A Computational Approach to the Number of Synthesis of Linkages

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This paper presents a number of systematically designed compliant topologies and discusses how the intrinsic kinematic behavior can be extracted from them. This is then applied to the number synthesis of linkages. Many techniques developed for number synthesis of linkages enumerate numerous possible kinematic chains, but few can select the best configuration among them. A systematic computational approach that can select the best configuration based on kinetostatic design specifications is presented here. This is a serendipitous result that transpired when two well-developed design techniques for compliant mechanisms were combined. A number of examples with nonintuitive design specifications are included to illustrate the new approach to the number synthesis. The examples also illustrate that the kinematic behavior is aptly captured in the elastic mechanics-based topology optimization method to compliant mechanism design. Dimensional synthesis is also accomplished in the same procedure, which is an added benefit of this approach. [DOI: 10.1115/1.1539513]

1 Introduction

The first step in the design of mechanisms is to select a kinematic configuration that best fulfills the intended mechanical task. The mechanism designer's creativity, experience, and subjective judgment are significant factors in this selection. Therefore, the best selection is not always accomplished. *Type synthesis*, which is the selection of the type of mechanism (linkage, gear, cam, belt, etc. or a combination thereof), and *number synthesis*, which is the determination of the number of various members and kinematic pairs that connect them, are two important techniques developed for systematizing the selection of the best kinematic configuration [1,2]. Many approaches to type and number syntheses [3–6] focus on the enumeration of all possible configurations with little attention given to the selection of the best configuration for a given task. Although rational approaches to type and number syntheses have been successfully applied to practical problems (e.g., [7] for casement window mechanisms), the number of enumerated configurations is often too large to evaluate all of them one by one to select the best configuration (see [1] and [3] for a review). For example, the number of configurations for n-link revolute-jointed kinematic chains are 1, 2, 16, 230, and 6856 for $n=4$, 6, 8, 10, 12 respectively if one degree of freedom is desired [8]. When kinematic inversions are considered, these numbers will be much larger.

Assessing the geometric capability [9], kinematic mappings [10], task-based selection criteria [11] and possibly a few others are techniques developed to help in the systematic evaluation of the numerous possible kinematic configurations. More recently, expert and knowledge-based systems have been successfully applied to automate the mechanism selection procedure (TADSOL [12], TYSES [13], IMSC [14], Mexpert [15], MinDwell [16], etc.). It should also be recognized that a configuration deemed to be the best based only on kinematic criteria might not be the best when the overall objective of the mechanical task is considered. The overall objective might include kinetostatics, geometric restrictions on the available space, manufacturability, performance at high speeds, and other quantitative criteria. A computational procedure that can search through all possible configurations to select the best one based on qualitative criteria of functional behavior as well as quantitative criteria of performance is still an eluding goal [3]. The purpose of this paper is to present such an approach for a restricted class of linkages consisting of revolute joints and for a few evaluation criteria. As explained next, this is a serendipitous result that transpired by combining two previously developed approaches to *compliant mechanism* synthesis.

Compliant mechanisms utilize elastic deformation of links to produce the desired motion. They can be constructed as a unitized material continuum without using kinematic pairs (see Fig. 1). Optimal design of the structural form of the material continuum for desired force-deflection behavior is one way to design them [17,18]. In this approach, the optimal structural form can be obtained from the quantitative kinetostatic description of the mechanical task. The optimization algorithm would search through the design space of all possible configurations allowed by a particular type of design parameterization. An objective function and constraints can be formulated in accordance with the selection criteria. Thus, in effect, this method selects the best configuration from among all possibilities without having to enumerate them explicitly. Designer's bias and prior judgment are therefore avoided in this systematic design method. It is briefly reviewed in Section 2.

The second approach to the design of compliant mechanisms is to represent compliant mechanisms as equivalent rigid-body linkages and applying the existing theory for rigid-body kinematic synthesis to an assumed kinematic configuration [19]. The premise for this is the *pseudo rigid-body model* that accurately describes the kinetostatic behavior of thin prismatic beams for a large range of deformation [20,21]. Using the pseudo rigid-body models, compliant mechanisms can be represented equivalently as rigid-body linkages with revolute joints and torsional springs. The latter can be designed for kinetostatic specifications and converted back to compliant mechanisms. The pseudo rigid-body model is briefly described in Section 3.

The two approaches to compliant mechanisms can be combined as follows. If the space available for a mechanism is parameterized using a *super structure* of Euler beam elements, the optimization method generates the optimal configuration that consists of thin prismatic beams and relatively rigid frame-like portions. This design can be interpreted kinematically and converted to a rigid-body linkage using the pseudo rigid-body model concept. This

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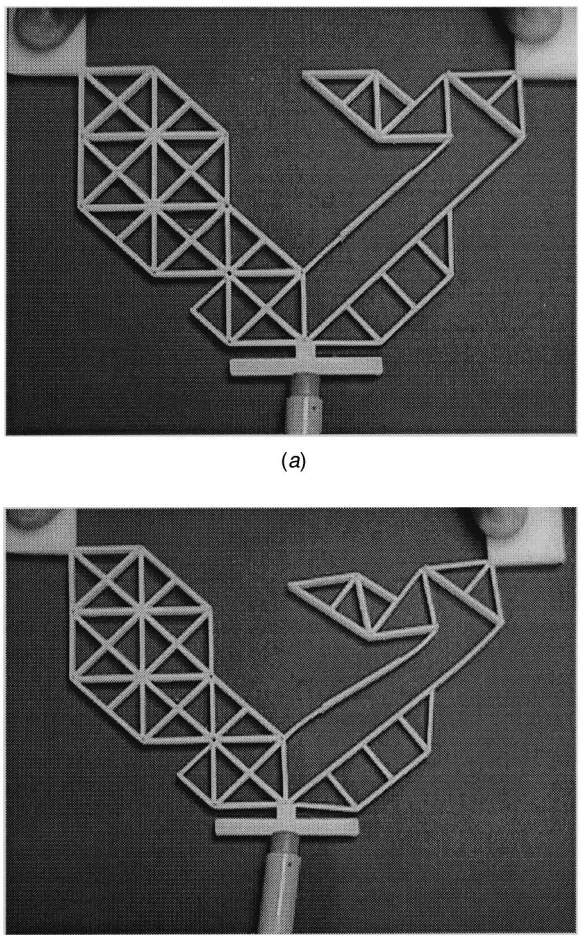

Fig. 1 A compliant displacement amplifying mechanism (*a*) undeformed (*b*) deformed

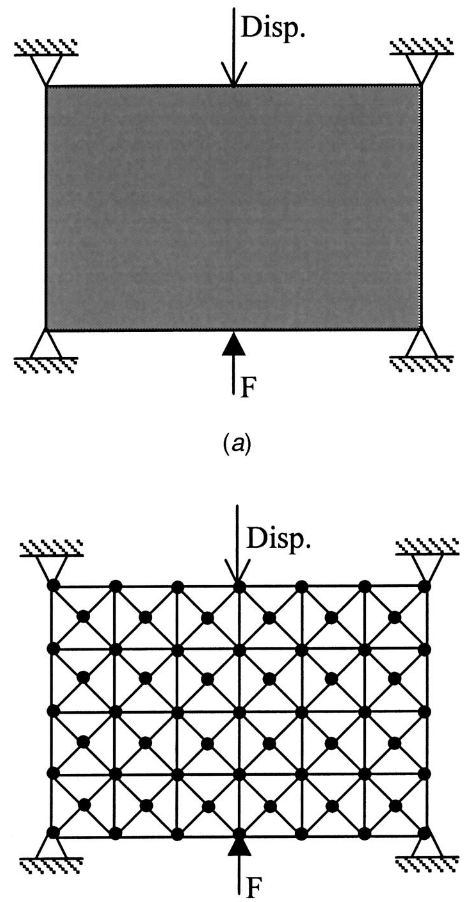

Fig. 2 Kinetostatic task specification for compliant mechanism synthesis (*a*) kinetostatic task specifications for compliant mechanisms (*b*) beam element-based super structure parameterization

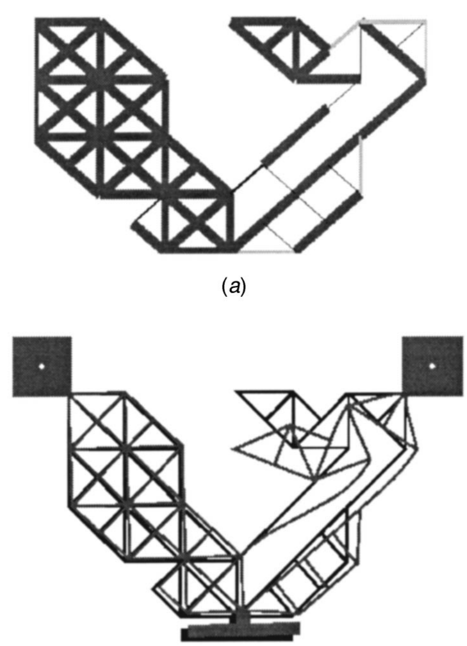

Fig. 3 (*a*) Optimized topology for the problem in Fig. 2(*b*) (*b*) deformed configuration superimposed on the undeformed configuration

determines the number of links and the joints that connect them. Thus, the number synthesis is accomplished systematically. In the same process, since the sizes of different links are determined, dimensional synthesis is also accomplished, which is an added benefit of this approach. This procedure is presented in Section 4.

In Section 5, a number of examples are presented and the highlights of the method described in this paper are discussed. Some concluding remarks are made in Section 6.

2 Design of Optimal Compliant Topologies

The desired mechanical task can be specified for rigid-link mechanisms in terms of three generic types of kinematic synthesis: (i) coordinated motion of two links according to a specified function, (ii) following a specified path at a point on a link when another link is moved, and (iii) guiding a link as a rigid body with translations and rotations as specified when another link is moved. In contrast, the tasks specification for a compliant mechanism should in general involve forces and motion. In other words, *ki-*

Table 1 Specifications for the problem in Fig. 2(*b*)

Design domain: 200×100 μm
Young's modulus$=160$ GPa
Beam thickness$=5$ μm
Lower limit on beam width$=b_l=1$e-4 μm
Upper limit on beam width$=b_u=5$ μm
Force$=F=500$ μN

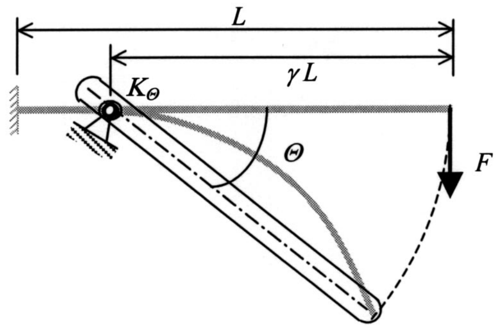

Fig. 4 Cantilever with a tip load and its pseudo rigid-body model

netostatic behavior needs to be specified. It should also be noted that since the application of force on an elastic object causes displacement, coordinating with a force is, in effect, equivalent to coordinating with the displacement at the force application point. Although it is possible to use input displacements instead of input forces, it is more convenient to use forces because it is more intuitive from the standpoint of practical applications. The same three generic types can now be specified as: (i) coordinating the deflection of an output point in a particular direction with an input force at another point (ii) following a path at an output point with an input force at another point, and (iii) guiding a portion of the compliant mechanism with an input force at a point. The first two types are discussed in [18,22,23], and others. In this paper, only the first type is considered for brevity.

Design of an elastic continuum involves three levels of hierarchy: *topology, shape*, and *size*. Topology refers to the way different portions of interest such as the force application point(s), output point(s), and fixed portion(s) are connected to each other. Topology also determines the number of holes in the structure. This is analogous to the number synthesis in rigid-body linkages. Determining the shape of each segment in a topology is the second level, and determining the size, i.e., width, thickness, etc., is the third level. The design method described next generates all three simultaneously. Specifications for the coordinated force-deflection design problem are graphically shown in Figs. 2(*a*) and 2(*b*) for the continuum and discretized models respectively. In the continuum model, point-wise fictitious density ρ is the design variable. The variable ρ takes different values depending on whether there is material at point or not. Instead of making ρ a material or no-material type binary variable, a smooth function is used so that continuous optimization methods can be applied. This can be done in at least three ways: (1) homogenization method [24], (2) solid isotropic material with penalty (SIMP) [25], (3) peak function [26]. The fictitious density function is used to interpolate the material properties such as Young's modulus as shown below for the latter two approaches:

$$\begin{aligned} E &= \rho^n E_0 \quad \text{(SIMP model)} \\ E &= E_0 e^{-((\rho-\mu/\sqrt{2}\sigma)^2} \quad \text{(peak function model)} \end{aligned} \tag{1}$$

The optimization algorithm determines ρ at every point, which in turn, determines where the material will be, and gives rise to the optimal structural form for the specifications. On the other hand,

Table 2 Specifications for examples in Figs. 6–9

Design domain: 0.1×0.1 m
Young's modulus=6.9 GPa
t_l=lower limit on beam thickness=1E-5 m
t_u=upper limit on beam thickness=0.0635 m
b=beam width=0.0635
Force=F=45 N

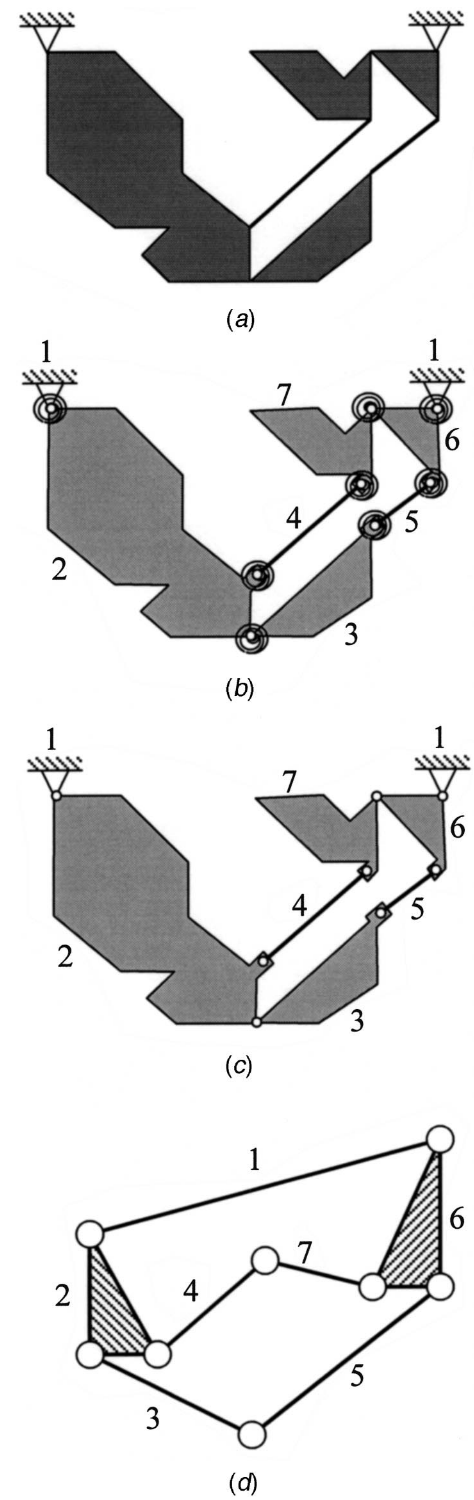

Fig. 5 (*a*) Interpretation in terms of rigid portions and thin beams (*b*) equivalent rigid-body linkage with torsional springs using pseudo rigid-body model (*c*) rigid-body linkage with two degrees of freedom (*d*) 7-link rigid-body kinematic chain with two degrees of freedom

in the discretized finite element model, the geometric design domain is parameterized using a super structure of Euler beam elements. The cross-section dimensions of the beam elements are used as the design variables. If lower (almost zero) and upper (as dictated by the manufacturability and the grid size) bounds are imposed on these design variables, the optimization algorithm de-

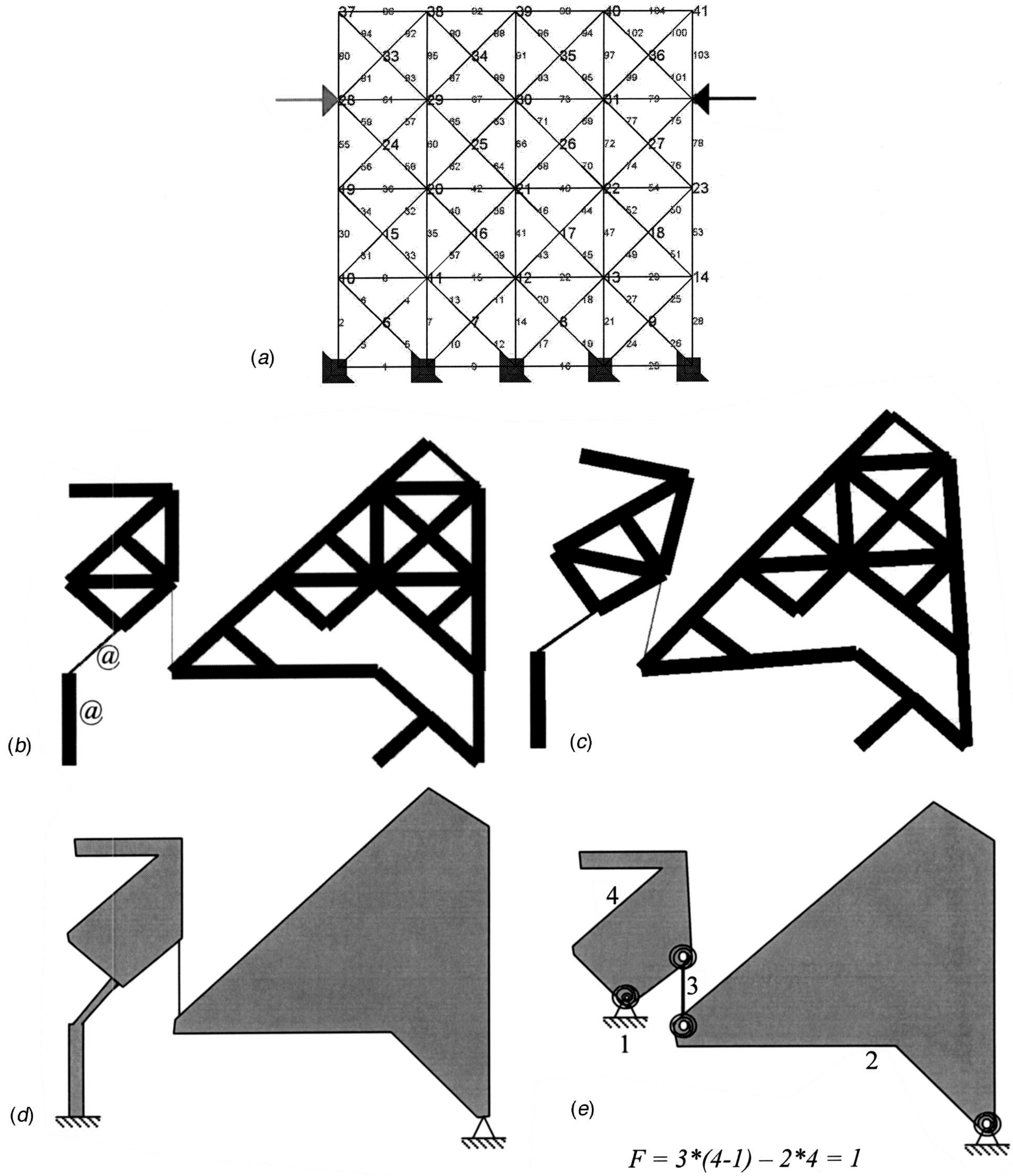

Fig. 6 Example 1 (*a*) task specifications (*b*) optimal topology (*c*) deformed configuration (*d*) interpreted compliant topology (*e*) equivalent rigid-link mechanism

termines which beam elements should stay and which ones should be removed (when the variable reaches the lower bound). This design parameterization is of interest in this paper although a similar procedure can also be presented for the fictitious density parameterization.

The objective function for the design problem takes many forms as discussed in the recent literature (see [18] for these references). In the examples presented in this paper, the optimal design problem is posed as follows:

$$\textit{Minimize} - \frac{\text{mutual strain energy (a measure of flexibility)}}{\text{strain energy (a measure of stiffness)}}$$

Subject to equilibrium equations

or mathematically for the continuum and discrete models as:

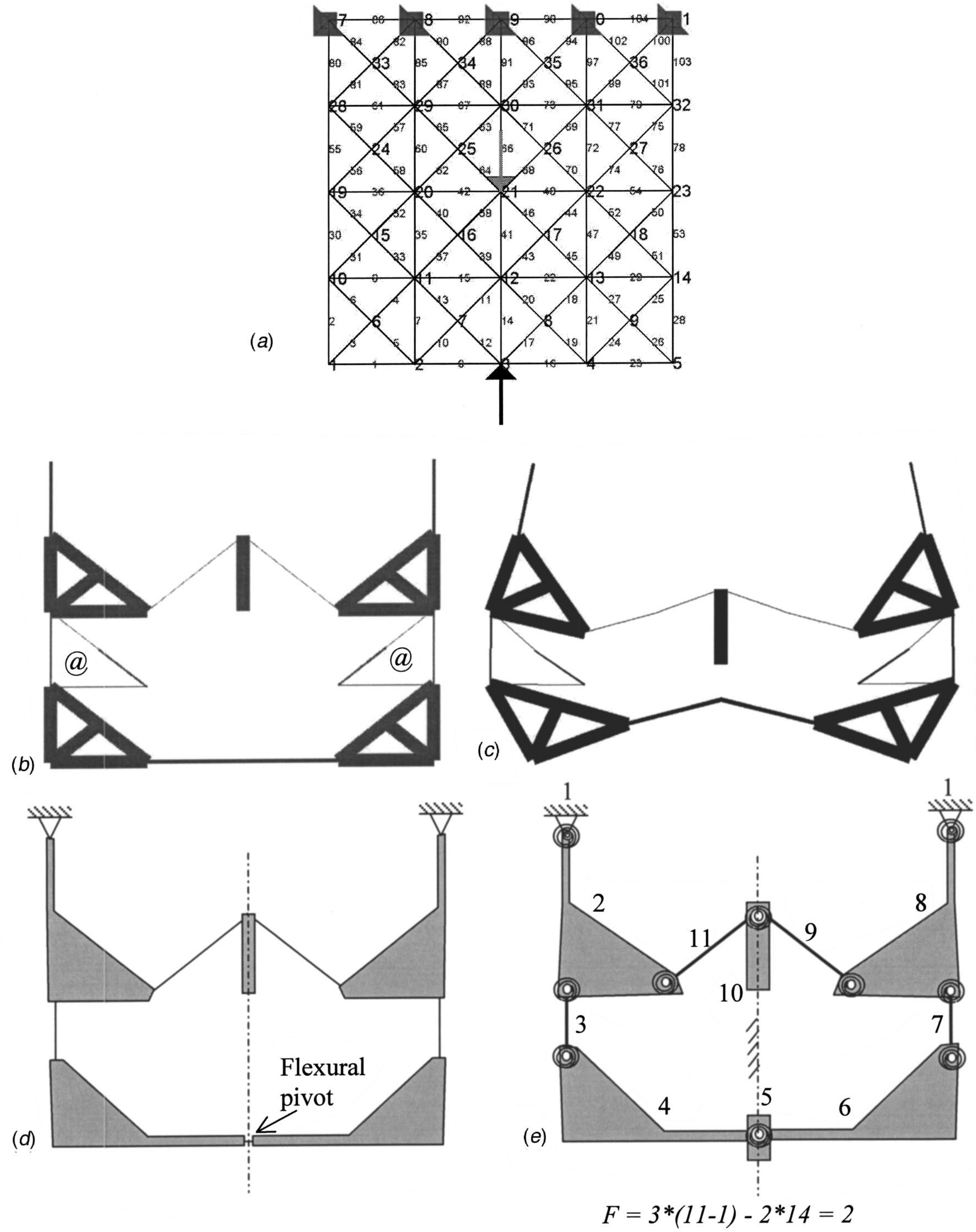

Fig. 7 Example 2 (*a*) task specifications (*b*) optimal topology (*c*) deformed configuration (*d*) interpreted compliant topology (*e*) equivalent rigid-link mechanism

$$\begin{aligned} &\text{Minimize} - \frac{\int_\Omega \varepsilon(u):E:\varepsilon(v)\,d\Omega}{\frac{1}{2}\int_\Omega \varepsilon(u):E:\varepsilon(u)\,d\Omega} \\ &\text{Subject to} \quad \int_\Omega \varepsilon(u):E:\varepsilon(w_u)\,d\Omega = \int_{\Gamma_{in}} F\cdot w_u\,d\Gamma \\ &\qquad\qquad \int_\Omega \varepsilon(v):E:\varepsilon(w_v)\,d\Omega = \int_{\Gamma_{out}} f\cdot w_v\,d\Gamma \end{aligned} \tag{2a}$$

$$\begin{aligned} &\text{Minimize} - \frac{\{\mathbf{v}\}^T[K]\{\mathbf{u}\}}{\frac{1}{2}\{\mathbf{u}\}^T[K]\{\mathbf{u}\}} \\ &\text{Subject to} \quad [K]\{\mathbf{u}\} = \{\mathbf{F}\} \\ &\qquad\qquad [K]\{\mathbf{v}\} = \{\mathbf{f}\} \end{aligned} \tag{2b}$$

where

u = deformation under the applied F

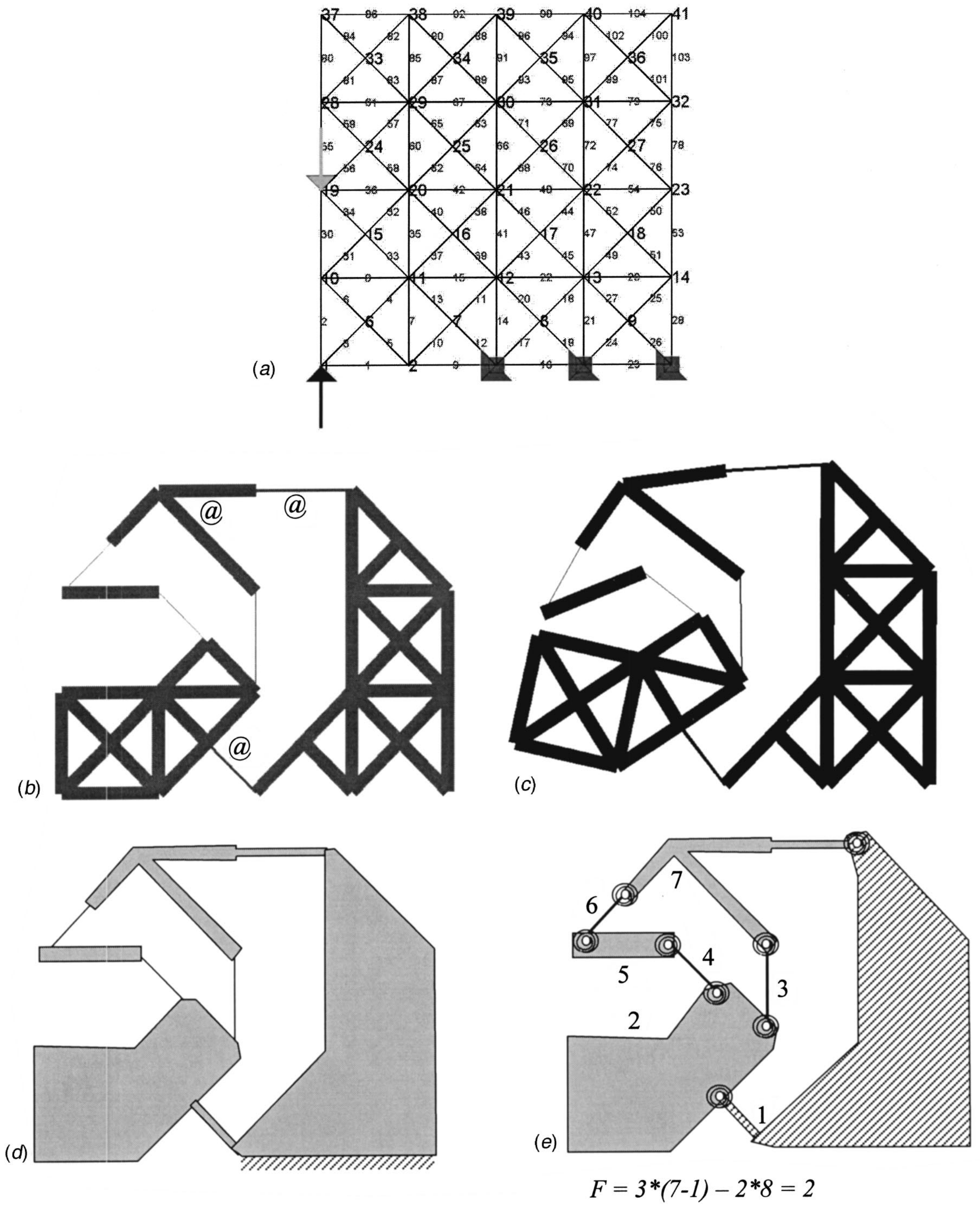

Fig. 8 Example 3 (*a*) task specifications (*b*) optimal topology (*c*) deformed configuration (*d*) compliant topology (*e*) equivalent rigid-link mechanism

v=deformation under the unit dummy load f applied at the output point in the direction of the desired displacement

ε=linear elastic strain tensor

E=stress-strain elasticity tensor

w_u=virtual displacement corresponding to u used to write the equilibrium equation in the weak form for the case of applied load F

w_v=virtual displacement corresponding to v used to write the equilibrium equation in the weak form for the case of the unit dummy load f

Ω=design domain

Γ=boundary of the design domain

{ }=discretized column vector of the corresponding variable

$[K]$=stiffness matrix of the discretized finite element model

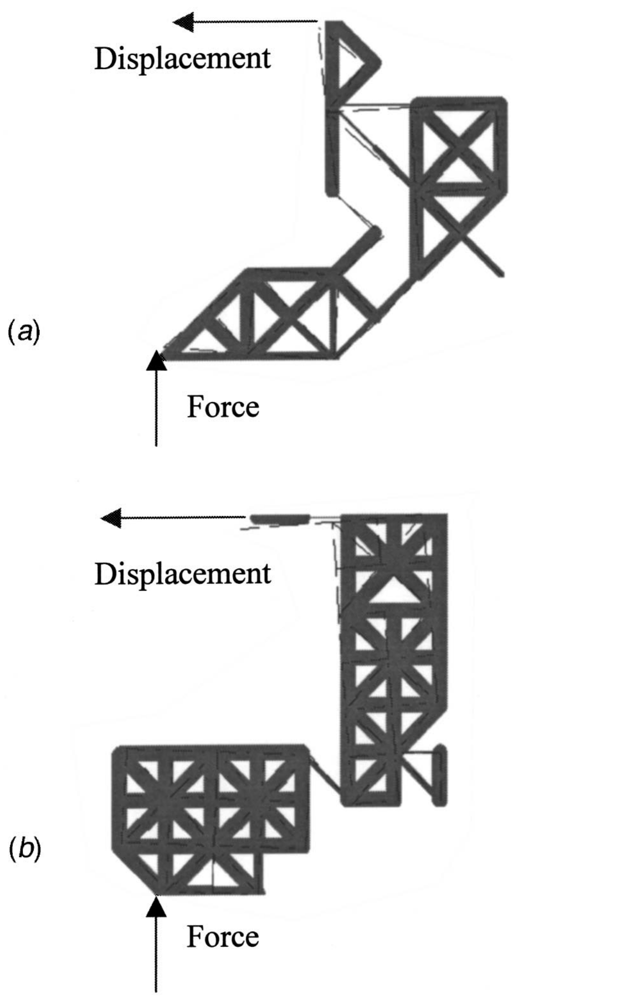

Fig. 9 An example to show the effect of size of the grid on the rsulting topologies. The grid size in (*b*) is half that of (*a*).

The mutual strain energy is essentially the output displacement, which is indicator of the flexibility of the mechanism. The larger this energy the more flexible the mechanism is. Therefore, a minus sign is included in the objective function to maximize this energy. The strain energy is in the denominator because it needs to keep the structure reasonably stiff at the same time. If the input force is specified, the strain energy is directly indicative of the input displacement. The reader is referred to [18] and [26] for a detailed description of this problem statement and the solution procedure.

For the quantitative data shown in Table 1 with uniform thickness for all the elements as the initial guess, the solution for the specifications shown in Fig. 2(*b*) is shown in Fig. 3(*a*) for the discretized model. The widths of the lines in this figure show the optimized relative out-of-plane thicknesses of the beam elements. The design intent in this example is to achieve a large ratio of the output displacement to the input displacement, i.e., the ratio of mutual strain energy to the strain energy as shown in Eqs. (2*a*) and (2*b*). It should be noted that the optimization algorithm did not use the fixed pivots at the bottom. The deformed configuration is shown in Fig. 3(*b*) where it can be seen that the output displacement is significantly amplified by a factor of 10. This compliant topology will be interpreted kinematically in Section 4 after discussing the pseudo rigid-body models.

3 Pseudo Rigid-Body Model

An important concept that is necessary to interpret compliant topologies is briefly reviewed here. This is concerned with modeling the deformation of thin flexible beams using rotating rigid links. In this regard, [27,20] showed that the end-point deflection behavior of a thin, prismatic cantilever beam is kinematically equivalent to a rotating rigid link over a large range of deformation. The length of the pseudo rigid link is shorter than the length of the beam with the pivot located some distance away from the actual fixed connection. Howell and Midha [20,21] systematically and extensively developed this to include accurate quantitative modeling of the kinetostatic behavior by including a torsional spring at the jointed end of the rigid link as illustrated in Fig. 4. This remarkable modeling technique, called the *pseudo rigid-body* model, has been effectively applied to many compliant design problems involving different end conditions [28]. The characteristic radius factor γ, and the torsional spring constant K_Θ, have been derived in convenient non-dimensional form. For a prismatic cantilever with a vertical load at the free end, these are given by [28]:

$$\gamma = 0.85 \quad K_\Theta = 2.25\frac{EI}{L} \tag{3}$$

where E is the Young's modulus, L is the length of the beam, and I is the moment of inertia. These values accurately predict the kinematic behavior up to $\Theta < 64.3$ deg, and kinetostatic behavior up to $\Theta < 58.5$ deg.

For a specialized boundary condition such as the fixed-fixed beam, developing a pseudo rigid-body model over a long range of deformation is complicated when the end moments are opposite in direction to the moments caused by the end forces. This is because an inflection point occurs in the middle of the beam and makes the elliptic integrals-based solution procedure difficult [29,30]. However, for at least short ranges of deformation, a pseudo rigid-body model could be developed by using elliptic integrals [30] or other methods of nonlinear deflection analysis. Such a model consists of a rigid link connected to the fixed portions using revolute joints with torsional springs. Furthermore, the small-length flexible connections between two rigid portions can also be modeled quantitatively using a revolute joint with a torsional spring [31].

4 Kinematic and Kinetostatic Interpretation of Optimized Compliant Topologies

Photographs of the macro-size ABS plastic prototype of the optimized solution in Fig. 3(*a*) are shown in Fig. 1. As can be seen clearly in the photographs, Fig. 3(*a*) and the deformed configuration of Fig. 3(*b*), there are portions in this mechanism that can be considered to be relatively rigid. Furthermore, these rigid portions are connected by thin beams. This interpretation is shown in Fig. 5(*a*) with four rigid portions connected with two beams. By using the pseudo rigid-body model, the compliant topology of Fig. 5(*a*) can be represented equivalently as in Fig. 5(*b*). This rigid-body linkage consists of two ternary links and five binary links including the fixed frame. There are torsional springs at all the joints because in a compliant topology no joint is free from resistance when the deformation occurs. By applying Grübler's formula, the kinematic degrees of freedom (dof) of this linkage can be found to be two. However, because of the presence of the torsional springs, this linkage will deform predictably with a single input force.* If one wishes to use a rigid-body linkage without torsional springs (as shown in Fig. 5*c*), then the two inputs to the linkage must be identified and coordinated to generate the same motion as in the compliant topology. The relationship between any two such inputs can be extracted from the deformed profile of the elastic model of the mechanism. The rotations of two ternary links about their

*It is also possible to compute the compliant degrees of freedom as discussed in [32,33], but that interpretation is not relevant for the current discussion.

respective fixed pivots constitute one possible set of two inputs for this linkage. Since the elastic analysis of the compliant topology gives these two rotations, coordinating them is straightforward. Thus, the rigid-body linkage retains the kinematic behavior of the compliant topology.

The corresponding kinematic chain with two degrees of freedom is shown in Fig. 5(*d*). This would have been one of many possible kinematic chains a traditional number synthesis technique would generate. Here, by using an optimization method and pseudo rigid-body model, the optimal linkage configuration is generated systematically. In the same process, the dimensions of the linkage are determined. As shown in this example, if the resulting configuration has more than one degree of freedom, the torsional springs can be used to achieve the same kinetostatic behavior as the optimized compliant topology. The procedure is summarized in the next section and more examples are presented.

5 Procedure and Examples

The procedure described above using the displacement amplifier is repeated for a number of other examples. This consisted of the following steps. For the quantitative data shown in Table 2 and the beam super structure consisting of 104 elements and 41 nodes, the compliant topology was obtained using PennSyn, the compliant topology design software [34]. PennSyn has a graphical user interface that enables the designer to specify any boundary conditions, forces, desired output displacements, grid size, and other parameters. It also animates the deformation of the solution topology to see which portions are deforming and which relatively move as rigid bodies. The compliant topology is then interpreted first as rigid portions and thin beams, and then using the pseudo rigid-body model, equivalent rigid-body linkages are constructed. This was done manually but can be automated. Finally, Grübler's formula was applied to obtain the kinematic degrees of freedom. A number of examples were solved which resulted in kinematic degrees of freedom of one to six. Four representative examples are shown in Figs. 6–9. In each figure, kinetostatic design specifications are shown in (a), the topology solution in (b) where relative thicknesses of the beam elements are shown as different line-widths, deformed configuration in (c), rigid-flexible interpretation in (d), and the equivalent rigid-body linkage in (e). The boundary conditions in Figs. 6(*a*)–8(*a*) should be recognized as follows. At a node, a triangle with the vertex facing upwards indicates the restriction of the y degree-of-freedom; a triangle with the vertex facing right indicates the restriction of the x degree-of-freedom, and a square indicates the restriction of the slope degree-of-freedom (i.e., the rotation). The solid arrow indicates the direction of the applied force and the gray arrow the direction of the desired displacement.

As mentioned earlier, the basis for the interpretation of rigid portions and thin beams is not only the optimized thicknesses of the beam elements but also the deformation pattern. An example of this can be seen in Fig. 6(*b*) where a thin element is interpreted as part of a rigid portion as there is no significant deformation between the elements marked with the ''@'' symbol. This results in a four-bar mechanism with one degree of freedom as shown in Fig. 6(*e*). This is one of the many examples that illustrates that a ''mechanism'' is needed to achieve a non-intuitive design specification such as the one in Fig. 6(*a*). A ''structure'' would simply deflect to the left when a force is applied to the left. Therefore, the optimization algorithm generated a compliant ''mechanism'' topology that is equivalent to a four-bar mechanism. The example in Fig. 7 also has a non-intuitive design specification and has symmetry. Fig. 7(*d*) does not include a compliant triangle marked with ''@'' symbol in Fig. 7(*b*) because the kinetostatic effect of this is included in the torsional springs shown in Fig. 7(*e*). Even if this triangle is included, the kinematic behavior and degrees-of-freedom would still be the same as can be easily verified. Note that a small-length flexural pivot is included at the middle of the bottom edge in Fig. 7(*d*). This is done to let the two thin beams deflect as shown in Fig. 7(*c*) when the input force is applied. This is consistent with the nomenclature developed for compliant mechanisms [32,33]. Two sliders are included in Fig. 7(*e*) to take the symmetry into account. This gave rise to two kinematic degrees of freedom, which can be easily seen to be the movements of the two sliders. However, when there are torsional springs, the motion of one of the sliders can predict the motion of all the links. If symmetry is ignored, the kinematic degrees of freedom will be six as there will be two less one degree-of-freedom joints in the application of the Grübler's formula.

The task specification in Fig. 8(*a*) is nonintuitive for a human designer to guess a candidate topology. PennSyn generated the topology shown in Fig. 8(*b*). Once again, a think element, marked with ''@'', is interpreted as part of a rigid portion based on the deformation pattern. As shown in Fig. 8(*e*), the equivalent rigid-body linkage has seven links with two degrees of freedom.

It should be noted that the optimization problem posed in Eq. (2) is nonconvex in that there are multiple local minima. The optimization algorithms used in PennSyn lead to a local minimum from an arbitrarily chosen initial guess. In most cases, the desired force-deflection relationship is satisfactorily met by the local minimum. Furthermore, the size of the grid also affects the outcome. An example is shown in Fig. 9. The same problem is solved for two grid sizes. The size of the grid for the solution in Fig. 9(*b*) is half of that of Fig. 9(*a*). The procedure of identifying the equivalent rigid-body linkage remains the same for any of the solutions obtained with the topology optimization procedure. The main conclusion is that a local optimum is found for given problem specifications, and finding a global solution is still an open issue.

Discussion

The scope of number synthesis considered in this paper is broader than the traditional connotation. Instead of considering only the kinematic specifications, forces are also included to design for kinetostatic behavior. Instead of merely enumerating the different possibilities, the optimization algorithm effectively searches through the space of all possible configurations and picks the best one. Pseudo rigid-body models provide an equivalent rigid-body linkage whose dimensions are also readily computed. If one wishes not to include the torsional springs when multiple degrees of freedom exist, sufficient quantitative information is available for coordinated multiple actuation so that the intended kinematic behavior is retained. As the optimal topology methods and pseudo rigid-body model theories are further developed, the procedure outlined here becomes more useful. If strength related criteria [35] and dynamic response characteristics are included in the topology optimization, the shapes of the links determined by the optimization also become relevant. It is also worth noting that pseudo rigid-body models for dynamic behavior are already underway [36]. Thus, the entire design of the linkage with optimal configuration, dimensions, and shapes of the links is accomplished systematically.

The principal motivation of this paper was to connect two approaches to compliant mechanisms as it serendipitously enables a systematic solution for the number synthesis of linkages. A question might still arise as to why one would use rigid-body linkages instead of compliant mechanisms. Notwithstanding numerous advantages of compliant mechanisms, designers might prefer rigid-body linkages because of various reasons. Some of these include: lack of a suitable material, concern over the fatigue life, and unwillingness to change the existing manufacturing setup. Even in those circumstances, the methods developed for compliant mechanisms are still useful as demonstrated in this paper.

6 Conclusions

There are many techniques of number synthesis that can enumerate all possible kinematic chains for a specified number of degrees of freedom and other design intents. But there are much fewer techniques that can evaluate them to pick the best configuration. In this paper, a novel perspective to number synthesis is proposed. A combination of the elastic mechanics-based topology optimization method and the pseudo rigid-body model made this approach possible. The computational procedure for number synthesis is outlined and illustrated with examples. In the end, not only the number of links and their connectivity are determined but also the dimensions of the links, their initial orientations, and sizes are obtained. The procedure is clearly extendable to many other situations. Another main conclusion is that the elastic mechanics-based topology optimization method accounts for the kinematic behavior in the same way as traditional kinematic synthesis techniques do, although it may not be apparent at first sight.

Acknowledgments

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