Sujitkumar V. Naik
e-mail: sujitkvn@gmail.com

Anupam Saxena[1]
e-mail: anupams@iitk.ac.in

Indian Institute of Technology,
Kanpur 208016, India

Ashok Kumar Rai
Sr. Software Engineer
Aon Hewitt,
Techbolevard Tower A, Sector 127,
Noida 201304, India
e-mail: rai284@gmail.com

B. V. S. Nagendra Reddy
Indian Institute of Technology,
Kanpur 208016, India
e-mail: bvsnagendrareddy@gmail.com

How to Choose From a Synthesized Set of Path-Generating Mechanisms

Partially compliant mechanisms inherit the attributes of fully compliant and rigid-body linkages and offer simpler, compact design alternatives to accomplish complex kinematic tasks such as tracing large nonsmooth paths. This paper describes qualitative and quantitative criteria that can be employed to select the linkage configuration. The proposed criteria are categorized as general or specific. General criteria pertain to often-used kinematic attributes whereas specific criteria address the application at hand. The veracity and viability of each mechanism are evaluated with respect to compactness, design simplicity, static and dynamic failure, number of rigid-body joints, relative ease of fabrication, and other relevant criteria. Three decision-making techniques, namely, Pugh decision matrix, analytic hierarchy process, and a variant of the Pugh decision matrix are used to perform the evaluation. An example of a displacement-delimited gripper with a prescribed large nonsmooth path is used to illustrate linkage selection. [DOI: 10.1115/1.4004608]

Keywords: linkage selection, ranking, criteria, path generating mechanisms, evaluation methods, decision matrix, partially compliant mechanisms

1 Introduction

From a set of monolithic, partially compliant, and rigid-link mechanisms designed for a prescribed task, choosing the best linkage may not always be straightforward. A monolithic mechanism, if available, is an undisputed choice as it is simple, and free from friction and backlash. The absence of hinges makes fully compliant mechanisms easier to fabricate, maintain, and miniaturize. On the contrary, high stresses due to large deformation and low fatigue life are some factors that may not favor their choice. If only partially compliant linkages are available, the one having minimum number of hinges can be chosen. A four-bar rigid-link mechanism may be preferred over a partially compliant linkage of comparable size if they offer the same functionality and if the number of hinges in the latter is significantly large. Thorough analysis may be required for more involved linkages in terms of criteria like functionality, compactness, complexity, failure, and many others. Deduction of a suitable linkage thereafter can further necessitate intricate decision making. Here, we present a systematic approach to linkage ranking and selection that employs Pugh's decision matrix, analytic hierarchy process, and variant of the Pugh method. Various design and selection approaches used previously for the three linkage types are presented as under.

1.1 Partially Compliant Mechanisms. The presence of one or more flexible links makes a partially compliant mechanism remarkably versatile. Such linkages can trace a variety of paths that may or may not have slope (C^1) discontinuities. A compact and topologically simpler, partially compliant linkage can trace a prescribed C^1 discontinuous (or nonsmooth) path accurately [1]. However, a rigid-body linkage designed to trace a similar path can be bulky and complex [2,3]. Shoup and McLarnan [4] present a basis to choose partially compliant mechanisms that are derived from single, closed-loop rigid-body linkages having degrees-of-freedom (dof) less than one. Winter and Shoup [5] note that the geometric capability of a four-bar linkage can be enhanced significantly if one of its links is flexible. They further envisage the existence of numerous design possibilities if multiple loops and flexible members are permitted. Sriram and Mruthyunjaya [6] perform dimensional synthesis of four-bar linkages with a single flexible link. Ramrakhyani et al. [7] perform concurrent topology and size synthesis of small deformation partially compliant mechanisms. Rai et al. [1] extend this approach to synthesize large-displacement mechanisms that can trace a range of prescribed nonsmooth paths.

1.2 Rigid-Link and Fully Compliant Mechanisms. Numerous techniques, e.g., shown in Refs. [8–11], can be used to synthesize four-bar rigid-link mechanisms. Artobolevskii [12] uses sliders and turning pairs extensively to enumerate path-generating mechanisms. Kempe [2] presents an analytical method using algebraic geometry to synthesize a generic, planar rigid-link mechanism that can trace any curve described by $f(x, y) = 0$. Kempe's linkages can be bulky and complex. The number of links is of the order $O(n^4)$ where n is the degree of the algebraic planar curve [3]. Genetic algorithms are employed in Refs. [13–15] for topology and dimensional synthesis of rigid-link mechanisms. Freudenstein and Maki concept of *separation of structure from function* [16] is used in Refs. [17,18] to generate valid, nonisomorphic closed chains.

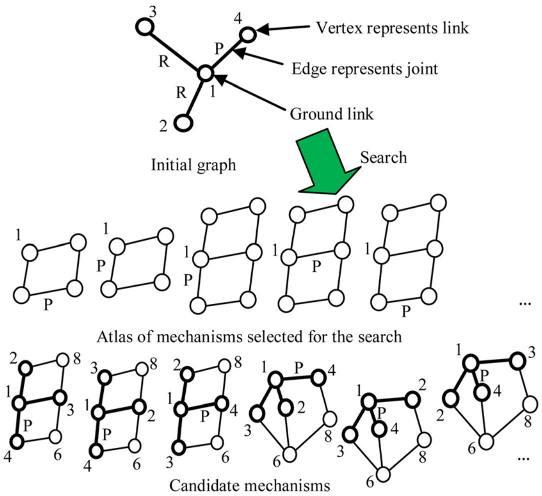
Extensive previous efforts over the last two decades focus on developing design methodologies for small and large deformation fully compliant mechanisms [7,19–29]. Complex paths can be traced by monolithic mechanisms if contact is permitted between various adjacent member-pairs during deformation. These contact interactions can either be prescribed (e.g., Ref. [29]) or determined through a systematic design process.

1.3 Unified Synthesis Method. Pucheta and Cardona [18] propose a type synthesis approach for rigid-body mechanisms wherein valid, nonisomorphic solutions are extracted from an atlas of mechanisms using an initial graph [Fig. 1(*a*)].

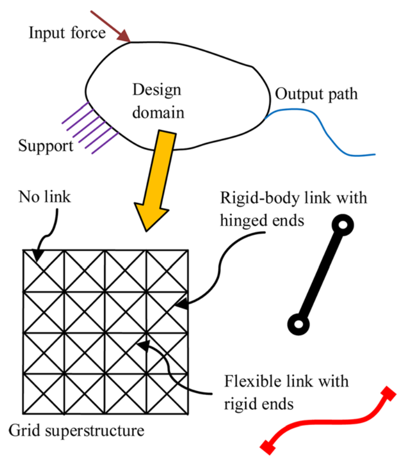
The approach involves enumeration and codification. Pucheta and Cardona [30] extend their method to synthesize compliant mechanisms by using two sequential stages of type (similar to [18]) and dimensional synthesis. Few mechanisms are identified in the type synthesis stage and their dimensions are optimized

[1]Corresponding author.

Contributed by the Mechanisms and Robotics Committee of ASME for publication in the Journal of Mechanical Design. Manuscript received September 15, 2010; final manuscript received June 16, 2011; published online September 15, 2011. Assoc. Editor: Ashitava Ghosal.



(a) Sequential Synthesis Method by Pucheta and Cardona [18]



(b) Concurrent synthesis with the Unified procedure [1]

Fig. 1 Comparison between the Synthesis methods: (*a*) Sequential Synthesis Method by Pucheta and Cardona. The initial graph specified by the user has four links (vertices 1–4) connected by revolute pairs (edges R) and a prismatic pair (edge P). A search is executed through the atlases that contain graphs of mechanisms consisting of revolute joints and one prismatic joint only. No other atlas is searched. Candidate mechanisms are selected based on the occurrence of the initial graph in the atlas. (*b*) Concurrent synthesis with the Unified procedure by Rai et al. A user prescribes only the design space, a monotonic input and the path to be traced. The design domain is represented as a grid superstructure. Each edge can be present or absent. If present, it can represent a deforming member with fixed ends or a rigid member with hinged ends. The link geometry comprising in-plane widths, out-of-plane thickness, end nodes, and slopes (only for deforming members) is evolved iteratively so that the prescribed kinematic goal is attained.

subsequently so that the kinematic errors are minimized. This is again performed in two steps wherein concepts similar to the pseudorigid-body model approach [31] are employed.

In contrast, Rai et al. [1] implement a unified approach that can concurrently determine the link-type, topology, and dimensions (shape and size) of path-generating mechanisms [Fig. 1(*b*)]. Monolithic, partially compliant, and rigid-link mechanisms can be synthesized using this approach. Rai et al. [1] show that nonsmooth paths with kinks at the desired locations can be traced by the resulting partially compliant mechanisms. Their approach can also be modified to design linkages for function and motion generation applications.

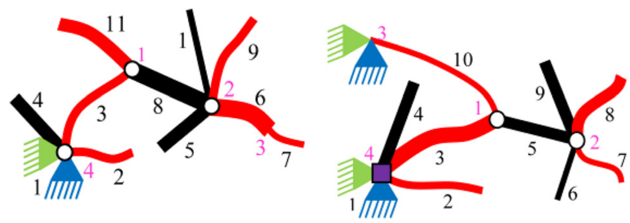
1.4 Linkage Selection.

Specific case studies for selection of mechanisms are presented previously, e.g., casement window [9], excavator, or mechanical shovel mechanisms [32], and single, closed-loop linkages with only three rigid-body joints [4]. For generic rigid-link mechanisms, Pucheta and Cardona [33] use *topological* and *qualitative* performance indicators to select the best linkage from a given set. Mechanisms are ranked via lexicographic preferences using one indicator at a time. The values of the topological indicators (e.g., number of links or joints) are compared directly whereas weights are assigned by the user to quantify the qualitative ones. The sum of the weights of qualitative indicators is augmented with the topological index. This *global index* is then employed to rank the rigid-link mechanisms.

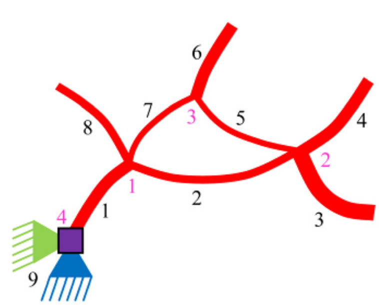
1.5 Motivation and Organization.

Given a set of generic rigid-body and compliant mechanisms that can perform a prescribed kinematic task, choosing the best linkage can depend on a variety of factors. These criteria can be formulated in terms of functionality, complexity, manufacturing ease, compactness, life, maintenance, precision, reliability, cost, and others. Such criteria are considered previously [4,33] but applied only to specific mechanism types. This work extends the use of quantitative and qualitative factors to rank generic rigid-link, partially and fully complaint mechanisms and choose the best linkage from amongst them. It further takes into purview additional criteria like the stresses induced in flexible links and fatigue failure to accommodate functional life and reliability of the linkages. Application-specific criteria are considered as well.

The paper is organized as follows. Section 2 enumerates various *general* and *specific* criteria to compare different linkage types. A brief description of Pugh's method [34], analytic



(a) $n_r = 5$, $n_f = 2$, $j_h = 6$ **(b) $n_r = 5$, $n_f = 2$, $j_h = 6$**



(c) $n_r = 0$, $n_f = 1$, $j_h = 0$

Fig. 2 Illustration of counting of links and joints in partially and fully compliant linkages. Rigid and flexible links are shown in black (dark) and red (gray), respectively. Hinges are shown as white circles. Triangles and square represent fixed displacement and no change-of-slope boundary conditions, respectively. (*a*) links 2, 3, 11, and 6, 7, 9 form two units of flexible members; (*b*) links 2, 3, 10, and 7 and 8 form two units of flexible members; (*c*) links 1–8 form a single unit of flexible links.

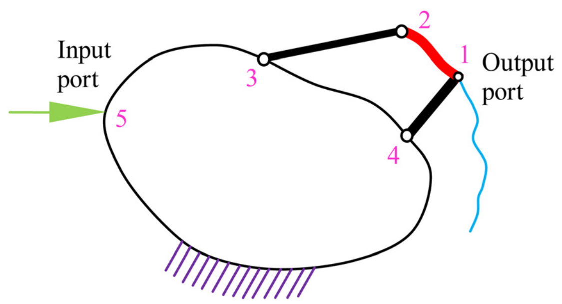


Fig. 3 Illustration of presence of redundant degrees-of-freedom in a partially compliant linkage

hierarchy process (AHP) [35], and variant of the Pugh method [36–38] is given in Sec. 3. In Sec. 4, partially compliant, monolithic, and rigid-link mechanisms synthesized for a prescribed *L*-shaped path are analyzed. Discussion is presented in Sec. 5, based on which the best linkage is identified. The paper concludes in Sec. 6.

2 Criteria to Compare Linkages

The criteria for comparing linkages can be classified into two categories—general and specific.

2.1 General Criteria. The General criteria to compare linkages are primarily topological and are elucidated as under.

(i) **Explicit Presence of Additional Degrees of Freedom (P_i):** To accommodate deformable members in a mechanism, Shoup and McLarnan [4] modify Grübler's mobility criterion as

$$N = 3(n_r + n_f - 1) - 2j_h \tag{1}$$

where N represents the *dof*, n_r and n_f are the number of rigid links and units of flexible links, respectively, and j_h is the number of hinges. A group of flexible links is considered as a single unit if they form a continuum across the linkage nodes. Computation of N is illustrated for various cases in Figs. 2(*a*)–2(*c*). The modified Grübler criterion usually gives the *dof* for partially or fully compliant linkages as zero or negative [1,4]. Although these linkages have infinite *dofs* due to the presence of flexible links, they become determinate once the forces acting on the members are known *a priori* [39–44].
The goal is not to employ the *dofs* computed from (1) in the evaluation process directly but rather, it is to use them to identify the presence of redundant *dofs*. Visual inspection of some linkages may reveal the explicit presence of redundant *dofs* such as a four-bar chain shown in Fig. 3. One of the nodes involved may be the output port. Presence of such a chain can also be verified by perturbing the output port and observing no displacement at the input node. Such a solution can be discarded either during or after the synthesis stage.

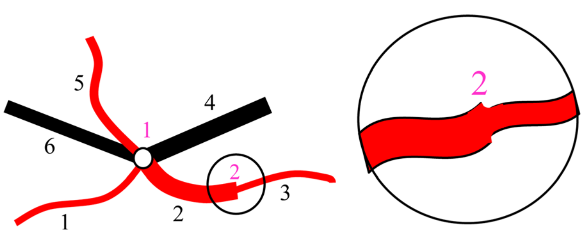


Fig. 4 Presence of a discontinuity in a flexible unit (Node 2)

(ii) **Number of links (N_l):** This topological criterion alludes to the complexity of a linkage [33]. Existence of less number of links is preferred as it suggests simpler linkage design, ease in assembly, time saving, reduced weight and hence the overall reduction in cost [31,p. 2]. This criterion is vital when choosing a fully compliant linkage over the rigid-body one with identical kinematic capability.

(iii) **Lengths of the longest and shortest links (L_m and L_s):** The length of the longest link along with the number of links influences the compactness or area of operation of a linkage. Usually, mechanisms with small link lengths are preferred. Links that are undesirably long can make a linkage bulky (e.g., Ref. [2]). Pucheta and Cardona [33] consider only the shortest length as a quantitative criterion. Short rigid links should have sufficient length to accommodate hinges to avoid manufacturing and assembly difficulties.

(iv) **Number of hinges (N_h):** Less number of hinges is desirable for reduced wear and need for lubrication [31,p. 2] for easier maintenance of the linkage. Precision is enhanced and, vibration and noise are reduced which improves the mechanism's reliability. Fabrication and assembly are also easier leading to reduced manufacturing time and the overall cost. This topological criterion is also decisive in selecting a fully compliant linkage over the rigid-body one.

(v) **Maximum number of hinges at a given location (N_{nh}):** It is desirable to have a uniform distribution of hinges over the entire rigid-link or a partially compliant mechanism. The pin lengths in such cases will be uniform and

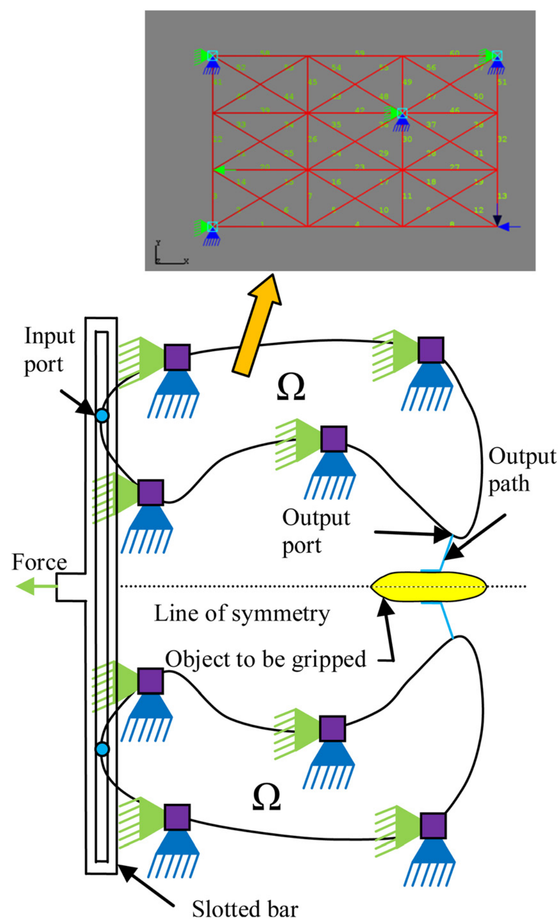


Fig. 5 Schematic diagram for the design of the displacement-delimited gripper

bending stresses in them will be lower. A number of hinges lumped at a particular node (location) suggests inferior design in terms of complexity, manufacturability, and operability. One hinge per node is ideal for a mechanism that has hinges. Pucheta and Cardona [33] do not consider this criterion.

(vi) **Compactness (C_m):** A linkage occupying as less space as possible, both in its rest and active states, is preferred. The total area spanned by a linkage is that swept between its initial and final positions. Pucheta and Cardona [33] employ permitted space violation condition as one of the qualitative criteria.

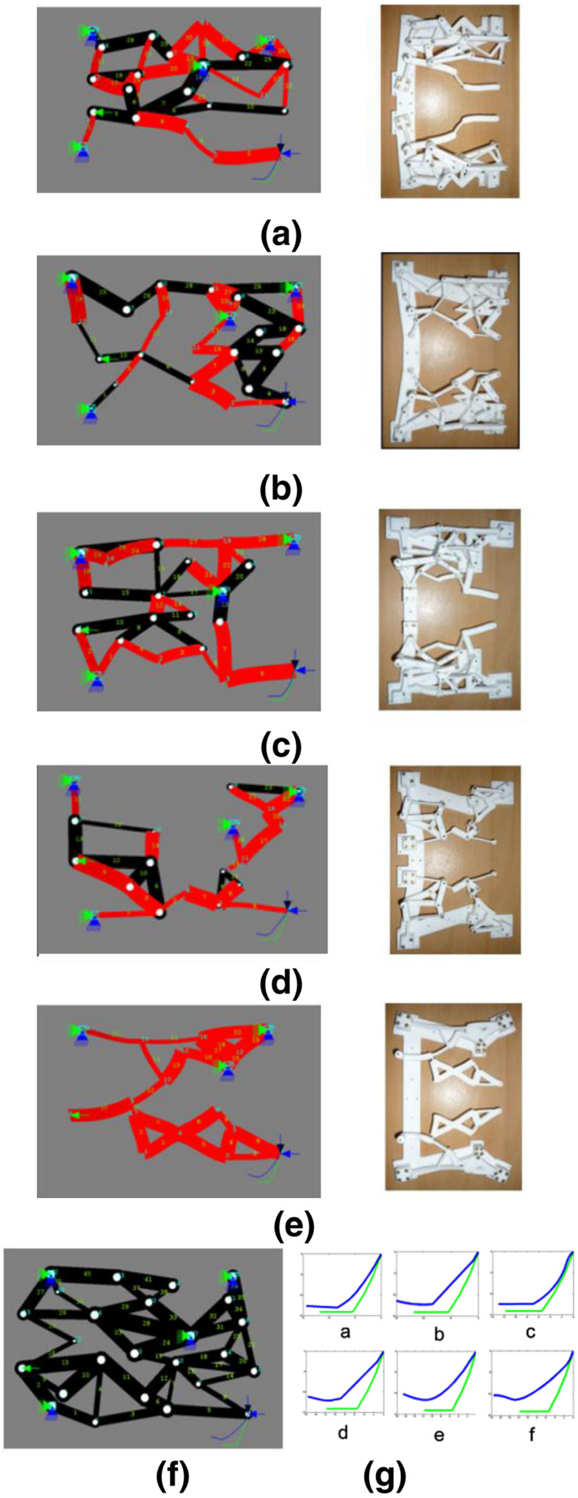


Fig. 6 (*a-f*) Synthesized solutions for the displacement-delimited gripper selected for comparison and the respective prototypes fabricated using Teflon; (*g*) shows the magnified prescribed path for all solutions in (*a-f*) in green (light gray), and traced path in blue (dark gray).

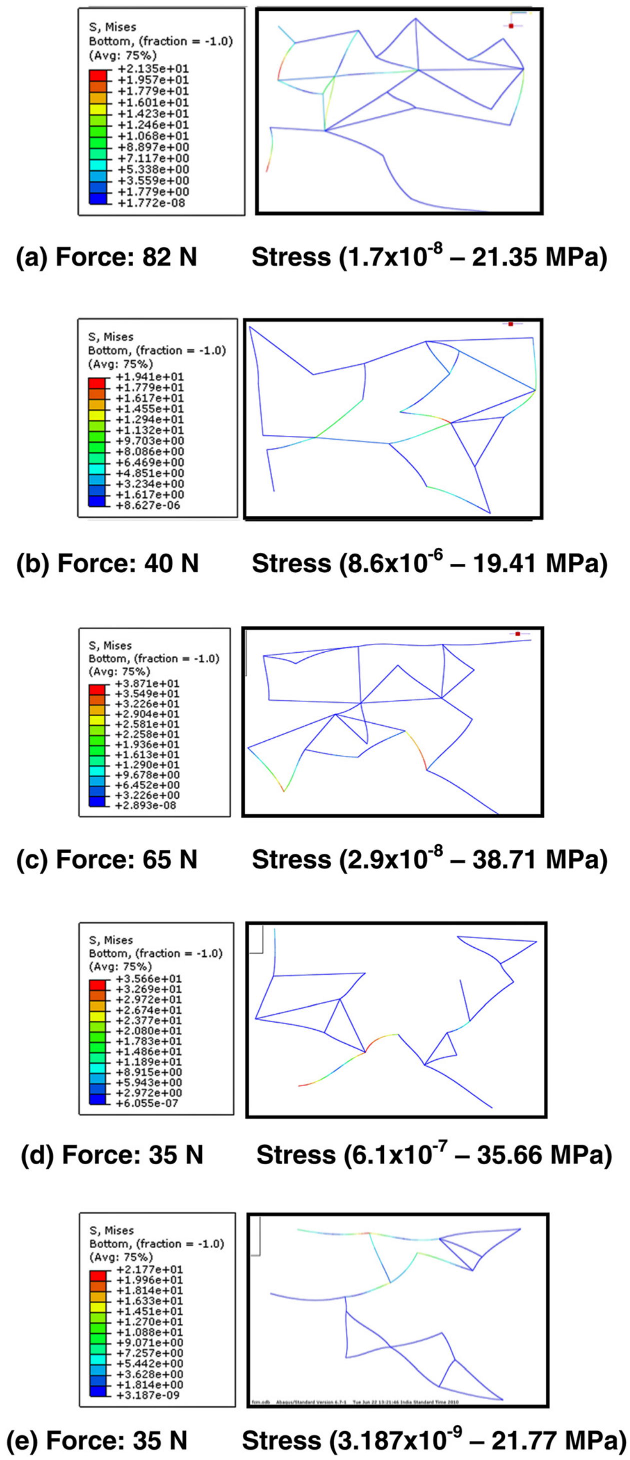


Fig. 7 (*a-e*) Quasi-static, large deformation stress analysis of M1–M5 in Figs. 6(*a*)–6(*e*) using ABAQUS®. (a) Force: 82 N–Stress (1.7×10^{-8}–21.35 MPa), (b) force: 40 N–stress (8.6×10^{-6}–19.41 MPa), (c) force: 65 N–stress (2.9×10^{-8}–38.71 MPa), (d) force: 35 N–stress (6.1×10^{-7}–35.66 MPa), (e) force: 35 N–stress (3.187×10^{-9}–21.77 MPa).

(vii) **Quasi-static stresses in flexible links (F_s):** For partially and fully compliant mechanisms, stresses in the deforming members should be within the permitted elastic limit(s). High stresses have negative effects on the durability or functional life of the linkage. Existence of stress concentration regions, i.e., those indicated by the sudden change in cross section (Fig. 4) can also be critical. These regions can be mitigated via radial fillets. However, local stiffness can get altered which can adversely affect the kinematic behavior of the mechanism.

(viii) **Fatigue failure (F_f):** Fatigue strength is an important strength-based criterion that can be used to estimate the functional life of a linkage. Incorporating fatigue failure as a constraint in linkage synthesis is not recommended. This is because generic simulation of fatigue failure can be difficult due to the uncertainties and numerous assumptions involved, and it can be temporally expensive. Instead, one can synthesize a set of linkages and then perform experimental fatigue tests to determine their life. This again is time-consuming. Criteria (vii) and (viii) are quite relevant in selection of partially and/or fully compliant mechanisms as many members may undergo considerable cyclic deformation to perform a given kinematic task repeatedly.

2.2 Specific Criteria. The specific criteria are uniquely relevant to the kinematic application under consideration and can vary with a prescribed set of goals. For a set of displacement-delimited gripper mechanisms [29] presented in Sec. 4, these criteria are as under:

(ix) **Kinematic performance (P_k):** An important application-specific criterion for a path-generating linkage requires that the path traced by the output port must be close to the path prescribed. The path specified for the gripper (Sec. 4) is *L*-shaped with a kink present at a desired location. The output port is deemed to approach a soft workpiece (e.g., a biological specimen) and later change its direction sharply after establishing contact. This sudden change in direction is imposed to prevent damage to the work piece due to high reaction forces. Mechanical advantage of the displacement-delimited gripper is thus negligible as the reaction forces are expected to be very small. In general, mechanical advantage should be considered as a separate criterion. Nevertheless, displacement-delimited linkages that do not have kinks in their output paths can be discarded using this criterion itself. It is assumed that some linkages that can trace the *L*-shaped path are available from the synthesis. Otherwise, linkages that trace smooth paths (i.e., those without the kinks) closest to the *L*-shaped path can be considered. Pucheta and Cardona [19,33] address this qualitative criterion in the dimensional synthesis stage.

(x) **Non-interference between the linkage portions across the line of symmetry (I_n):** Due to the symmetry in the gripper mechanism, only its top symmetric part is designed. It is expected that members of the two symmetric halves do not interfere after assembly and during deformation. Thus, after the linkages are synthesized, this criterion is employed to reject solutions wherein interference is observed.

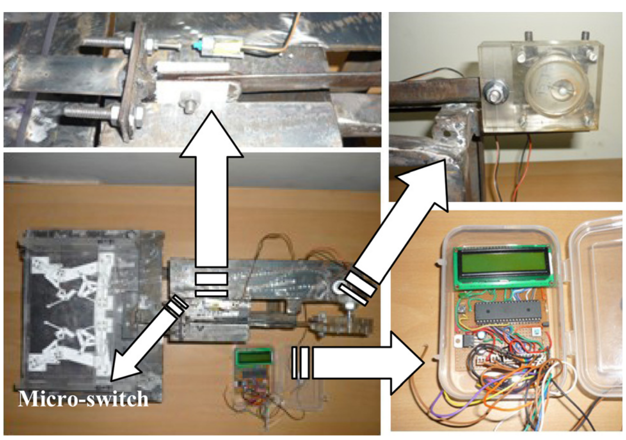


Fig. 8 Bottom left: Fatigue testing machine, top left: guide way, top right: eccentric mounted on a motor, bottom right: enlarged view of the counter circuit

3 Evaluation Methods for Synthesized Linkages

A number of techniques can be employed to choose the best linkage using the criteria in Sec. 2. Most rely on user input to convert qualitative (subjective) assessments into the respective quantitative (objective) ones. Some of these methods are described below:

3.1 Pugh Decision Matrix (PDM). In PDM [34], linkages are organized in columns and the criteria are placed in rows. One linkage from the set is chosen as the *datum* and the others are compared with it. The ratings of -1, 0, or $+1$ used imply that the linkage is worse than, equal to, or better than the datum, respectively. The overall positive and negative ratings are recorded separately for each linkage.

3.2 Analytic Hierarchy Process. In AHP [35], pair-wise comparison among all n criteria is performed first. Ratings a_{ij} for all criteria are arranged in n rows and n columns. a_{ij} is chosen as 1 if a user reckons criteria Q_i and Q_j to be equally important. a_{ij} is chosen as 3, 5, 7, or 9 if Q_i is marginally strong, strong, very strong, or extremely strong, respectively, compared to Q_j. $a_{ij}=2$, 4, 6, or 8 are the intermediate ratings. a_{ji} is the reciprocal of a_{ij} as it compares Q_j with Q_i. Also, $a_{ii}=1$ which is consistent with the rating scheme. Weights W_i for each criterion can be extracted from $\mathbf{A}_{n\times n}=[a_{ij}]$ in two ways. (a) One can use the normalized principal eigenvector for W_i. (b) Alternatively, one can normalize the columns of $\mathbf{A}$ and then compute the average of the elements of each row, which can be used as W_i.

Next, pair-wise comparison of all solutions is performed with respect to each criterion similar to the above. The linkage ratings with respect to that criterion are determined and normalized. The overall rating RO_i for the ith linkage is obtained as

$$\mathrm{RO}_i = \sum_{j=1}^{n} Rw_{ij}W_j, \quad i = 1, \ldots, m \tag{2}$$

where Rw_{ij} is the normalized rating for the ith linkage with respect to Q_j and m is the number of linkages.

3.3 Variant of Pugh Decision Matrix. This method [36–38] combines the PDM and AHP above. It employs the criteria weights from the AHP but the linkage ratings from PDM. The overall positive and negative ratings for each linkage are computed using Eq. (2) and the best linkage is one with the highest net positive rating.

4 Example: Displacement-Delimited Gripper

The design specifications for the displacement-delimited gripper are shown in Fig. 5. The linkage is deemed to be bounded within a square region of 100 mm $\times$ 60 mm. An L-shaped path of length 22 mm is prescribed for the output port as shown. The material chosen is Teflon with elastic modulus of 535 N mm^{-2} and yield stress of 23 N mm^{-2}. Fully, partially compliant, and rigid-

Table 1 Data for analysis

(a) Data for analysis based on General criteria. Allowable yield stress for Teflon: 23 MPa

Linkage	N_l	L_m	L_s	N_h	N_{nh}	C_m	F_s (MPa)	F (N)
M1	15	42.0	9.52	24	04	8023	21.35	82
M2	19	42.3	16.2	26	03	8435	19.41	40
M3	14	36.5	14.8	22	04	8143	38.71	65
M4	12	39.3	10.5	16	03	9258	35.66	35
M5	01	–	–	00	00	7855	21.77	35
M6	41	46.6	8.69	61	05	8752	–	–

(b) Result of the fatigue test

Linkage	Distance between the output ports (mm)		Difference (mm)
	Before test	After test	
M1	43	35	7
M2	47	41	6
M3	55	40	15
M4	45	39	6
M5	36	30	6

(c) Data for analysis based on specific criteria

Linkage	Kinematic performance (P_k)	Interference of linkage portions across the line of symmetry (I_n)
M1	Yes	No
M2	Yes	No
M3	Yes	No
M4	Yes	No
M5	No	No
M6	No	No

Key: $\mathbf{N_l}$—Number of links, $\mathbf{L_m}$—Length of the longest link in mm, $\mathbf{L_s}$—Length of the shortest link in mm, $\mathbf{N_h}$—Number of hinges, $\mathbf{N_{nh}}$—Maximum number of hinges at a given node, $\mathbf{C_m}$—Compactness in terms of the area swept in mm^2, $\mathbf{F_s}$—Maximum stress induced in N/mm^2, and **F**—The input force magnitude.

body linkages are synthesized using the unified procedure [1]. Presence/absence of the links and link-type (rigid or deformable) is controlled by the binary design variables. In-plane widths, nodal positions, end slopes of deforming members, and magnitudes of input forces are other real valued decision variables. The in-plane widths are sought between (2, 8) mm and end slopes between (−0.3, 0.3) radians. The out-of-plane thickness for each existing member is chosen as 6 mm. The input force is monotonic and bounded within (5, 100) N. Repositioning of all nodes (except for the output port) is allowed within local rectangular regions of size 15×10 mm^2.

Fourier shape descriptors (FSDs) are used to compare the path shapes [1,45]. For better accuracy, the first 20 coefficients are used. The least squared differences between the FSDs of the design and desired paths, and the same of the path lengths are minimized in the weighted linear combination setting. The coefficients for shape errors are chosen as 10 each and that for the discrepancy in path length is 0.1 [1].

The existence of a corner point on the path cannot be verified by the optimized value of the objective. This is because FSDs are approximate and capture the path's shape based on the discrete placement of points. One can use fine discretization of the paths and compute slopes using finite differences. A sudden change between two consecutive slopes can be used as a quantitative measure for the kinematic criterion (P_k) to detect a corner point.

Thirty solutions each for fully compliant, partially compliant, and rigid-body linkages are generated separately using the stochastic hill-climber search. 10,000 generations are employed for each linkage type. When synthesizing fully and partially compliant linkages, the CPU times with Intel Pentium 4 processor (2.60 GHz) are about 15.3 h and 13.2 h, respectively. For rigid-body linkages, the CPU time with Intel Core 2 Duo E7300 processor (2.66 GHz) is about 1.1 h.

First, a subset of linkages is extracted from each of the three groups for which the objective value is lower than a specified threshold. These extracted solutions are subjected to the kinematic performance criterion ($\mathbf{P_k}$, Sec. 2, ix). All solutions with kinks in their respective output paths are chosen invariably. A fully compliant and a rigid-link mechanism with low objective values are also considered.

The chosen linkages are shown in Figs. 6(*a*)–6(*f*) and termed as M1–M6, respectively, for ease in the following analysis. M1–M4 in Figs. 6(*a*)–6(*d*) are the partially compliant mechanisms while M5 and M6 in Figs. 6(*e*) and 6(*f*) are the fully compliant and rigid-link mechanisms respectively. M1–M4 are metamorphic partially compliant mechanisms as their effective topology is altered during deformation [46, 47]. Near the kink, a few links in

Table 2 Pugh decision matrix

Linkage	M1	M2	M3	M4	M5	M6
Criteria		Datum				
$\mathbf{P_k}$	0	0	0	0	−1	−1
$\mathbf{N_l}$	+1	0	+1	+1	+1	−1
$\mathbf{L_m}$	+1	0	+1	+1	0	−1
$\mathbf{L_s}$	0	0	0	0	0	0
$\mathbf{N_h}$	+1	0	+1	+1	+1	−1
$\mathbf{N_{nh}}$	−1	0	−1	0	+1	−1
$\mathbf{C_m}$	+1	0	+1	−1	+1	−1
$\mathbf{I_n}$	0	0	0	0	0	0
$\mathbf{F_s}$	−1	0	−1	−1	0	0
$\mathbf{F_f}$	−1	0	−1	0	0	+1
+ve rating	4	0	4	3	4	1
−ve rating	3	0	3	2	1	6
Net rating	1	0	1	1	3	−5

Table 3 AHP hierarchy

Goal: To determine the best solution from the synthesized set of linkages											
Criterion	P_k	N_l	L_m	L_s	N_h	N_{nh}	C_m	I_n	F_s	F_s	Overall rating
(Criterion) Normalized weights →	0.2853	0.0493	0.0151	0.1812	0.0409	0.0278	0.0151	0.1812	0.1220	0.0820	
(Solution) Normalized ratings ↓											
M1	0.2600	0.0982	0.0712	0.1667	0.0876	0.0941	0.2671	0.1667	0.0333	0.0644	0.1601
M2	0.2600	0.0534	0.0712	0.1667	0.0640	0.1769	0.0794	0.1667	0.3000	0.1384	0.1949
M3	0.2600	0.1403	0.1833	0.1667	0.1263	0.0907	0.1807	0.1667	0.0333	0.0250	0.1609
M4	0.1682	0.2000	0.1189	0.1667	0.2380	0.1769	0.0268	0.1667	0.0333	0.1384	0.1505
M5	0.0258	0.4863	0.5090	0.1667	0.4629	0.4092	0.3965	0.1667	0.3000	0.1384	0.1837
M6	0.0258	0.0218	0.0465	0.1667	0.0212	0.0521	0.0495	0.1667	0.3000	0.4954	0.1498

Sample calculation: Overall normalized rating of the mechanism in Fig 6a: $0.2853 \times 0.2600 + 0.0493 \times 0.0982 + 0.0151 \times 0.0712 + 0.1812 \times 0.1667 + 0.0409 \times 0.0876 + 0.0278 \times 0.0941 + 0.0151 \times 0.2671 + 0.1812 \times 0.1667 + 0.1220 \times 0.0333 + 0.0820 \times 0.0644 = 0.1601$

the respective mechanisms get locked and remain in that position while the rest of the path is traced. Prototypes of the partially and fully compliant mechanisms (M1–M5) are also shown. These are further analyzed for quasi-static, large deformation stresses using ABAQUS®. The deformed configuration and stress contours for each solution are depicted in Figs. 7(*a*)–7(*e*).

4.1 Fatigue Test. A testing unit, (Fig. 8) consisting of a frame and a guide way, is designed to perform fatigue test on partially and fully compliant mechanisms (M1–M5). An eccentric is mounted on a 30 rpm motor shaft at one end of the machine. The slider, connected to the eccentric via the connecting rod, reciprocates along the guide way to cyclically actuate the mechanism at the other end. A counter circuit is provided to register the number of cycles. A microswitch, placed at one of the dead center positions of the slider, sends a pulse once per slider stroke. The number of cycles is incremented by one for every pulse received from the microswitch.

Each linkage in Figs. 6(*a*)–6(*e*) is subjected to cyclic push-and-pull type loading for a prescribed 15,000 cycles. Each test lasts for about 8 h. No rupture failure is observed in any linkage. However, permanent plastic deformation is witnessed. The difference between the initial and final positions of the output port for the same position of input port is considered as an indicator of fatigue failure for a linkage.

For the chosen displacement-delimited grippers in Fig. 6, Table 1(*a*) shows all topological attributes (ii–vi) described in Sec. 2 and values of the maximum quasi-static stresses. Table 1(*b*) shows the distances between the output ports for each linkage before and after the fatigue test. Table 1(*c*) verifies the kinematic performance (ix) and interference (x) criteria.

4.2 Selection Using PDM. All linkages in Fig. 6 are analyzed using the Pugh decision matrix (Table 2). Linkage M2 is chosen as the datum as it traces the prescribed output path (Sec. 2, ix) with high fidelity and fulfills the $\mathbf{L}_s$ and $\mathbf{F}_s$ criteria (Secs. 2, iii, vii) as well.

Explanation for the ratings in Table 2 is as follows. The output paths for mechanisms M5 and M6 do not have kinks. Hence, compared to the datum, they are worse in kinematic performance ($\mathbf{P_k}$) and are rated as −1. Linkages M1, M3, and M4 trace nonsmooth paths and are therefore considered the same as the datum and rated as 0.

With regard to other general criteria in Sec. 2, linkages with least $\mathbf{N}_l$, $\mathbf{L}_m$, $\mathbf{N}_h$, $\mathbf{N}_{nh}$, and $\mathbf{C}_m$ are preferred. Each linkage is compared with the datum and using Table 1, ratings are recorded in Table 2. For $\mathbf{L}_m$, linkage M5 is rated as zero (same as datum) as it is a fully compliant mechanism and the length of the longest link has no relevance. The shortest link ($\mathbf{L}_s$) in each mechanism is of length ≥7 mm and considered manufacturable. Hence for ($\mathbf{L}_s$), all linkages are rated zero, the same as the datum. The symmetric parts of the datum as well as the other linkages do not interfere across the line of symmetry. Thus, all ratings corresponding to the specific criterion $\mathbf{I_n}$ are zero.

The maximum quasi-static stress exceeds the yield strength for Teflon for the input load suggested by the synthesis algorithm for some linkages. Hence, the loads are reduced slightly in magnitude. It is ensured that with the revised loads, linkages still trace a portion of the path beyond the prescribed kink. The maximum stresses [$\mathbf{F_s}$, Table 1(*a*)] in cases of the datum M2 and solution M5 are within the yield strength of 23 MPa. Hence, they are rated zero. Linkages M3 and M4 are rated as −1. M1 is rated as −1 as well since the output port does not traverse beyond the kink. Mechanism M6 is a rigid-body linkage and is thus rated as 0.

In the datum linkage, a difference of 6 mm is observed in the positions of its output port before and after the fatigue test [$\mathbf{F}_f$, Table 1(*b*)]. Greater this difference, the worse the linkage is and vice versa. Once all ratings are known, the positive and negative values are added separately to give the overall respective ratings. The net rating for each linkage is the sum of the positive and negative ratings. For linkages M1–M6, these ratings are computed in the last three rows of Table 2.

Table 4 Pair-wise comparison of the criteria (AHP)

	P_k	N_l	L_m	L_s	N_h	N_{nh}	C_m	I_n	F_s	F_f	Weight	Normalized weight
$\mathbf{P_k}$	1	6	8	3	6	7	8	3	4	5	1.0000	0.2853
$\mathbf{N_l}$	1/6	1	6	1/5	2	3	6	1/5	1/5	1/4	0.1727	0.0493
$\mathbf{L_m}$	1/8	1/6	1	1/7	1/5	1/4	1	1/7	1/7	Q	0.0530	0.0151
$\mathbf{L_s}$	1/3	5	7	1	5	6	7	1	3	5	0.6351	0.1812
$\mathbf{N_h}$	1/6	1/2	5	1/5	1	3	5	1/5	1/5	1/4	0.1432	0.0409
$\mathbf{N_{nh}}$	1/7	1/3	4	1/6	1/3	1	4	1/6	1/6	1/5	0.0975	0.0278
$\mathbf{C_m}$	1/8	1/6	1	1/7	1/5	1/4	1	1/7	1/7	1/6	0.0530	0.0151
$\mathbf{I_n}$	1/3	5	7	1	5	6	7	1	3	5	0.6351	0.1812
$\mathbf{F_s}$	1/4	5	7	1/3	5	6	7	1/3	1	3	0.4277	0.1220
$\mathbf{F_f}$	1/5	4	6	1/5	4	5	6	1/5	1/3	1	0.2873	0.0820

Table 5 Pair-wise comparison of linkages with respect to kinematic performance (AHP)

P_k	M_1	M_2	M_3	M_4	M_5	M_6	Rating	Normalized rating
M1	1	1	1	2	9	9	1.0000	0.2600
M2	1	1	1	2	9	9	1.0000	0.2600
M3	1	1	1	2	9	9	1.0000	0.2600
M4	1/2	1/2	1/2	1	9	9	0.6469	0.1682
M5	1/9	1/9	1/9	1/9	1	1	0.0993	0.0258
M6	1/9	1/9	1/9	1/9	1	1	0.0993	0.0258

Table 6 Pair-wise comparison of linkages with respect to number of links (AHP)

N_l	M1	M2	M3	M4	M5	M6	Rating	Normalized rating
M1	1	3	1/2	1/3	1/5	7	0.2019	0.0982
M2	1/3	1	1/4	1/5	1/6	5	0.1099	0.0534
M3	2	4	1	1/2	1/5	8	0.2885	0.1403
M4	3	5	2	1	1/5	8	0.4112	0.2000
M5	5	6	5	5	1	9	1.0000	0.4863
M6	1/7	1/5	1/8	1/8	1/9	1	0.0448	0.0218

Table 8 Pair-wise comparison of linkages with respect to length of the shortest link (AHP)

L_s	M1	M2	M3	M4	M5	M6	Rating	Normalized rating
M1	1	1	1	1	1	1	0.4082	0.1667
M2	1	1	1	1	1	1	0.4082	0.1667
M3	1	1	1	1	1	1	0.4082	0.1667
M4	1	1	1	1	1	1	0.4082	0.1667
M5	1	1	1	1	1	1	0.4082	0.1667
M6	1	1	1	1	1	1	0.4082	0.1667

4.3 Selection Using AHP. All linkages in Fig. 6 are now ranked using the analytic hierarchy process (Table 3) (Sec. 3.2). Pair-wise comparison among the criteria and the mechanisms is performed and depicted in Tables 4–14, respectively, in the Appendix. The entries of the principal eigenvector of the matrix in Table 4 is employed as weights for the criteria. The procedure for criterion rating is explained below with the kinematic criterion ($\mathbf{P}_k$) as reference.

For the displacement-delimited gripper, ($\mathbf{P}_k$) is rated as 6, that is, between strong and very strong in comparison to ($\mathbf{N}_l$) and ($\mathbf{N}_h$). ($\mathbf{P}_k$) is considered between very strong and extremely strong in comparison to the link length ($\mathbf{L}_m$) and compactness criterion ($\mathbf{C}_m$). Thus, its rating is 8 with respect to both. ($\mathbf{P}_k$) is rated as 3 compared to the length of the shortest link ($\mathbf{L}_s$) and interference ($\mathbf{I}_n$) criteria. With respect to ($\mathbf{N}_{nh}$), ($\mathbf{F}_s$), and ($\mathbf{F}_f$), ($\mathbf{P}_k$) is rated as 7, 4, and 5, respectively. The ratings of all criteria in comparison with ($\mathbf{P}_k$) are the respective reciprocals and placed in the transposed cell positions. These indicate that other criteria are relatively weak compared to the kinematic requirement of tracing the prescribed path with high fidelity.

Similarly, the remaining criteria are compared pair-wise with ratings shown in Table 4. These values are based on how a user construes the relative importance between any two criteria. Accordingly, quantitative interpretations of these qualitative assessments may vary. The principal eigenvector and the normalized weights (relative importance) for each criterion are depicted in the penultimate and last columns of Table 4, respectively.

Pair-wise comparison of all linkages is performed next for each criterion. In Table 5, linkages are compared with each other for ($\mathbf{P}_k$). Mechanism M1 is considered equally important as linkages M2 and M3. This is because they satisfy the kinematic criterion equally well. Thus, it is rated as 1 each (the first three entries, second row in Table 5). M1 is between equally and marginally strong compared to M4 and hence the corresponding rating is 2. M1 is deemed extremely strong compared to linkages M5 and M6. Hence, it is rated as 9 each. The ratings of all other linkages in comparison with M1 are the reciprocals of the above respective values and placed in the transposed cell positions. The remaining linkages are compared likewise and the ratings are as shown in Table 5.

In a similar manner, one-to-one comparison is made for all linkages for each topological, qualitative and strength, general and specific criterion. The ratings are assessed subjectively and shown in Tables 6–14. For instance, explanation for comparison between linkages M2 and M4 in the respective tables (4th row, 2nd column) in the appendix are as follows. Linkage M4 is stronger (rating 5) because of comparatively small number of links (Table 6). It is marginally better (rating 2) because the length of its longest link is slightly smaller (Table 7). Both mechanisms have the shortest links larger than the minimum specified and so the corresponding entry is 1 (Table 8). Referring to Table 9, linkage M4 is stronger (rating 5) because the number of hinges is less by 10 (Table 1). In Table 10, the two mechanisms have the rating 1 as the number of hinges per node is the same. In Table 11, linkage M2 is between marginally strong and strong (column 7, Table 1) with regard to criterion ($\mathbf{C}_m$) and hence its rating is 4. Correspondingly, in the 4th × 2nd cell in Table 11, the rating is 1/4. All linkages in Fig. 6 satisfy the interference criterion (rating 1, Table 12). Linkage M2 is significantly better and rated 9 (Table 13) in terms of ($\mathbf{F}_s$). This is because, for mechanism M4, the maximum stress exceeds the permissible limit. The two linkages are the same in respect of fatigue failure (Table 1).

For each comparison table, the normalized principal eigenvector is computed just as that for the matrix in Table 4. The normalized ratings of the criteria and the mechanisms with respect to each criterion are consolidated in Table 3, giving the hierarchy for the AHP. Table 3 also shows the overall normalized rating RO_i (Eq. 2) for each linkage solution.

4.4 Selection Using Variant of PDM. All linkages in Fig. 6 are analyzed using the variant of Pugh decision matrix. Linkage M2 is chosen as the datum and the solutions are rated (Table 15) as in Sec. 4.2. The weight for each criterion is taken from Table 4. Table 15 also depicts the positive, negative, and the overall ratings for each linkage. To compute these, Eq. (2) is employed with appropriate modifications.

5 Discussion

5.1 The Best Displacement-Delimited Gripper Mechanism. Table 2 suggests that solution M5, the fully compliant mechanism, is the best by PDM with the net positive rating of 3. Partially compliant linkages M1, M3, and M4 follow with the net ratings of 1 each. The AHP suggests the partially compliant mechanism M2 as the best (Table 3) with the overall normalized rating of 0.195. This is closely followed by linkages M5 and M3 with ratings 0.18 and 0.16, respectively. The variant of Pugh's method suggests M2 as the best solution as well with the net rating of zero.

In general, the three methods of systematic selection may not rank the linkage solutions identically. This is because of the difference in the scales used (e.g., ±1, 0 for PDM, 1–9 for AHP) to

Table 7 Pair-wise comparison of linkages with respect to length of the longest link (AHP)

L_m	M1	M2	M3	M4	M5	M6	Rating	Normalized rating
M1	1	1	1/3	1/2	1/6	2	0.1398	0.0712
M2	1	1	1/3	1/2	1/6	2	0.1398	0.0712
M3	3	3	1	2	1/5	4	0.3602	0.1833
M4	2	2	1/2	1	1/5	3	0.2336	0.1189
M5	6	6	5	5	1	6	1.0000	0.5090
M6	1/2	1/2	1/4	1/3	1/6	1	0.0913	0.0465

Table 9 Pair-wise comparison of linkages with respect to number of hinges (AHP)

N_h	Ml	M2	M3	M4	M5	M6	Rating	Normalized rating
Ml	1	2	1/2	1/4	1/5	7	0.1893	0.0876
M2	1/2	1	1/3	1/5	1/6	7	0.1382	0.0640
M3	2	3	1	1/3	1/5	8	0.2729	0.1263
M4	4	5	3	1	1/4	8	0.5140	0.2380
M5	5	6	5	4	1	9	1.0000	0.4629
M6	1/7	1/7	1/8	1/8	1/9	1	0.0457	0.0212

Table 10 Pair-wise comparison of linkages with respect to maximum number of hinges at a given node (AHP)

N_{nh}	Ml	M2	M3	M4	M5	M6	Rating	Normalized rating
Ml	1	1/2	1	1/2	1/4	2	0.2300	0.0941
M2	2	1	2	1	1/3	4	0.4324	0.1769
M3	1	1/2	1	1/2	1/5	2	0.2217	0.0907
M4	2	1	2	1	1/3	4	0.4324	0.1769
M5	4	3	4	3	1	5	1.0000	0.4092
M6	1/2	1/4	1/2	1/4	1/5	1	0.1274	0.0521

Table 12 Pair-wise comparison of linkages with respect to interference criterion (AHP)

I_n	M1	M2	M3	M4	M5	M6	Rating	Normalized rating
M1	1	1	1	1	1	1	0.4082	0.1667
M2	1	1	1	1	1	1	0.4082	0.1667
M3	1	1	1	1	1	1	0.4082	0.1667
M4	1	1	1	1	1	1	0.4082	0.1667
M5	1	1	1	1	1	1	0.4082	0.1667
M6	1	1	1	1	1	1	0.4082	0.1667

compare both, the criteria and solutions. A user may assign different preferences (weights) to the criteria which will affect the solution rankings.

The selection methods may not concur with the initial perception a user may have of the best solution. Rather, linkage choice after systematic selection will be more trustworthy. For instance, before the foregoing analysis is performed, mechanisms M1 and M3 are perceived as candidates for the best solution based on visual observation and the criteria values in Table 1. This is because the paths traced are very close to that prescribed ($\mathbf{P}_k$), the number of links ($\mathbf{N}_l$) and hinges ($\mathbf{N}_h$) are relatively less, and both are compact ($\mathbf{C}_m$). While the fully compliant mechanism M5 is best in terms of simplicity and compactness, it does not trace the prescribed nonsmooth path accurately. This mechanism has the highest net rating by Pugh's method as it is better than its partially compliant and rigid-body counterparts in terms of many general criteria. Linkages M1 and M3 are suggested by PDM as the second best choices.

The individual preference for each criterion is not considered in PDM. However, in the AHP or the variant of PDM, weightage for each criterion is taken into account and thus the fully compliant mechanism M5 is pushed lower in the ranking. Linkages M3 and M1 are ranked third and fourth, respectively. The variant of PDM suggests M1 and M3 to be the third best with the same rating. Thus a linkage, initially thought of as the best, may or may not be so after systematic ranking analysis.

Any decision-making process is prone to subjectivity. For measurable criteria like $\mathbf{N}_l$, $\mathbf{L}_m$, $\mathbf{L}_s$, $\mathbf{N}_h$, $\mathbf{N}_{nh}$, $\mathbf{C}_m$, $\mathbf{F}_s$, and $\mathbf{F}_f$, user inputs in the assessment tables are well informed but still subjective. Bias gets introduced when design assessments are transformed in terms of "grades" or "scales". Criterion like kinematic performance ($\mathbf{P}_k$) used here may need qualitative assessment too. As noted in Sec. 4, better ways can be used to verify the presence of a kink on the path. It is always preferred that all selection criteria should be quantified objectively. However, in general, subjective evaluation cannot always be avoided.

Plastic deformation of the mechanisms is considered as an indicator of fatigue failure. However, obtaining the results with actual rupture is possible with increased number of cycles. Choice of a different material can also be explored to give better insight into fatigue failure as it is an important criterion in the selection of the best linkage.

5.2 Automation of Rating Generation. Automating the weight generation for each criterion and rating generation for each linkage with respect to a criterion is possible only if the basis of pair-wise comparison is objective. The grading scheme chosen is also a factor that may make the objective pair-wise comparison difficult and involved. In AHP, for instance, numerous comparison tables are generated.

General objective criteria like $\mathbf{N}_l$, $\mathbf{L}_m$, $\mathbf{L}_s$, $\mathbf{N}_h$, $\mathbf{N}_{nh}$, and $\mathbf{C}_m$ can all be rated automatically in the Pugh's decision method as -1, 0, or $+1$ provided the datum solution is identified by the user. The rating for the partially and fully compliant linkages with respect to strength related criteria $\mathbf{F}_s$ and $\mathbf{F}_f$ can be automated as well. Likewise, even the interference conditions can be automatically determined. However, the user needs to qualitatively assess the preferences for the kinematic performance criterion, after which it is reckoned that the selection process can be automated.

Automation in case of the AHP or Variant of PDM is not possible until the user specifies the relative importance between the general and specific, topological, qualitative, and strength-based criteria. In AHP, comparing solutions one-to-one with respect to each measure again requires user inputs to rate the solutions on a scale of 1–9 (as opposed to ± 1, 0 in PDM) which makes automation difficult.

In case of the variant of PDM, the relative significance between the criteria needs to be established by the user. Compared to the AHP, most user choices are avoided here. It is observed for the example presented that both AHP and the variant of PDM yield the same linkage solution (M2) as the best. This, in general, may not be true. AHP is more comprehensive compared to PDM or its variant as it allows for gray assessment via the 1–9 scale as opposed to the binary ± 1, 0 scale. In general, a combination of the AHP, PDM and its variant can help converge to the best linkage solution.

Some or many criteria discussed herein can be considered as constraints during linkage synthesis. However, in presence of many constraints, finding good mechanism solutions in the design space that conform to the prescribed kinematic performance (the main objective in synthesis) can be difficult. It is therefore suggested that only essential objectives and constraints should be used in synthesis and later, other general and specific criteria can be used in the proposed systematic selection procedure to determine the best linkage. While path-generating mechanisms are considered in this paper, the proposed selection method is applicable to linkages synthesized using any kinematic (function, path, motion generation) objective.

Table 11 Pair-wise comparison of linkages with respect to compactness (AHP)

C_m	M1	M2	M3	M4	M5	M6	Rating	Normalized rating
M1	1	4	2	8	1/2	6	0.6738	0.2671
M2	1/4	1	1/3	4	1/5	2	0.2002	0.0794
M3	1/2	3	1	7	1/3	5	0.4557	0.1807
M4	1/8	1/4	1/7	1	1/9	1/3	0.0677	0.0268
M5	2	5	3	9	1	7	1.0000	0.3965
M6	1/6	1/2	1/5	3	1/7	1	0.1249	0.0495

Table 13 Pair-wise comparison of linkages with respect to quasi-static stresses induced in the links (AHP)

F_s	M1	M2	M3	M4	M5	M6	Rating	Normalized rating
M1	1	1/9	1	1	1/9	1/9	0.1111	0.0333
M2	9	1	9	9	1	1	1.0000	0.3000
M3	1	1/9	1	1	1/9	1/9	0.1111	0.0333
M4	1	1/9	1	1	1/9	1/9	0.1111	0.0333
M5	9	1	9	9	1	1	1.0000	0.3000
M6	9	1	9	9	1	1	1.0000	0.3000

Table 14 Pair-wise comparison of linkages with respect to fatigue failure (AHP)

F_f	M1	M2	M3	M4	M5	M6	Rating	Normalized rating
M1	1	1/3	5	1/3	1/3	1/6	0.1299	0.0644
M2	3	1	7	1	1	1/5	0.2794	0.1384
M3	1/5	1/7	1	1/7	1/7	1/8	0.0504	0.0250
M4	3	1	7	1	1	1/5	0.2794	0.1384
M5	3	1	7	1	1	1/5	0.2794	0.1384
M6	6	5	8	5	5	1	1.0000	0.4954

Table 15 Variant of the Pugh decision matrix

Solution No.		**M1**	**M2**	**M3**	**M4**	**M5**	**M6**
Criteria	Weights		Datum				
$\mathbf{P_k}$	0.28	0	0	0	0	−1	−1
$\mathbf{N_l}$	0.05	+1	0	+1	+1	+1	−1
$\mathbf{L_m}$	0.02	+1	0	+1	+1	0	−1
$\mathbf{L_s}$	0.18	0	0	0	0	0	0
$\mathbf{N_h}$	0.05	+1	0	+1	+1	+1	−1
$\mathbf{N_{nh}}$	0.03	−1	0	−1	0	+1	−1
$\mathbf{C_m}$	0.02	+1	0	+1	−1	+1	−1
$\mathbf{I_n}$	0.18	0	0	0	0	0	0
$\mathbf{F_s}$	0.12	−1	0	−1	−1	0	0
$\mathbf{F_f}$	0.08	−1	0	−1	0	0	+1
+ve rating	-	0.14	0	0.14	0.12	0.15	0.08
−ve rating	-	0.23	0	0.23	0.15	0.28	0.45
Net rating	-	−0.09	0	−0.09	−0.03	−0.13	−0.37

6 Closure

This paper mainly deals with the identification and implementation of various criteria that can be considered to determine the best linkage. The selection process is demonstrated for an example problem of the displacement-delimited gripper synthesized for a prescribed large nonsmooth path. Linkage selection is performed via (i) PDM, (ii) AHP, and (iii) variant of PDM where different criteria are deemed either equal in importance or are assigned preferences through the weights obtained from AHP. The merit function used in the synthesis method [1] is independent of the choice of the selection procedure. It is noted that these selection methods, individually, may or may not yield the same linkage as the best solution. It is recommended that all three should be employed to confirm that the best mechanism solution suggested by one is indeed the best in all. Also, the resulting best mechanism may not concur with the initial perception of the user. Systematic linkage selection is essential to choose the best design from a given set so that the utility of the mechanism is fully realized. In future, we intend to explore the possibility of automating all selection procedures and integrating them with the synthesis step.

Acknowledgment

Anupam Saxena sincerely acknowledges IGM, RWTH-Aachen University and the Alexander von Humboldt Foundation for their support. He also expresses his sincere thanks to Professor Panos Papalambros and Ben Palumbo for their suggestions in improving the presentation of this manuscript.

Appendix

Pair-wise comparison of the criteria and solutions per criterion using the analytic hierarchy process is given in Tables 4–14.

References

[1] Rai, A. K., Saxena, A., and Mankame, N. D., 2009, "Unified Synthesis of Compact Planar Path-Generating Linkages With Rigid and Deformable Members," Struct. Multidiscip. Optim., **41**, pp. 1–17.

[2] Kempe, A. B., 1876, "On a General Method of Describing Plane Curves of the nth Degree by Linkwork," Proc. London Math. Soc., **7**, pp. 213–216.

[3] Gao, X.-S., Zhu, C.-C., Chou, S.-C., and Ge, J.-X., 2001, "Automated Generation of Kempe Linkages for Algebraic Curves and Surfaces," Mech. Mach. Theory, **36**, pp. 1019–1033.

[4] Shoup, T. E., and McLarnan, C. W., 1971, "A Survey of Flexible Link Mechanisms Having Lower Pairs," J. Mech., **6**, pp. 97–105.

[5] Winter, S. J., and Shoup, T. E., 1972, "The Displacement Analysis of Path-Generating Flexible-Link Mechanisms," Mech. Mach. Theory, **7**, pp. 443–451.

[6] Sriram, B. R., and Mruthyunjaya, T. S., 1995, "Synthesis of Path Generating Flexible-Link Mechanisms," Comput. Struct., **56**(4), pp. 657–666.

[7] Ramrakhyani, D. S., Frecker, M. I., and Lesieutre, G. A., 2009, "Hinged Beam Elements for the Topology Design of Compliant Mechanisms Using the Ground Structure Approach," Struct. Multidiscip. Optim., **37**(6), pp. 557–567.

[8] Hunt, K. H., *Kinematic Geometry of Mechanisms* (Oxford University, New York, 1978).

[9] Erdman, A. G., and Sandor, G. N., 1984, *Mechanism Design: Analysis and Synthesis*, Prentice-Hall, Englewood Cliffs, NJ, Vol. 1.

[10] Sandor, G. N., and Erdman, A. G., 1984, *Advanced Mechanism Design: Analysis and Synthesis*, Prentice-Hall, Englewood Cliffs, NJ, Vol. 2.

[11] Hartenberg, R. S., and Denavit, J., *Kinematic Synthesis of Linkages* (McGraw-Hill, New York, 1964).

[12] Artobolevskii, I. I., *Mechanisms for the Generation of Plane Curves* (Pergamon, New York, 1964).

[13] Sedlaczek, K., Gaugele, T., and Eberhard, P., 2005, "Topology Optimized Synthesis of Planar Kinematic Rigid Body Mechanisms," Multibody Dynamics, ECCOMAS Thematic Conference, Madrid, Spain.

[14] Liu, Y., and McPhee, J., 2007, "Automated Kinematic Synthesis of Planar Mechanisms With Revolute Joints," Mech. Based Des. Struct. Mach., **35**(4), pp. 405–445.

[15] Kawamoto, A., 2005, "Path-Generation of Articulated Mechanisms by Shape and Topology Variations in Non-Linear Truss Representation," Int. J. Numer. Methods Eng., **64**, pp. 1557–1574.

[16] Tsai, L.-W., *Mechanism Design: Enumeration of Kinematic Structures According to Function* (CRC, Boca Raton, 2001).

[17] Chen, D.-Z., and Pai, W.-M., 2005, "A Methodology for Conceptual Design of Mechanisms by Parsing Design Specifications," ASME J. Mech. Des., **127**, pp. 1039–1044.

[18] Pucheta, M., and Cardona, A., 2007, "An Automated Method for Type Synthesis of Planar Linkages Based on a Constrained Subgraph Isomorphism Detection," Multibody Syst. Dyn., **18**, pp. 233–258.

[19] Kota, S., Joo, J., Li, Z., Rodgers, S. M., and Sniegowski, J., 2001, "Design of Compliant Mechanisms: Applications to MEMS," Analog Integr. Circuits Signal Process., **29**, pp. 7–15.

[20] Saxena, A., and Ananthasuresh, G. K., 2003, "A Computational Approach to the Number Synthesis of Linkages," ASME J. Mech. Des., **125**, pp. 110–118.

[21] Saxena, A., 2005, "Topology Design of Large Displacement Compliant Mechanisms With Multiple Materials and Multiple Output Ports," Struct. Multidiscip. Optim., **30**, pp. 477–490.

[22] Saxena, A., 2005, "Synthesis of Compliant Mechanisms for Path Generation Using Genetic Algorithm," ASME J. Mech. Des., **127**, pp. 745–752.

[23] Rai, A. K., Saxena, A., and Mankame, N. D., 2007, "Synthesis of Path Generating Compliant Mechanisms Using Initially Curved Frame Elements," ASME J. Mech. Des., **129**, pp. 1056–1063.

[24] Ananthasuresh, G. K., Kota, S., and Gianchandani, Y., 1994, "A Methodical Approach to the Design of Compliant Micro-Mechanisms," Technical Digest, Solid-State Sensor and Actuator Workshop, June 13–16, Transducers Research Foundation, Inc., Hilton Head Island, South Carolina, pp. 189–192.

[25] Nishiwaki, S., Frecker, M. I., Min, S., and Kikuchi, N., 1998, "Topology Optimization of Compliant Mechanisms Using the Homogenization Method," Int. J. Numer. Methods Eng., **42**, pp. 535–559.

[26] Pederson, C. B. W., Buhl, T., and Sigmund, O., 2001, "Topology Synthesis of Large-Displacement Compliant Mechanisms," Int. J. Numer. Methods Eng., **50**, pp. 2683–2705.

[27] Rahmatalla, S., and Swan, C. C., 2005, "Sparce Monolithic Compliant Mechanisms Using Continuum Structural Topology Optimization," Int. J. Numer. Methods Eng.., **62**, pp. 1579–1605.

[28] Saxena, A., 2008, "A Material-Mask Overlay Strategy for Continuum Topology Optimization of Compliant Mechanisms Using Honeycomb Discretization," ASME J. Mech. Des., **130**, 082304.

[29] Mankame, N. D., and Ananthasuresh, G. K., 2004, "Topology Optimization for Synthesis of Contact-Aided Compliant Mechanisms Using Regularized Contact Modeling," Comput. Struct., **82**, pp. 1267–1290.

[30] Pucheta, M. A., and Cardona, A., 2010, "Design of Bistable Compliant Mechanisms Using Precision-Position and Rigid-Body Replacement Methods," Mech. Mach. Theory, **45**, pp. 304–326.

[31] Howell, L. L., *Compliant Mechanisms* (Wiley, New York, 2001).

[32] Sardain, P., 1997, "Linkage synthesis: Topology Selection Fixed by Dimensional Constraints, Study of an Example," Mech. Mach. Theory, **32**(1), pp. 91–102.

[33] Pucheta, M. A., and Cardona, A., 2009, "Automated Classification of Synthesized Linkage Mechanisms," *Asociación Argentina de MecánicaComputacional, XXVIII*, pp. 3241–3250.

[34] Board on Manufacturing and Engineering Design, 2001, "Theoretical foundations for decision making in engineering design," (National Academy Press, Washington D.C.).

[35] Bhushan, N., and Rai, K., *Strategic Decision Making—Applying the Analytic Hierarchy Process* (Springer–Verlag, Berlin, 2004).

[36] Obaidat, W., and Dwairi, A., "Concept Evaluation and Selection," accessed at http://www.slideshare.net/QRCE/concept-evaluation-and-selection-presentation, Last accessed on June 10, 2010.
[37] "Pugh Method or Decision—Matrix Method," accessed at http://www.enge.vt.edu/terpenny/Smart/Virtual_econ/Module2/pugh_method.htm, Last accessed June 10, 2010.
[38] Process MA, "Pugh matrix," accessed at http://www.processma.com/resource/pugh_matrix.htm, Last accessed on June 10, 2010.
[39] Murphy, M. D., Midha, A., and Howell, L. L., 1994, "On the Mobility of Compliant Mechanisms," *23rd ASME Biennial Mechanisms Conference on Machine Elements and Machine Dynamics*, DE-Vol. 71, pp. 475–479.
[40] Murphy, M. D., Midha, A., and Howell, L. L., 1994, "The Topological Analysis of Compliant Mechanisms," *23rd ASME Biennial Mechanisms Conference on Machine Elements and Machine Dynamics*, DE-Vol. 71, pp. 481–489.
[41] Murphy, M. D., Midha, A., and Howell, L. L., 1994, "Type Synthesis of Compliant Mechanisms Employing a Simplified Approach to Segment Type," *23rd ASME Biennial Mechanisms Conference on Mechanism Synthesis and Analysis*, DE-Vol. 70, pp. 51–60.
[42] Murphy, M. D., Midha, A., and Howell, L. L., 1994, "Methodology for the Design of Compliant Mechanisms Employing Type Synthesis Techniques With Example," *23rd ASME Biennial Mechanisms Conference on Mechanism Synthesis and Analysis*, DE-Vol. 70, pp. 61–66.
[43] Howell, L. L., and Midha, A., 1995, "Determination of the Degrees of Freedom of Compliant Mechanisms Using the Pseudo-Rigid-Body Model Concept," *Proceedings of the Ninth world Congress on the Theory of Machines and Mechanisms*, Milano, Italy, Vol. 2, pp. 1537–1541.
[44] Ananthasuresh, G. K., and Howell, L. L., 1996, "Case Studies and a Note on the Degrees-of-Freedom in Compliant Mechanisms," Proceedings of the 1996 ASME Mechanisms Conference, Paper No. 96-DETC/MECH-1217.
[45] Mankame, N. D., and Ananthasuresh, G. K., 2007, "Synthesis of Contact-Aided Compliant Mechanisms for Non-Smooth Path Generation," Int. J. Numer. Methods Eng., **69**, pp. 2564–2605.
[46] Li, D., Zhang, Z., and McCarthy, J. M., 2011, "A Constraint Graph Representation of Metamorphic Linkages," Mech. Mach. Theory, **46**, pp. 228–238.
[47] Li, D., Zhang, Z., and Chen, G., 2011, "Structural Synthesis of Compliant Metamorphic Mechanisms Based on the Adjacency Matrix Operations," Chin. J. Mech. Eng. Vol. **24**, No. 4, pp. 1–7.