Proof of formula for derivatives of a matrix within a Frobenius norm

For matrices $\mathbf{W}$, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ defined such that the following equation makes sense, we have

$$\frac{\partial \|\mathbf{AWB}+\mathbf{C}\|_2^2}{\partial \mathbf{W}} = 2\mathbf{A}^\top(\mathbf{AWB}+\mathbf{C})\mathbf{B}^\top \tag{1}$$

Proof: let's write $\mathbf{Z} = \mathbf{AWB}+\mathbf{C}$.
$\|\mathbf{AWB}+\mathbf{C}\|_2^2 = \|\mathbf{Z}\|_2^2 = \sum_i \sum_j (Z_{i,j})^2$, and

$$\frac{\partial \|\mathbf{Z}\|_2^2}{\partial \mathbf{W}} = \sum_i \sum_j \frac{\partial (Z_{i,j})^2}{\partial \mathbf{W}} \tag{2}$$

We have $Z_{i,j} = \sum_k A_{i,k}(\mathbf{WB})_{k,j} + C_{i,j}$, or

$$Z_{i,j} = \sum_k A_{i,k}\left(\sum_l W_{k,l}B_{l,j}\right) + C_{i,j} \tag{3}$$

In Equation (2), for any $W_{m,n}$, we have

$$\frac{\partial (Z_{i,j})^2}{\partial W_{m,n}} = 2Z_{i,j}\frac{\partial Z_{i,j}}{\partial W_{m,n}} \tag{4}$$

Using Equation (3),

$$\frac{\partial Z_{i,j}}{\partial W_{m,n}} = A_{i,m}B_{n,j} \tag{5}$$

Combining equations (2), (4) and (5), we have

$$\frac{\partial \|\mathbf{Z}\|_2^2}{\partial W_{m,n}} = \sum_i \sum_j (2Z_{i,j}A_{i,m}B_{n,j}) = 2\sum_i (\mathbf{A}^\top)_{m,i}\left(\sum_j Z_{i,j}(\mathbf{B}^\top)_{j,n}\right)$$

and finally

$$\frac{\partial \|\mathbf{Z}\|_2^2}{\partial W_{m,n}} = 2(\mathbf{A}^\top\mathbf{Z}\mathbf{B}^\top)_{m,n}$$

Q.E.D.