$

This means that the upper bound of the stride is a function of the period. Here, the maximum stride S_{max} is determined as follows by the maximum period T_{max} in which the robot can walk stably as described in Section 4:

$$S_{max} = \frac{U_{limit}}{48J} l{T_{max}}^2. \tag{14}$$

In addition, by substituting (11) we obtain into (13):

$$V_G \leq \frac{U_{limit}}{24J} lT. \tag{15}$$

(15) means that decreasing the period to increase the speed is not a better way with respect to the maximum speed and it is desirable to walk with as large a period and large a stride as possible. But there also exits a structural limitation of stride S_{limit}:

$$S_{limit} = 2l. \tag{16}$$

There are two cases according to which condition is effective, (14) or (16).

Type a: The case where $S_{max} < 2l$ is obtained because U_{limit} is relatively smaller than the other parameters (J, l, etc.).

Type b: The case where $S_{max} > 2l$ is obtained because U_{limit} is relatively larger than the other parameters (J, l, etc.).

In the case of type a, (16) can be ignored and the maximum speed V_{Gmax} is determined by the maximum period T_{max} (shown as the point A in Fig. ??) from (15) and expressed as follows:

$$V_{Gmax} = \frac{U_{limit}}{24J} l T_{max}. \tag{17}$$

In the case of type b, the stride is limited by (16) and there exists a period which maximizes the speed (shown as point B in Fig. ??):

$$T = \sqrt{\frac{96J}{U_{limit}}} \qquad (< T_{max}). \tag{18}$$

Then the maximum speed V_{Gmax} is written as

$$V_{Gmax} = \sqrt{\frac{U_{limit}}{6J} l^2}. \tag{19}$$

In the case of Collie-2, the physical values are

$$U_{limit} = 1.15\ N\ m, \quad J = 0.0153\ Kg\ m^2, \quad l = 0.3\ m,$$

$$S_{max} = 0.3\ m \quad for \quad T_{max} = 0.8\ s.$$

Therefore it can be said that Collie-2 belongs to type a. On the contrary, as it is observed that animals (for an example, a dog) walk with a constant stride at any speed, it can be said that animals belong to type b.

6 ENERGY CONSUMPTION

6.1 Definition of energy consumption

There are many kinds of energy consumed in the walk. For example,

(a) kinetic energy,

(b) energy loss by friction,

(c) energy loss by collision between the legs and the floor; and

(d) Joule thermal loss by the current in the motors.

Which term is dominant depends on the structure of the quadruped, the actuator, etc.

Here we consider only (a) and (d) according to the assumptions described in Section 2.1. We define the kinetic energy as

$$E_m = \int_0^T \sum_{Leg=FR}^{HL} \sum_{i=1}^{5} \delta(u_i \dot{\theta}_i) dt, \tag{20}$$

$$\delta(x) = \begin{cases} x & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$$

and the Joule thermal loss as

$$E_e = \int_0^T \sum_{Leg=FR}^{HL} \sum_{i=1}^{5} R_i (\frac{u_i}{G_i K_i})^2 dt, \tag{21}$$

where R_i, G_i, and K_i are the armature resistance, gear ratio, and torque factor, respectively, and the torque u_i is obtained by solving (3) for a given trajectory.

The calculation results in Collie-2 are listed in Table **??**. They indicate that in the case of robots where a high torque is generated by the large current in the motor, the energy consumed during walking is mainly the Joule thermal loss. Therefore we use the Joule thermal loss defined by (21) as the consumed energy.

The energy consumption (the energy consumed in walking a unit distance) is defined as

$$P = \frac{E_e}{V_G T}. \tag{22}$$

The parameters used in planning the walk can be calculated under the condition of minimizing P as described in the following sections.

6.2 Parameters minimizing the consumed energy

6.2.1 Period.

The energy consumption P can be represented by the period T and speed V_G as follows (Appendix D):

$$P = C_{sw} \frac{V_G}{T^2} + C_{up} \frac{1}{V_G T^4} + C_{sp} T^2 V_G + \frac{C_{body}}{V_G}. \tag{23}$$

Each term has the following meaning:

1) energy to accelerate or decelerate the forward motions of the swinging legs;

2) energy to accelerate or decelerate the lifting motions of the swinging legs;

3) energy to compensate the gravity according to the angles of the supporting legs;

4) energy to compensate the weight of the body.

The coefficients (C_{sw}, C_{up}, etc.) can be obtained by dividing the value of P calculated from (21) and (22) into each term.

T_{opt} which minimizes P can be obtained by solving $dP/dT = 0$. The results calculated for trot and pace are shown in Fig. **??**. The reason why T_{opt} exists can be explained as follows:

> As T increases, the third term increases because S increases. As T decreases, the first and second terms increase because the acceleration and deceleration of the swinging leg become larger.

6.2.2 Trajectory.

Since (3) is very complicated, it is difficult to determine the trajectory which minimizes (21) while satisfying (3). Therefore, the motion of the robot is conveniently separated into the following:

(1) the roll motion;

(2) the pitch motion of the swinging leg; and

(3) the pitch motion of the supporting leg.

Motions (1) and (2) are mainly decided by T alone and are hard to optimize. However, the pitch motion of the supporting legs as shown in Fig. **??** can be optimized locally. Let the simplified equation of motion be represented in the form of a linear combination of x_i and $\ddot{x}_i$ as follows:

$$u_j = \sum_{k=1}^{2}(h_{jk}\ddot{x}_k + g_{jk}x_k) \quad (j = 1 \ldots 2). \tag{24}$$

Minimizing (21) is equivalent to minimizing (25):

$$E = \int_0^{\frac{T}{2}} \sum_{j=1}^{2} {u_j}^2 dt. \tag{25}$$

Fig. **??** shows the motion obtained using Galerkin's method. The height h increases to reduce the load of the weight and the floor reaction force.

6.2.3 Torques.

To determine the necessary torque u for the planned trajectory x from (3), another condition must be introduced as mentioned in Section 2.3 One of the conditions is

$$\sum_{i=1}^{k} {u_i}^2 \rightarrow min. \tag{26}$$

This condition minimizes the energy consumption defined in Section 6.1 for a given x. Moreover, this condition prevents the horizontal reaction forces from becoming large by the unnecessary straddle and helps satisfy condition (b.2) mentioned in Section 2.2. Torque u satisfying (3) and (26) can be obtained by solving the following equations using Lagrange's multiplier method:

$$\begin{bmatrix} QC & 0 \\ 2I & -(QC)^t \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = \begin{bmatrix} Q(A\ddot{x} - g) \\ 0 \end{bmatrix}. \tag{27}$$

7 RESULTS OF CALCULATION

Fig. **??** shows the energy consumption P calculated from (21) and (22) for trot , pace and the static walk (crawl gait, $\alpha = 0.75$). In each figure, the point located on the extreme right for a given period is the maximum speed attainable for the period.

8 EXPERIMENT AND DISCUSSION

8.1 Results of experiments

Fig. ?? shows the results of experiments corresponding to Fig. ??. Fig. ?? shows photographs of Collie-2 walking in trot and pace.

8.2 Effect of the period

8.2.1 Period and stability.

In trot, the walk becomes unstable as the stride and period increase. In pace, the walk becomes unstable as the period increases. When the period is larger than 0.9 s, Collie-2 cannot continue to walk stably.

8.2.2 Period and maximum speed.

The maximum speed, V_{Gmax}, of Collie-2 increases as the period increases for the reason given in Section 5.

8.2.3 Period and energy consumption.

The period $T = 0.8$ s which makes the energy consumption P minimum in trot in Fig. ?? coincides with the calculated results of Fig. ??.

8.3 Comparison of the gaits

8.3.1 Gaits and energy consumption.

Trot is the best when considering the energy consumption.

8.3.2 Gaits and maximum speed.

In the static walk, the maximum speed is very low because the duty factor cannot exceed 0.75. In trot, increasing the stride causes instability. Then T_{max} must be decreased to keep the dynamic walk within bounds of stability. This causes the maximum speed to decrease.

8.3.3 Condition of gait selection.

We can say the follows from the previous discussions. In the case of Collie-2, trot is better in view of the energy consumption at the speed where both trot and pace are possible. Pace is required at the speed where trot is not possible.

9 CONCLUSION

Walking is a complicated motion with many degrees of freedom. There are many kinds of parameters in planning and realizing the walk (for example, the duty factor and phase difference between the legs which determine the gait, speed, period, stride, height of body, the condition needed to determine torques, etc.). In order to take advantage of the many degrees of freedom, some indices must be introduced in the walk. In this paper, three criteria have been pointed out as useful indices for evaluating the dynamic walk of a quadruped robot. They are '*stability*','*maximum speed*' and '*energy consumption*'. The relations between these three criteria and the parameters for the dynamic walk were formulated. The following conclusions have been obtained and verified by experiments using Collie-2:

(1) The period is a very significant parameter. It is desirable to walk with a longer period and a wider stride in order to increase the maximum speed. But there exists a maximum value of th period at which stable dynamic walk can be realized. The maximum speed is limited by this maximum period. There exists a period in which energy consumption becomes a minimum for a given speed.

(2) The three basic gaits are trot, pace and bound. The gait greatly affects these criteria. Trot is desirable as long as it is available. Otherwise, pace is recommended.

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Appendix

A REDUCING THE EQUATIONS OF MOTION

Consider the elimination of $E^t f$ from (29) under (28):

$$e(x) = 0 \quad (\in \mathbf{R}^m) \tag{28}$$

$$A(x)\ \ddot{x} = Cu + g(x) + E^t f \quad (\in \mathbf{R}^n) \tag{29}$$

$$E = \partial e/\partial x = [e_{n-m+1}, .., e_n]^t. \tag{30}$$

By differentiating (28), $E\dot{x} = 0$ is derived. Upon further differentiation, the following is derived:

$$\dot{E}\dot{x} + E\ddot{x} = 0 \tag{31}$$

We express this in general as

$$h(x, \dot{x}) + E\ddot{x} = 0 \tag{32}$$

Here we choose the vectors $e_1 \cdots e_{n-m}$ which are independent of $e_{n-m+1} \cdots e_n$ and of each other, and let

$$E_p = [e_1 \cdots e_{n-m}]^t, \quad E^* = [e_1 \cdots e_n]^t. \tag{33}$$

Then we can get the following:

$$E^* = \begin{bmatrix} E_p \\ E \end{bmatrix} \qquad E^*\dot{x} = \begin{bmatrix} E_p\dot{x} \\ 0 \end{bmatrix} \qquad E^*\ddot{x} = \begin{bmatrix} E_p\ddot{x} \\ -h \end{bmatrix} \tag{34}$$

From (29), we get

$$\ddot{x} = A^{-1}(Cu + g) + A^{-1}E^t f. \tag{35}$$

Substituting (35) into (32), we obtain

$$h + EA^{-1}(Cu + g) + EA^{-1}E^t f = 0. \tag{36}$$

From this, we obtain

$$f = -(EA^{-1}E^t)^{-1}\{EA^{-1}(Cu + g) + h\}. \tag{37}$$

We can determine the constraint forces f from (37) when the torques u are given. Furthermore, by substituting (37) into (35), we get

$$\ddot{x} = \{A^{-1} - A^{-1}E^t(EA^{-1}E^t)^{-1}EA^{-1}\}(Cu + g) - A^{-1}E^t(EA^{-1}E^t)^{-1}h \tag{38}$$

We can determine the acceleration $\ddot{x}$ from (38) when the torques u are given.

When multiplying both sides of (38) by E^*, the part derived by multiplying E is automatically satisfied. Therefore, the relation between x and u is reduced to the following equation with $n - m$ dimension which is obtained by multiplying (38) by E_p:

$$QCu = Q(A\ddot{x} - g), \tag{39}$$

where

$$P = A^{-1} - A^{-1}E^t(EA^{-1}E^t)^{-1}EA^{-1}, \qquad Q = E_pP. \tag{40}$$

B SIMPLIFYING THE EQUATIONS OF MOTION AROUND THE ROLL AXIS

The equation of motion of the inverted pendulum shown in Fig. ??b is written as

$$\begin{bmatrix} J_1 + m_1{p_1}^2 + m_2{l_1}^2 & m_2 l_1 p_2 \cos(\xi_1 - \xi_2) \\ m_2 l_1 p_2 \cos(\xi_1 - \xi_2) & J_2 m_2 {p_2}^2 \end{bmatrix} \begin{bmatrix} \ddot{\xi}_1 \\ \ddot{\xi}_2 \end{bmatrix}$$

$$= \begin{bmatrix} -u_2 \\ u_2 \end{bmatrix} + \begin{bmatrix} (m_1 p_1 + m_2 l_1) g \sin\xi_1 \\ m_2 p_2 g \sin\xi_2 \end{bmatrix}. \tag{41}$$

To lift the swinging legs, the angle of the body must satisfy $\xi_2 \geq 3/2\pi$. Considering the collision with the floor, the following trajectory is used here:

$$\xi_2 = \xi_{2max} \sin^2(\frac{\pi}{T_0} t) + \frac{3\pi}{2}, \tag{42}$$

$$0 < \xi_{2max} < 0.1(radian), \quad T_0 = T/2. \tag{43}$$

For simplicity, we let $\xi_{2max} = 0$. Then by eliminating u from (41) and linearizing it, we get the following:

$$\ddot{\xi}_1 = a\xi_1 - b, \tag{44}$$

$$J = J_1 + m_1{p_1}^2 + m_2{l_1}^2, \ a = \frac{(m_1 p_1 + m_2 l_1) g}{J}, \ b = \frac{m_2 p_2 g}{J}. \tag{45}$$

C FORMULATING THE MAXIMUM SPEED

Using the simple one-link model (Fig. ??), the maximum inertial torque U_{max} needed to accelerate and decelerate the swinging leg is formulated as follows.

U_{max} is proportional to the maximum acceleration of the link:

$$U_{max} = J \times max(\ddot{\theta}), \tag{46}$$

where

J : inertia of the swinging leg,

$\theta(t)$: angle of the swinging leg,

$max(\ddot{\theta})$: maximum value of $\ddot{\theta}(t)$ $\quad (0 \leq t \leq T_0, T_0 = T/2)$.

Let $\theta(t)$ be expressed as a cubic function satisfying the continuity of the angle and angular velocity at the time $t = 0$, $t = T_0$ as follows:

$$\theta(0) = \frac{s}{2l}, \quad \dot{\theta}(0) = \frac{V_G}{l} = \frac{S}{lT_0}, \tag{47}$$

$$\theta(T_0) = -\theta(0), \quad \dot{\theta}(T_0) = \dot{\theta}(0). \tag{48}$$

Then the maximum acceleration $max(\ddot{\theta})$ is written as

$$\begin{aligned} Max(\ddot{\theta}) &= \ddot{\theta}(0) \quad or \quad \ddot{\theta}(T_0) \\ &= |\frac{6(2\theta(0) + T_0\dot{\theta}(0))}{{T_0}^2}| \end{aligned}$$

$$= \frac{48S}{lT^2}. \tag{49}$$

By substituting (49) into (46), the following is obtained:

$$U_{max} = \frac{48JS}{lT^2}. \tag{50}$$

As shown above, the maximum torque U_{max} is directly proportional to the stride S and inversely proportional to the square of the period T.

D FORMULATING THE ENERGY CONSUMPTION

The consumed energy E_e is defined by (21). From this, we can say that E_e is proportional to the square of the torque and to the period. Here we assume that the consumed energy E_e defined by (21) can be divided into four terms described in Section 6.2.1. We write this as follows:

$$E_e = E_{swing} + E_{up} + E_{support} + E_{body}. \tag{51}$$

Each term can be represented by the speed V_G and the period T through simple consideration of the dynamics as follows.

(a) *Energy to swing the legs forward.* We assume that the inertia torque to swing a leg forward is directly proportional to the stride S and inversely proportional to the square of the period T throughout the motion. Then we obtain

$$E_{swing} = Const. \times (\frac{S}{T^2})^2 \times T = C_{sw}\frac{{V_G}^2}{T}. \tag{52}$$

(b) *Energy to swing the legs up.* Like to E_{swing}, we assume that the inertia torque to swing a leg up is inversely proportional to the square of the period T throughout the motion for a given swinging up height ΔH. Then we obtain

$$E_{up} = Const. \times (\frac{1}{T^2})^2 \times T = C_{up}\frac{1}{T^3}. \tag{53}$$

(c) *Energy to keep the supporting legs from falling down.* As it is not necessary to accelerate or decelerate the body and supporting legs in the steady walk, the energy to compensate for gravity is dominant. We assume that the torque to support a leg is directly proportional to the stride S when the stride is small. Then we obtain

$$E_{support} = Const. \times S^2 \times T = C_{sp}T^3{V_G}^2. \tag{54}$$

(d) *Energy to keep the body from falling down.* We assume that the torque to support the body including swinging legs is constant in the two-leg supporting phase of pace. Then we obtain

$$E_{body} = Const. \times T = C_{body}T. \tag{55}$$

From (51)$\sim$(55) and (22) we obtain (56)

$$P = C_{sw}\frac{V_G}{T^2} + C_{up}\frac{1}{V_G T^4} + C_{sp}T^2V_G + \frac{C_{body}}{V_G}. \tag{56}$$

It is confirmed by calculation that the assumptions made here are valid and that the coefficients of (56) are constant independently of the speed V_G and the period T. The results are shown in Table 3.