CSC 481: Horn Clauses

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- While refinements to Resolution or other inference procedures may help in some cases, the problem of first-order entailment is inherently intractable
- Horn Clauses are less expressive than full First-Order Logic, but there exist very efficient algorithms to check entailment
- We will start by studying resolution with Horn clauses in the context of propositional logic, then briefly talk about statements involving predicates, functions and equality.

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  - Clauses with one positive literal and no negative literals (“facts”)

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- Clauses with one positive literal and one or more negative literals ("rules")
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  - Can be rewritten as an implication: $a \land b \land c \rightarrow d$
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    - Ex: $\neg d \vee \neg e$

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- Types of Horn Clauses:

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  - Clauses with no positive literals ("goal clauses")
    - Ex: $\neg d \vee \neg e$
    - The empty clause [] is a special case of goal clause

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  - $sun \lor rain$

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  - $sun \vee rain$
  - $beach \rightarrow coconut \vee ice\_cream$

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  - Forward chaining: from premises to conclusions
  - Backward chaining: from a desired conclusion to its preconditions

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  - SLD derivation (see next slide)

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  - c_i is negative for all i
  - $c_1 \in S$
  - $c_n = \alpha$
  - c_{i+1} resolves from c_i and some clause in S

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- In other words, we get a “chain” where each new clause c_i resolved with some clause originally in S to produce the next clause c_{i+1}
- SLD stands for Selected literals, Linear pattern over Definite clauses
- Every proof in Horn clauses admits an SLD derivation
  - But not every correct derivation is SLD

Example

1. Consider the following KB and try to prove $KB \vdash E$:

   1. $A \rightarrow B$

   2. $C \rightarrow D$

   3. $B \wedge D \rightarrow E$

   4. A

   5. C

   6. $\neg E$

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6. $\neg E$

SLD derivation:

7. $\neg B \vee \neg D$ (3,6)
8. $\neg B \vee \neg C$ (2,7)
9. $\neg B$ (5,8)
10. $\neg A$ (1,9)
11. [] (4,10)

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Non SLD derivation:

7. B (1,4)
8. D (2,5)
9. $\neg B \vee \neg D$ (3,6)
10. $\neg D$ (7,9)
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KB	SLD derivation:		Non SLD derivation:	
1. $A \to B$	7. $\neg B \vee \neg D$	(3,6)	7. B	(1,4)
2. $C \to D$	8. $\neg B \vee \neg C$	(2,7)	8. D	(2,5)
3. $B \wedge D \to E$	9. $\neg B$	(5,8)	9. $\neg B \vee \neg D$	(3,6)
4. A	10. $\neg A$	(1,9)	10. $\neg D$	(7,9)
5. C	11. []	(4,10)	11. []	(8,10)
6. $\neg E$				

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Non SLD derivation:

7. B	(1,4)
8. D	(2,5)
9. $\neg B \vee \neg D$	(3,6)
10. $\neg D$	(7,9)
11.[]	(8,10)

Exercise

- Consider the following KB (AIMA Ch. 7.5.4), and try to prove $KB \vdash Q$:

  1. $P \rightarrow Q$
  2. $L \wedge M \rightarrow P$
  3. $B \wedge L \rightarrow M$
  4. $A \wedge P \rightarrow L$
  5. $A \wedge B \rightarrow L$
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A. By forward chaining: start with the facts A and B and try to produce Q

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    A. Only resolve facts with a negative of other clauses

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A. By forward chaining: start with the facts A and B and try to produce Q
   A. Only resolve facts with a negative of other clauses

B. By backward chaining: start with the negated query $\neg Q$ and try to produce the empty clause t

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   A. Only resolve facts with a negative of other clauses

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   A. Only resolve goals with the positive of other clauses

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3. $B \wedge L \rightarrow M$
4. $A \wedge P \rightarrow L$
5. $\cancel{A} \wedge \cancel{B} \rightarrow L$
6. A
7. B

$8. \neg Q$ (query)

$9. L$ (5,6,7)

Note: we could also do this in two steps, writing first

$8. B \rightarrow L (5,6)$

Then

$9. L (7,8)$

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10. M (3,7,9)

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10. M (3,7,9)
11. P (2,9,10)

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4. $A \wedge P \rightarrow L$
5. $\cancel{A} \wedge \cancel{B} \rightarrow L$
6. A
7. B

8. $\neg Q$ (query)
9. L (5,6,7)
10. M (3,7,9)
11. P (2,9,10)
12. Q (1,11)

Note: Here, it was possible for us to derive L again from 4,6,11, but an efficient algorithm should check that this fact had been generated before and not waste more time processing it.

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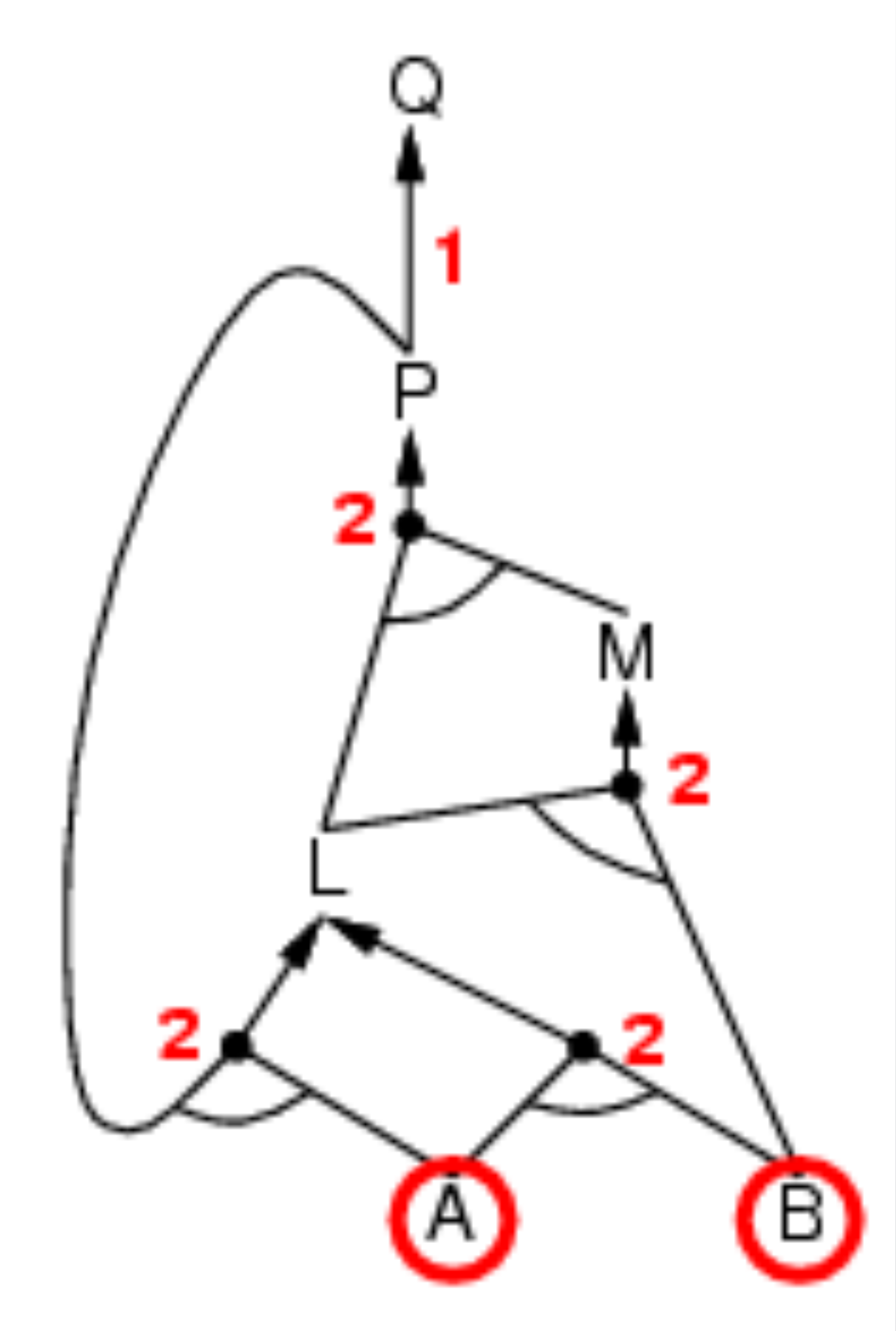
4. $A \wedge P \to L$
5. $\cancel{A} \wedge \cancel{B} \to L$
6. A
7. B

8. $\neg Q$	(query)
9. L	(5,6,7)
10. M	(3,7,9)
11. P	(2,9,10)
12. Q	(1,11)
13. []	(8,12)

Forward chaining example



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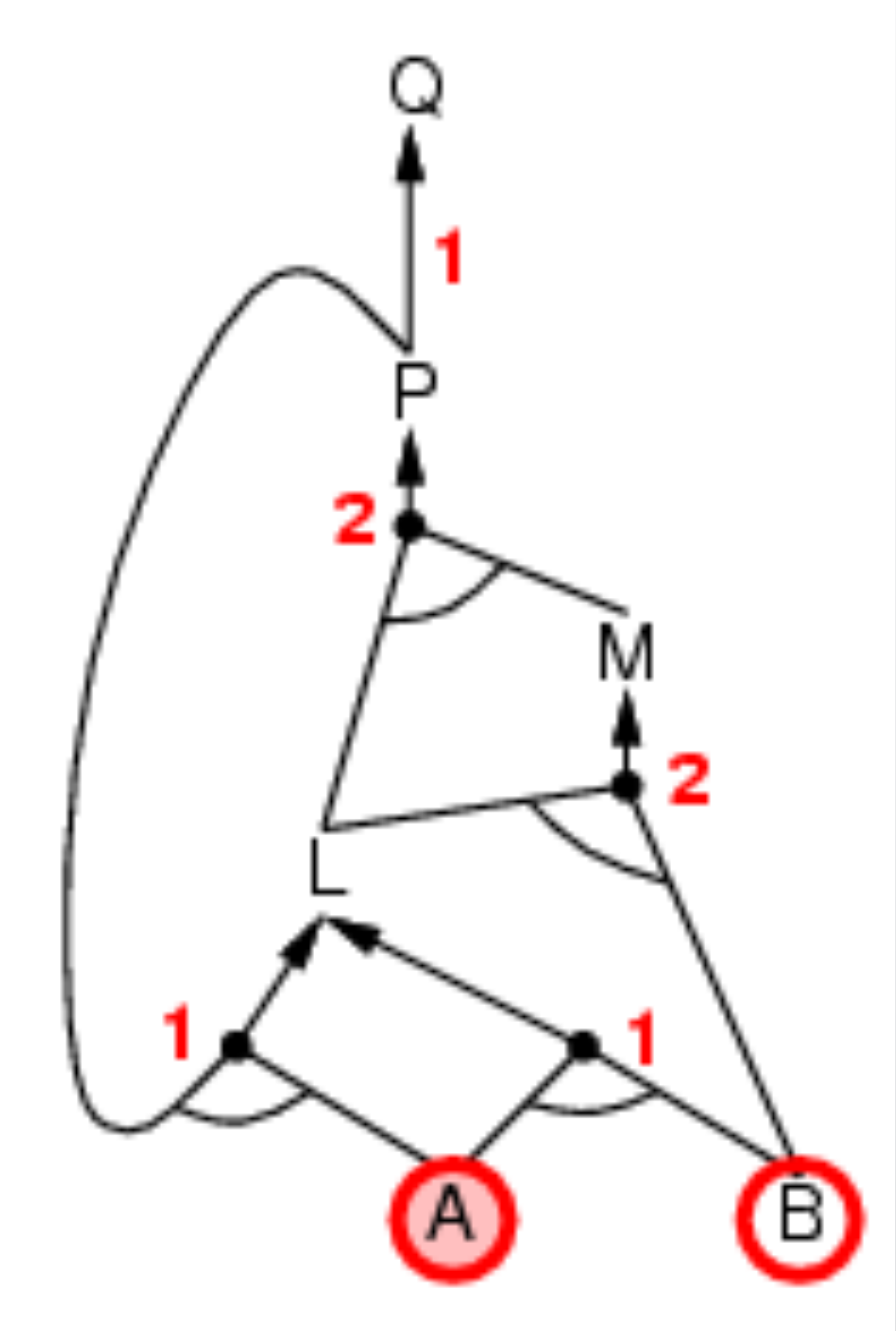

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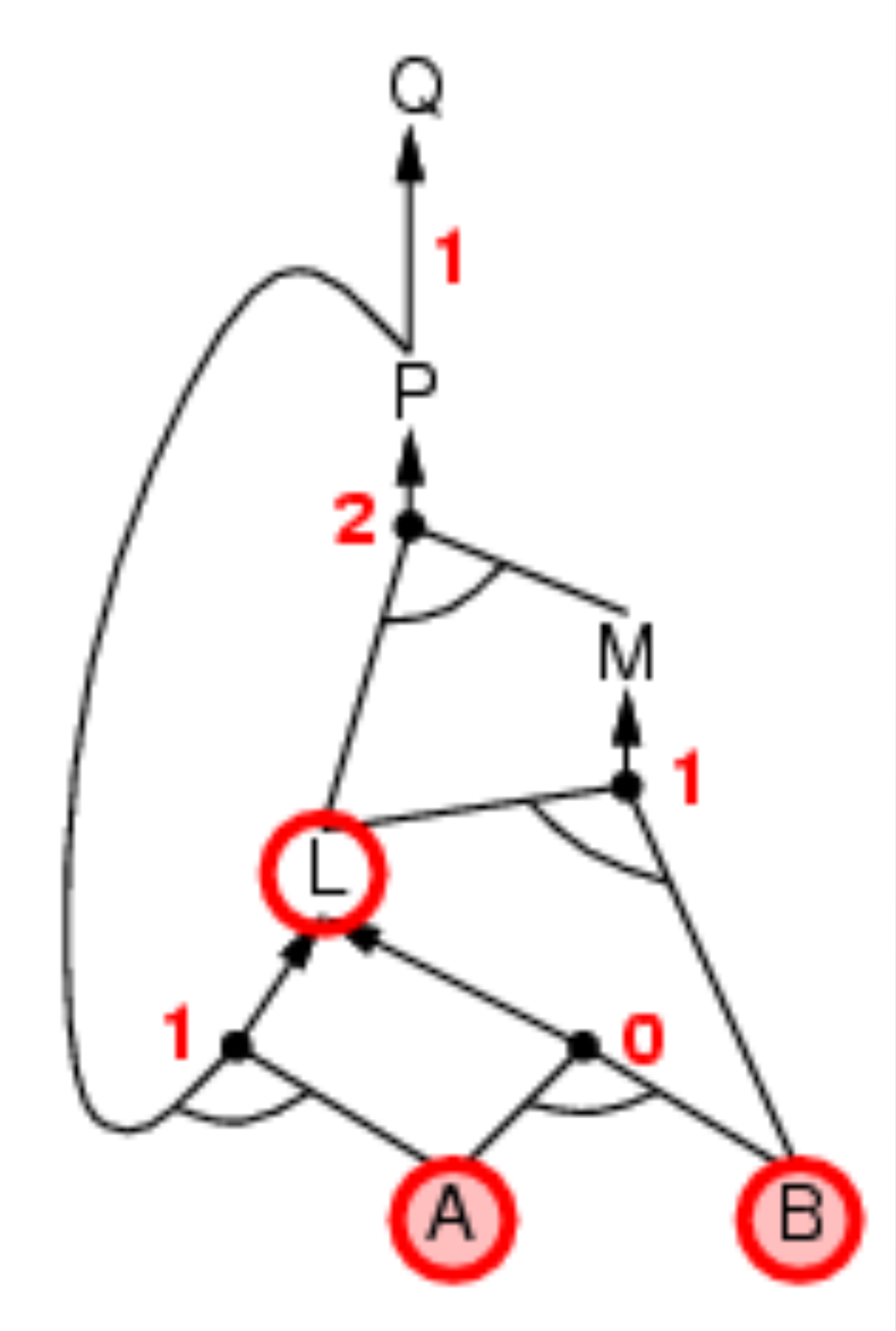

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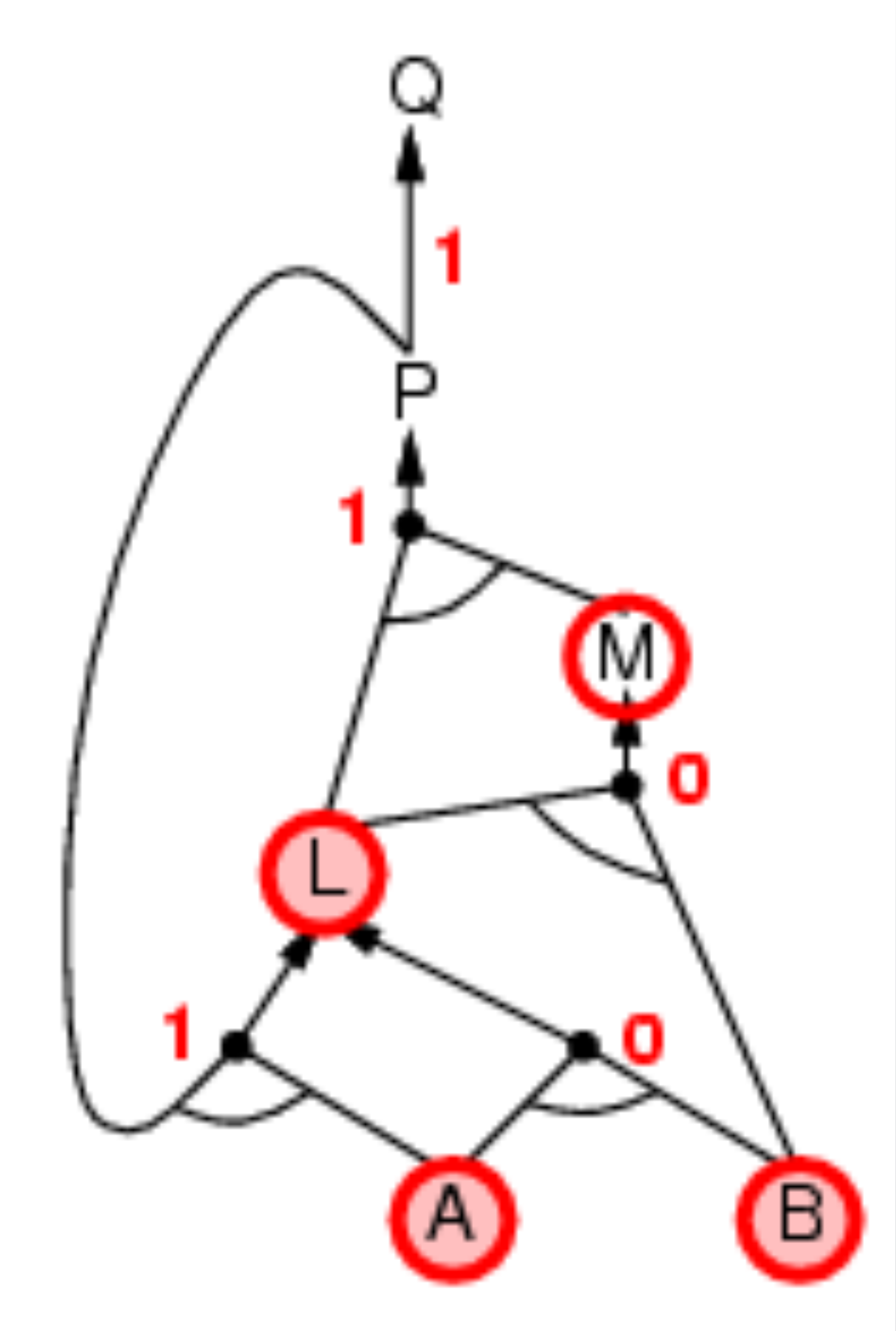

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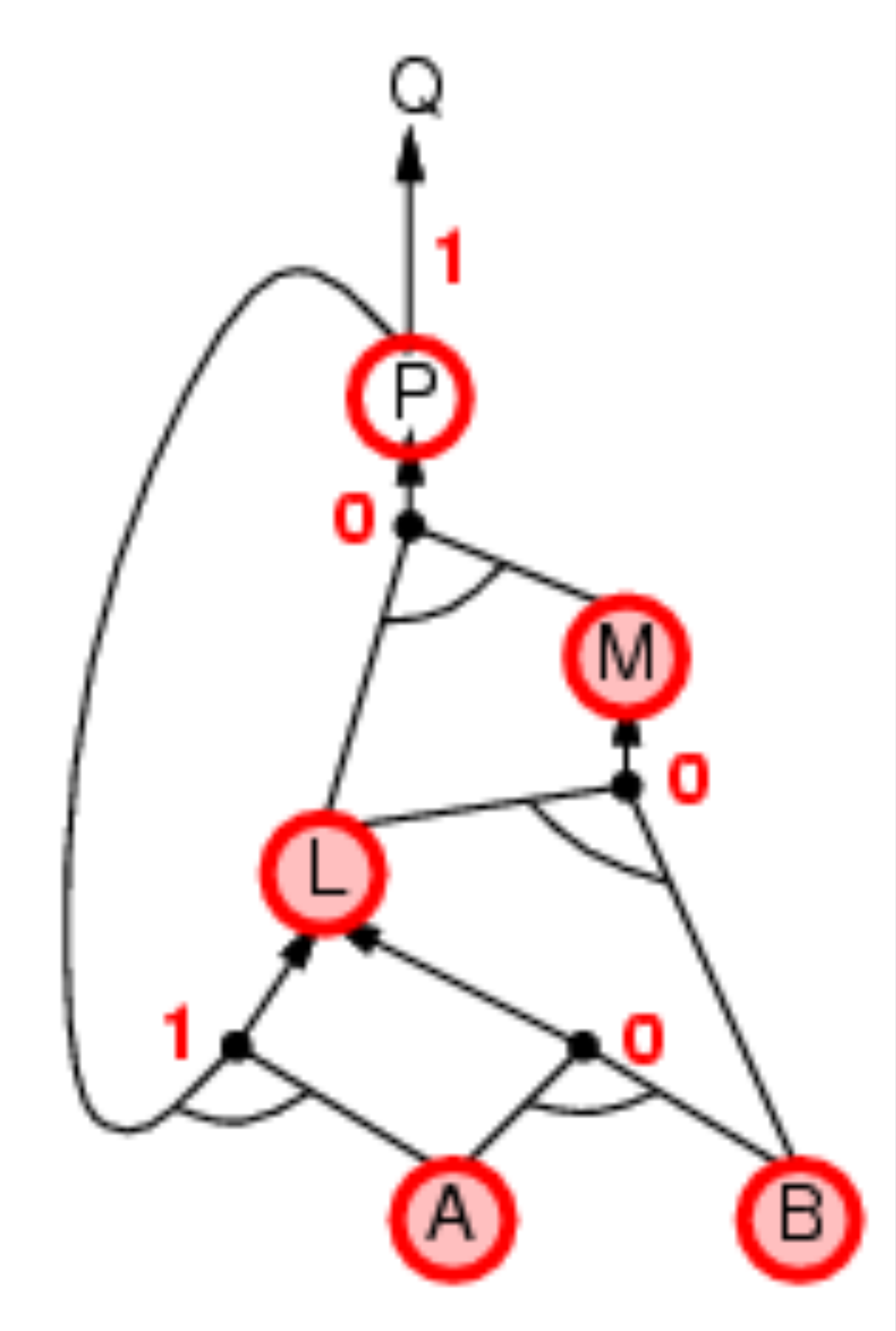

http://aima.eecs.berkeley.edu/slides-ppt/

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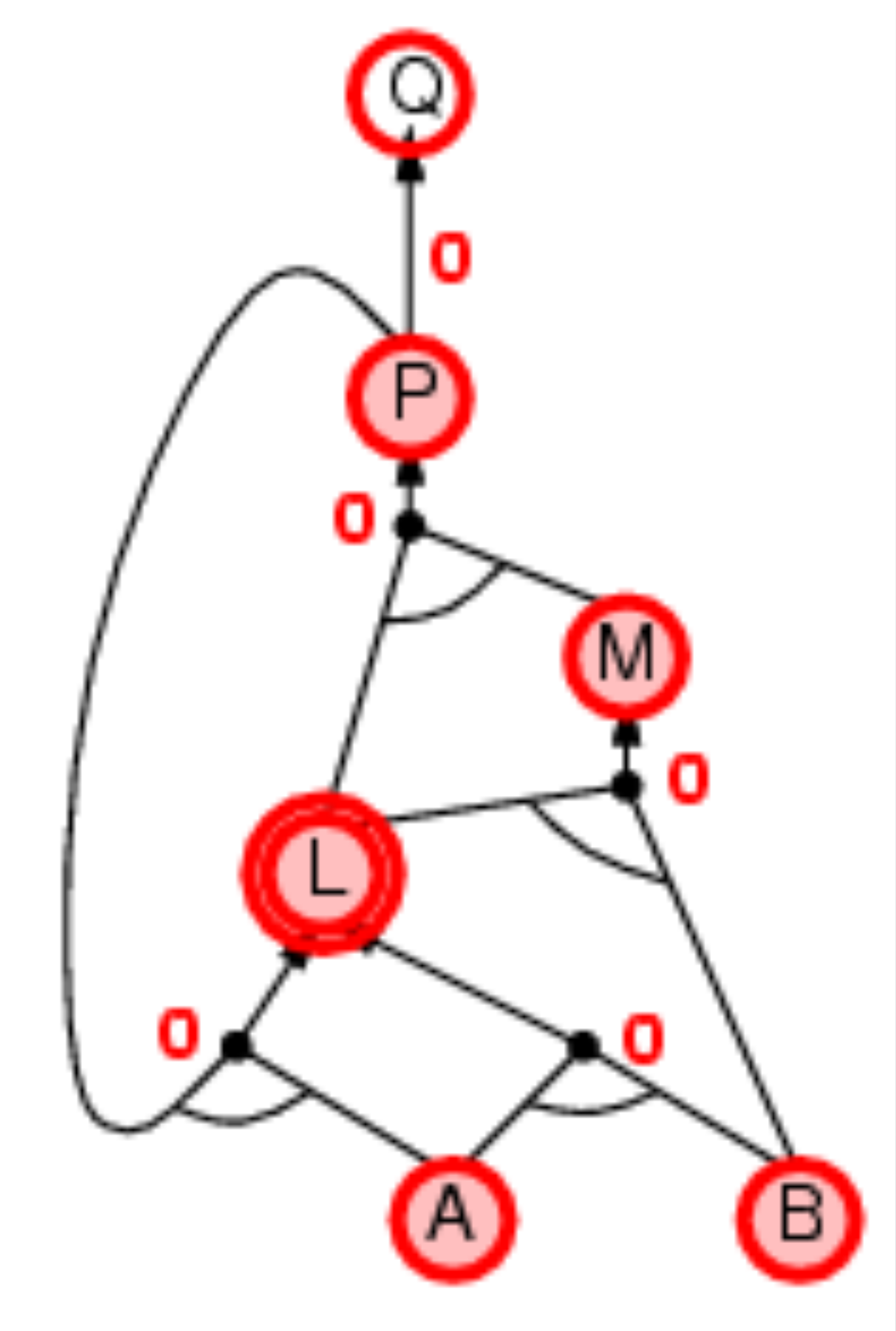

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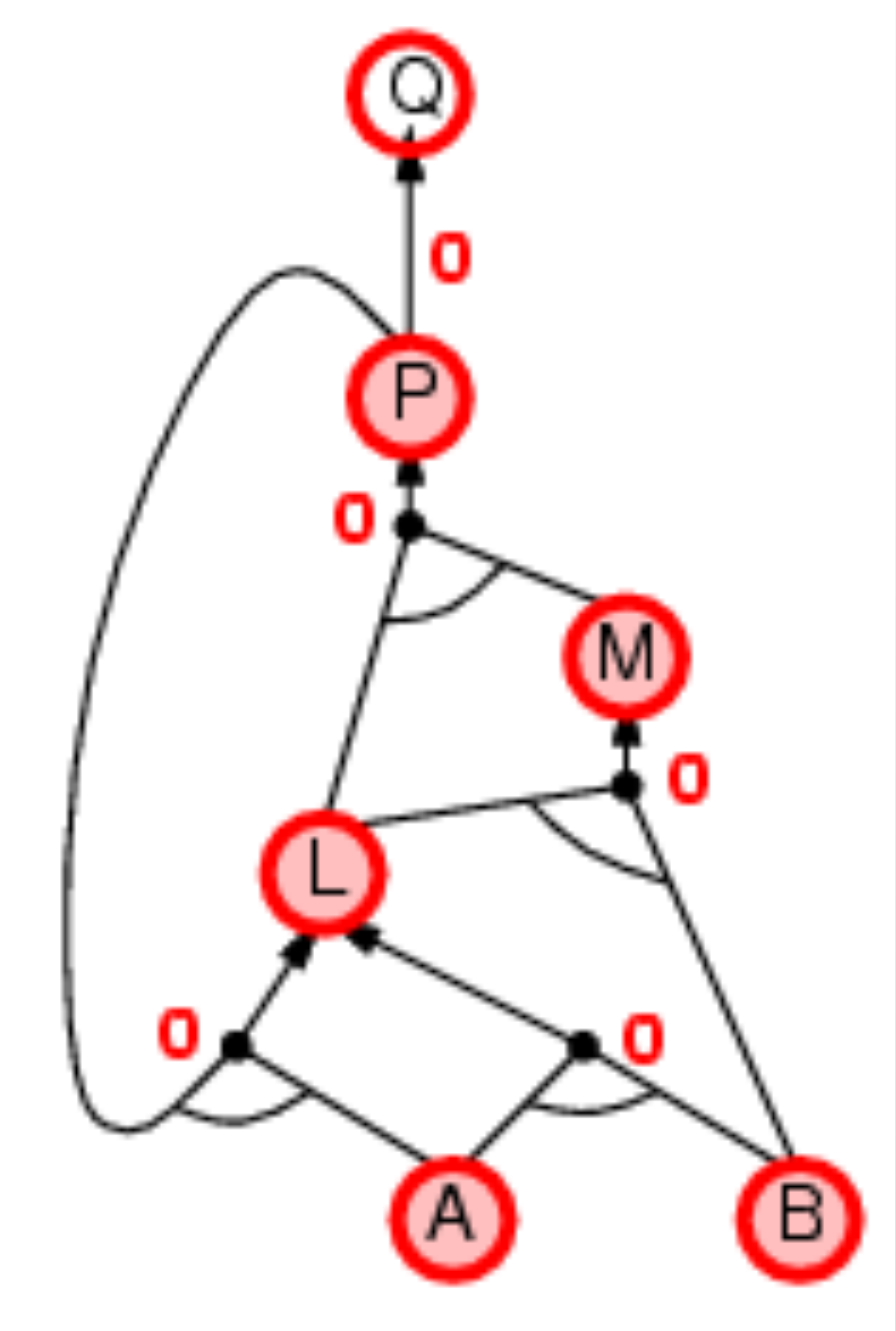

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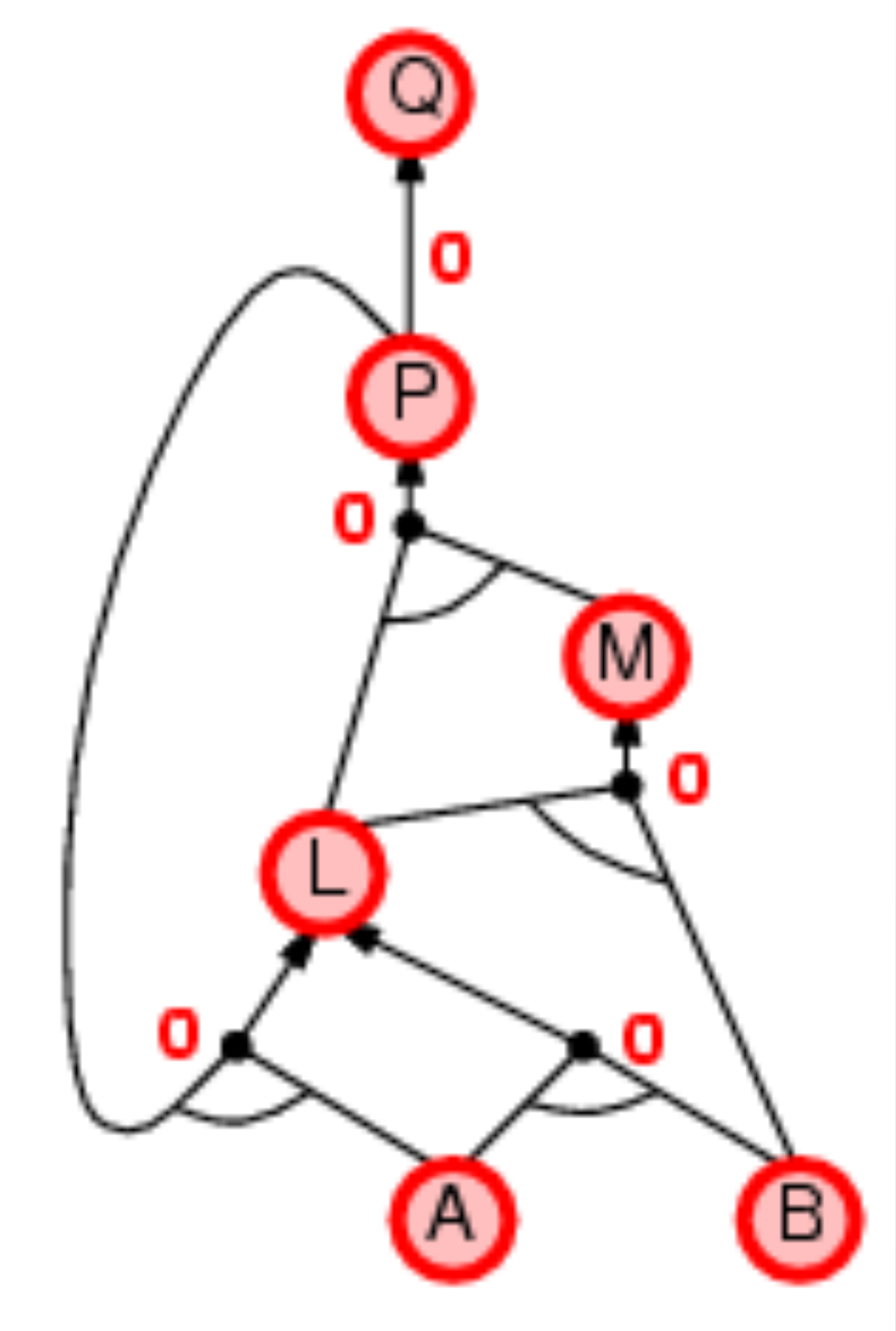

Backward

- Consider the following KB (AIMA Ch. 7.5.4), and try to prove $KB \vdash Q$:

1. $\neg P \lor Q$
2. $\neg L \lor \neg M \lor P$
3. $\neg B \lor \neg L \lor M$
4. $\neg A \lor \neg P \lor L$
5. $\neg A \lor \neg B \lor L$
6. A
7. B

8. $\neg Q$ (query)

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- Consider the following KB (AIMA Ch. 7.5.4), and try to prove $KB \vdash Q$:

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3. $\neg B \lor \neg L \lor M$
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8. $\neg Q$ (query)
9. $\neg P$ (1,8)

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- Consider the following KB (AIMA Ch. 7.5.4), and try to prove $KB \vdash Q$:

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3. $\neg B \vee \neg L \vee M$
4. $\neg A \vee \neg P \vee L$
5. $\neg A \vee \neg B \vee L$
6. A
7. B

8. $\neg Q$ (query)
9. $\neg P$ (1,8)
10. $\neg L \vee \neg M$ (2,9)

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7. B

8. $\neg Q$ (query)
9. $\neg P$ (1,8)
10. $\neg L \vee \neg M$ (2,9)
11. $\neg B \vee \neg L$ (3,10)

Note: resolving (5,10) actually gives us)
$\neg B \vee \neg L \vee \neg L$
But we can eliminate ("collect") the repeated $\neg L$ term (see book sec 4.1)

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8. $\neg Q$ (query)

9. $\neg P$ (1,8)

10. $\neg L \vee \neg M$ (2,9)

11. $\neg B \vee \neg L$ (3,10)

12. $\neg A \vee \neg B$ (5,11)

Note: we could instead resolve (4,10) to get

$$\neg A \vee \neg B \vee \neg P$$

But this would lead us to a dead end: after resolving this with 6 (A) and 7 (B), we would get

$$\neg P$$

Which is the exact same goal we've already attempted to solve from clause 9. Ideally, our algorithm would detect this repeated goal and move to another available clause such as 5.

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- Consider the following KB (AIMA Ch. 7.5.4), and try to prove $KB \vdash Q$:

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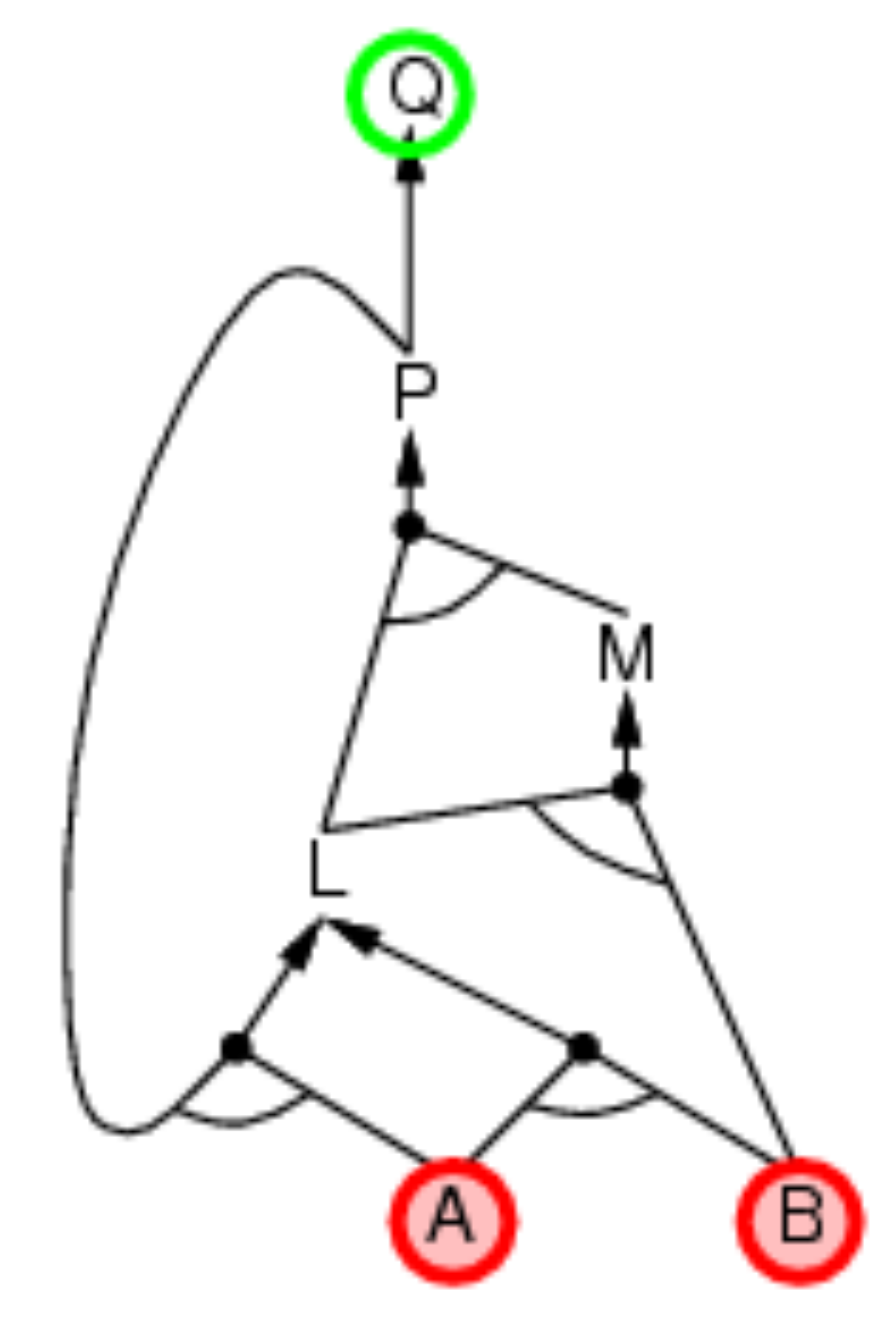
8. $\neg Q$ (query)
9. $\neg P$ (1,8)
10. $\neg L \lor \neg M$ (2,9)
11. $\neg B \lor \neg L$ (3,10)
12. $\neg A \lor \neg B$ (5,11)
13. $\neg B$ (6,12)
14. [] (7,13)

Backward chaining example



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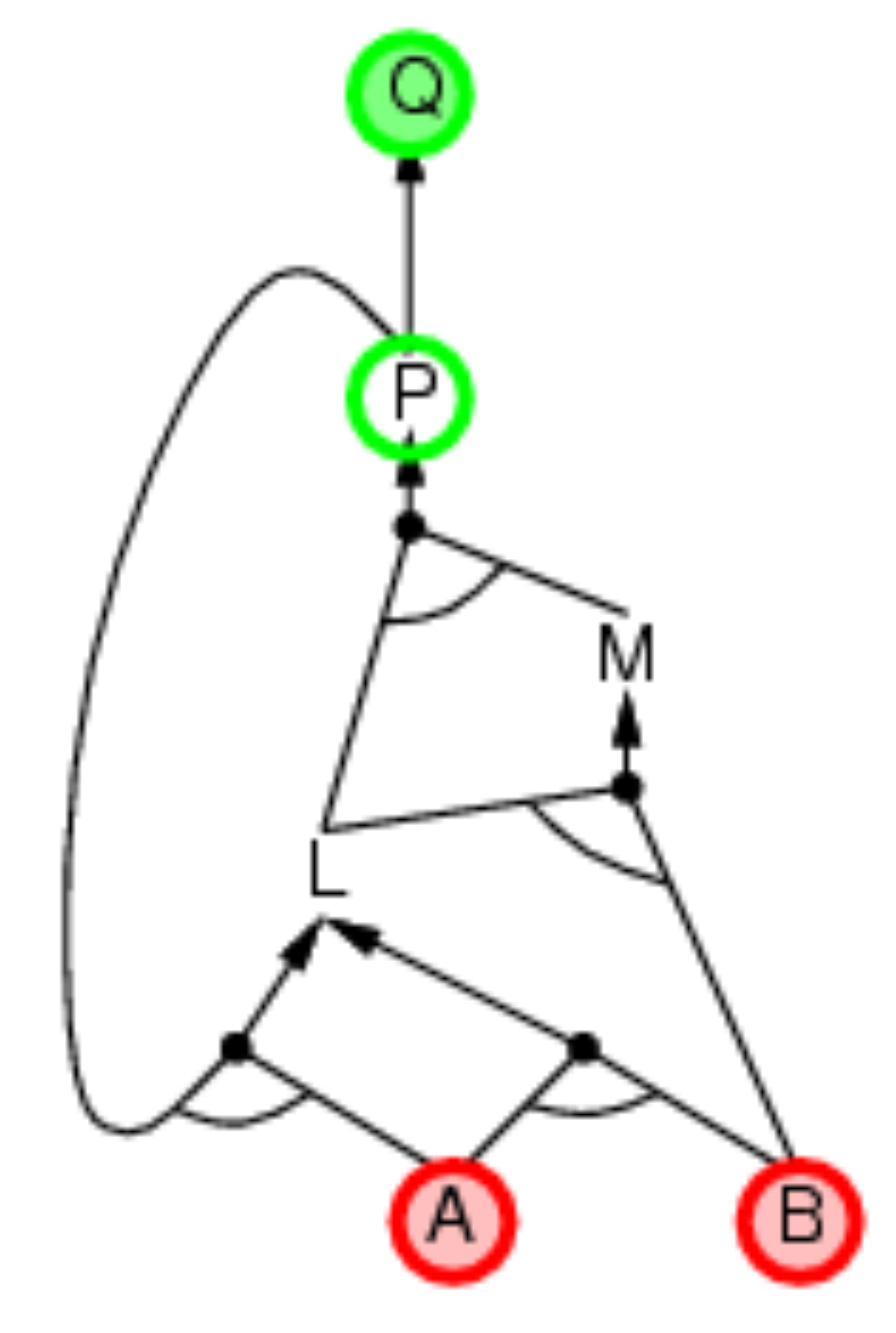

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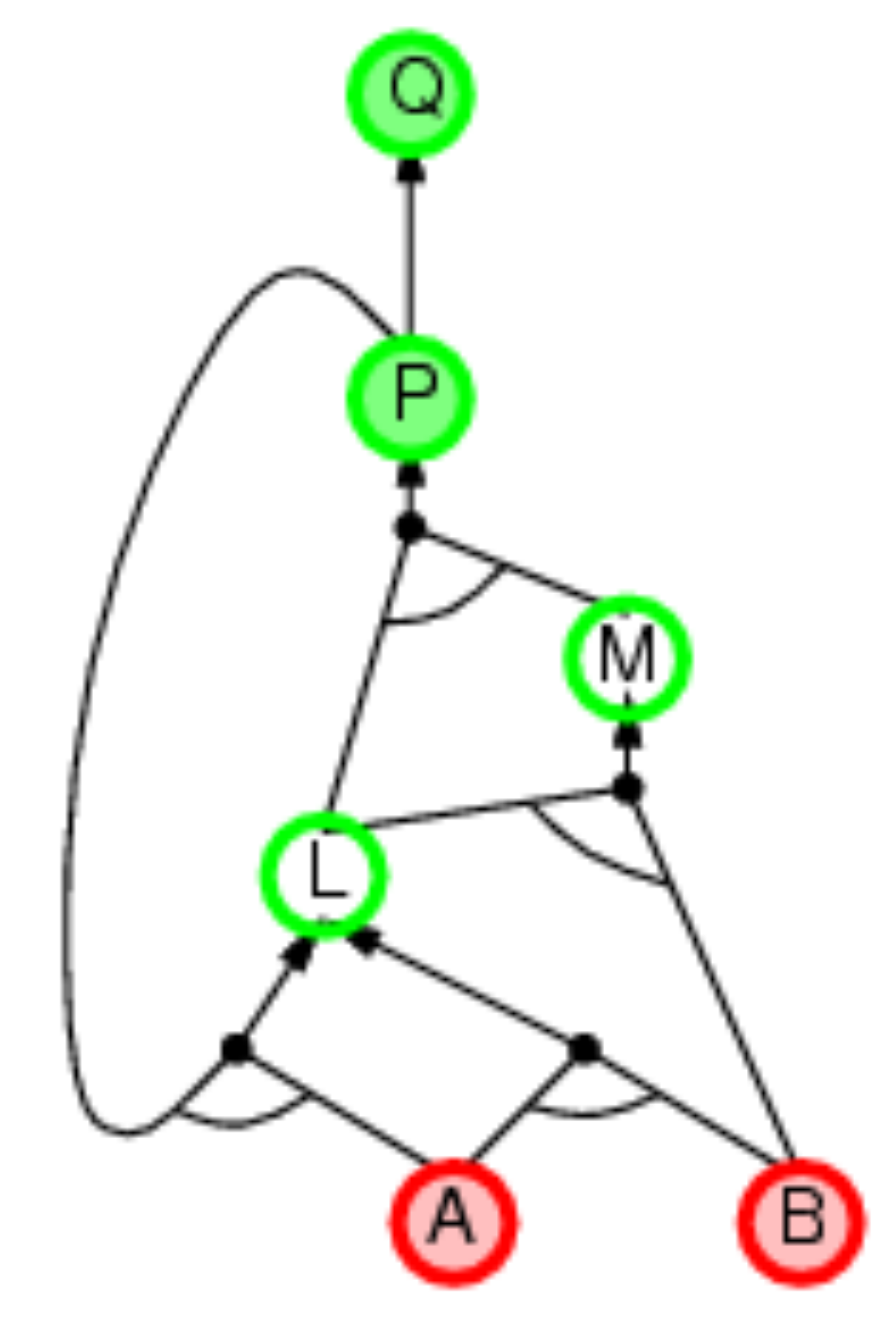

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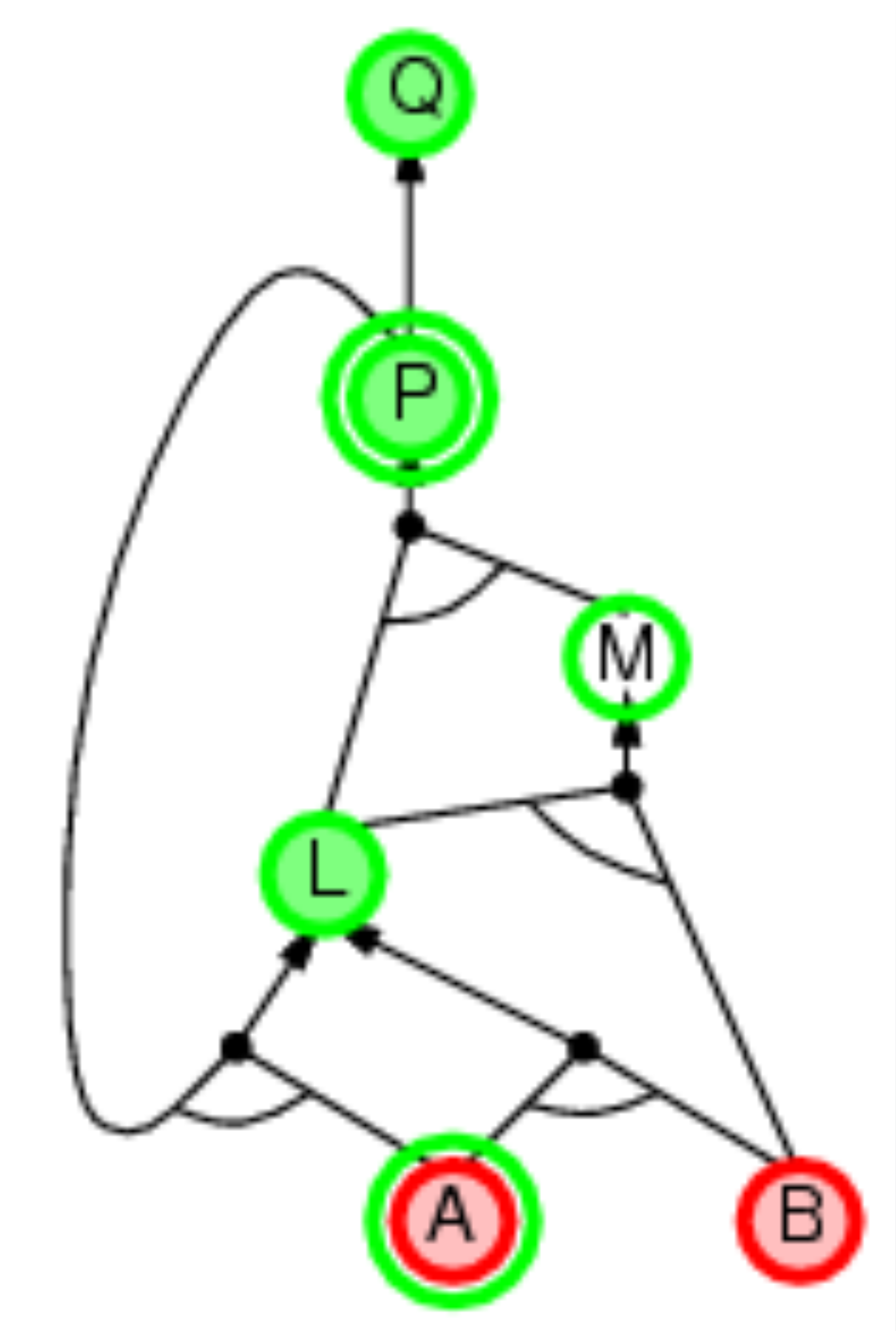

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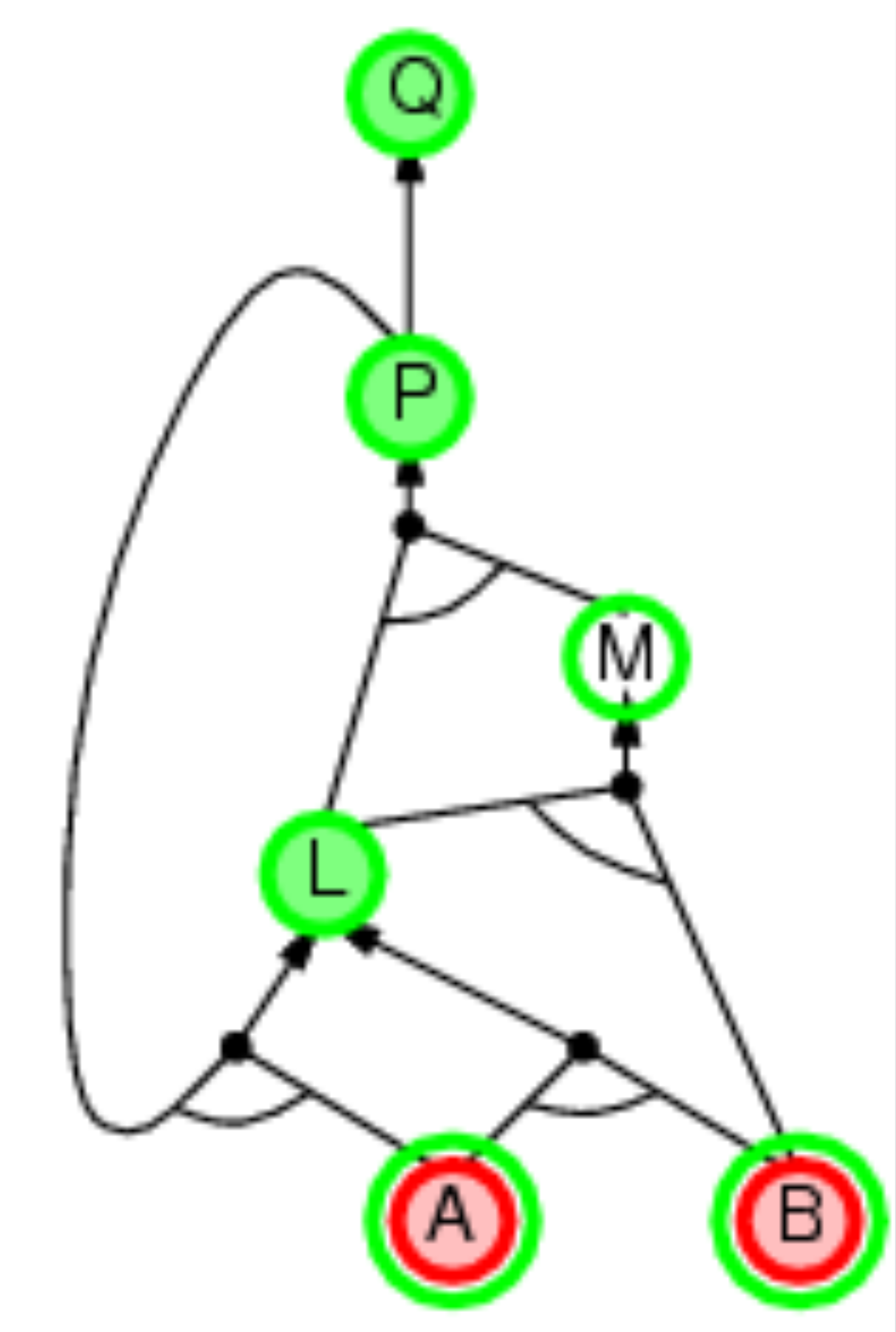

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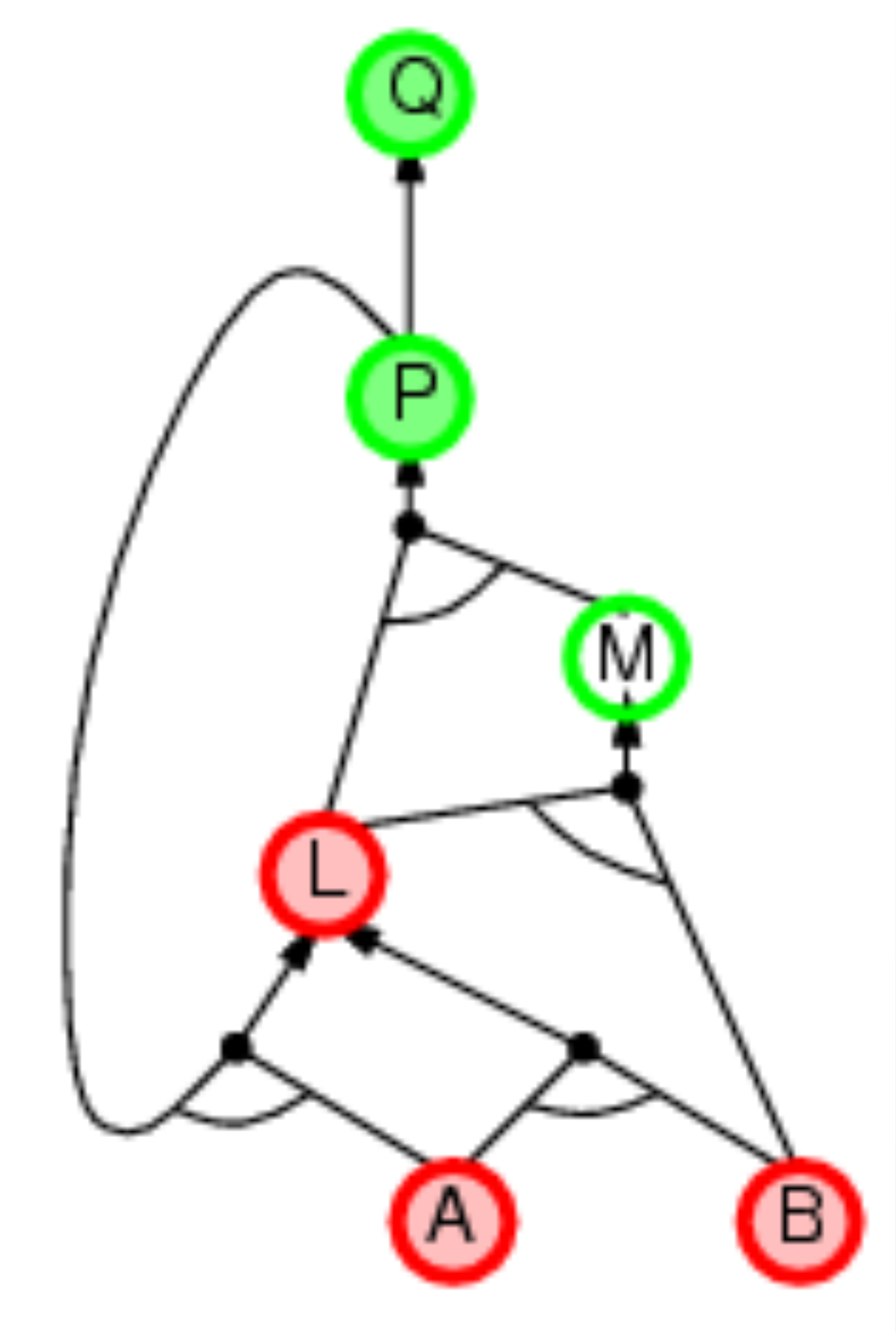

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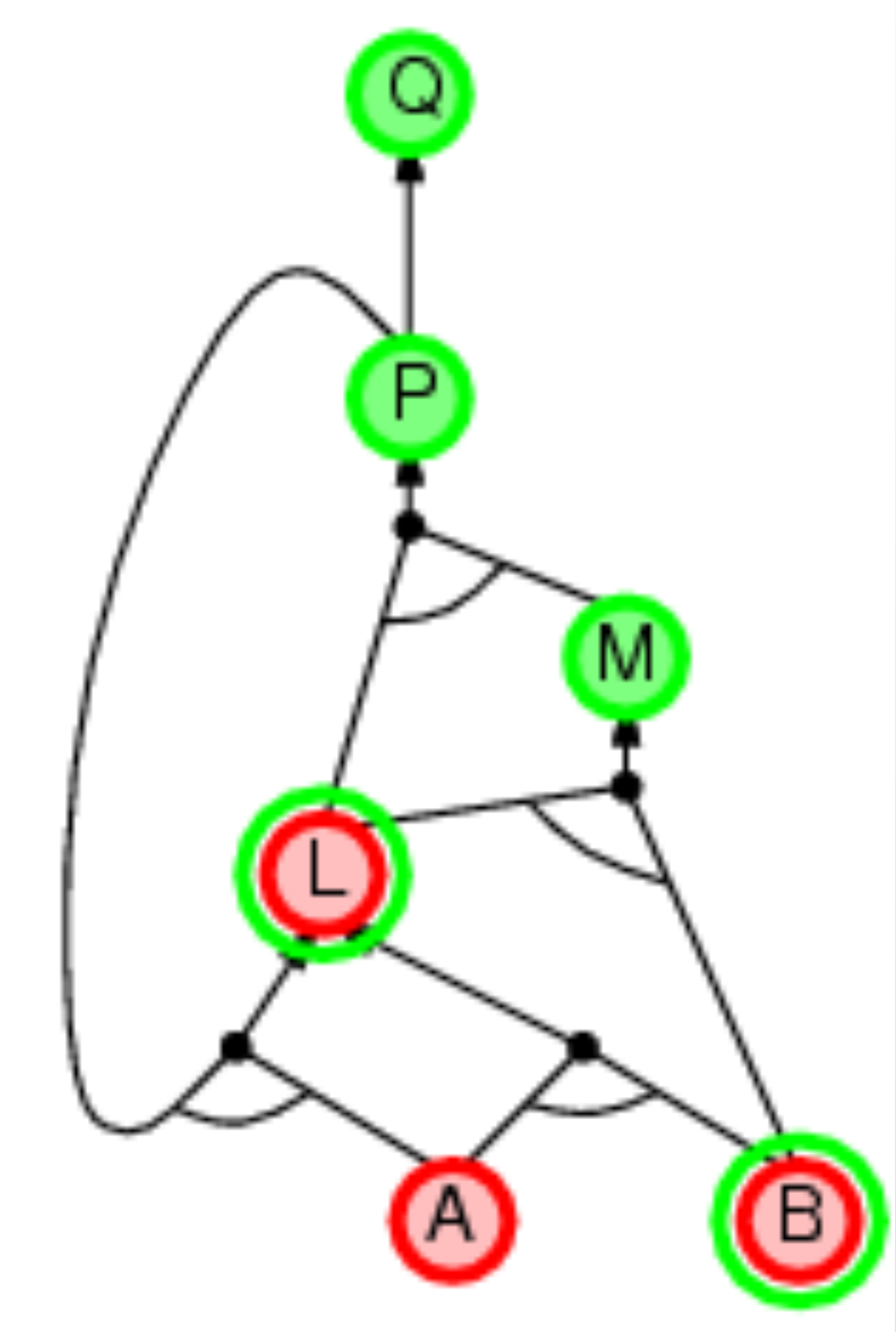

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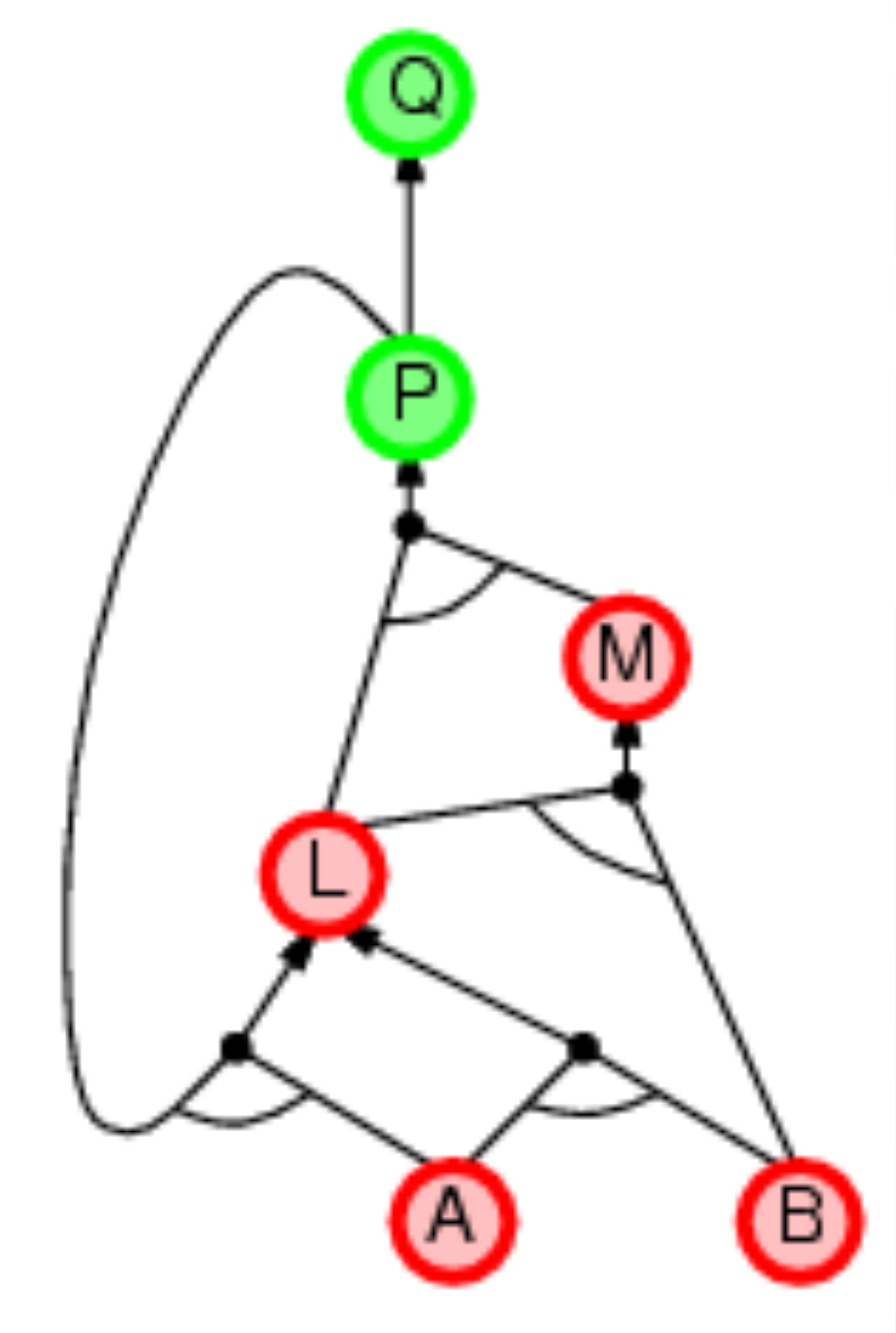

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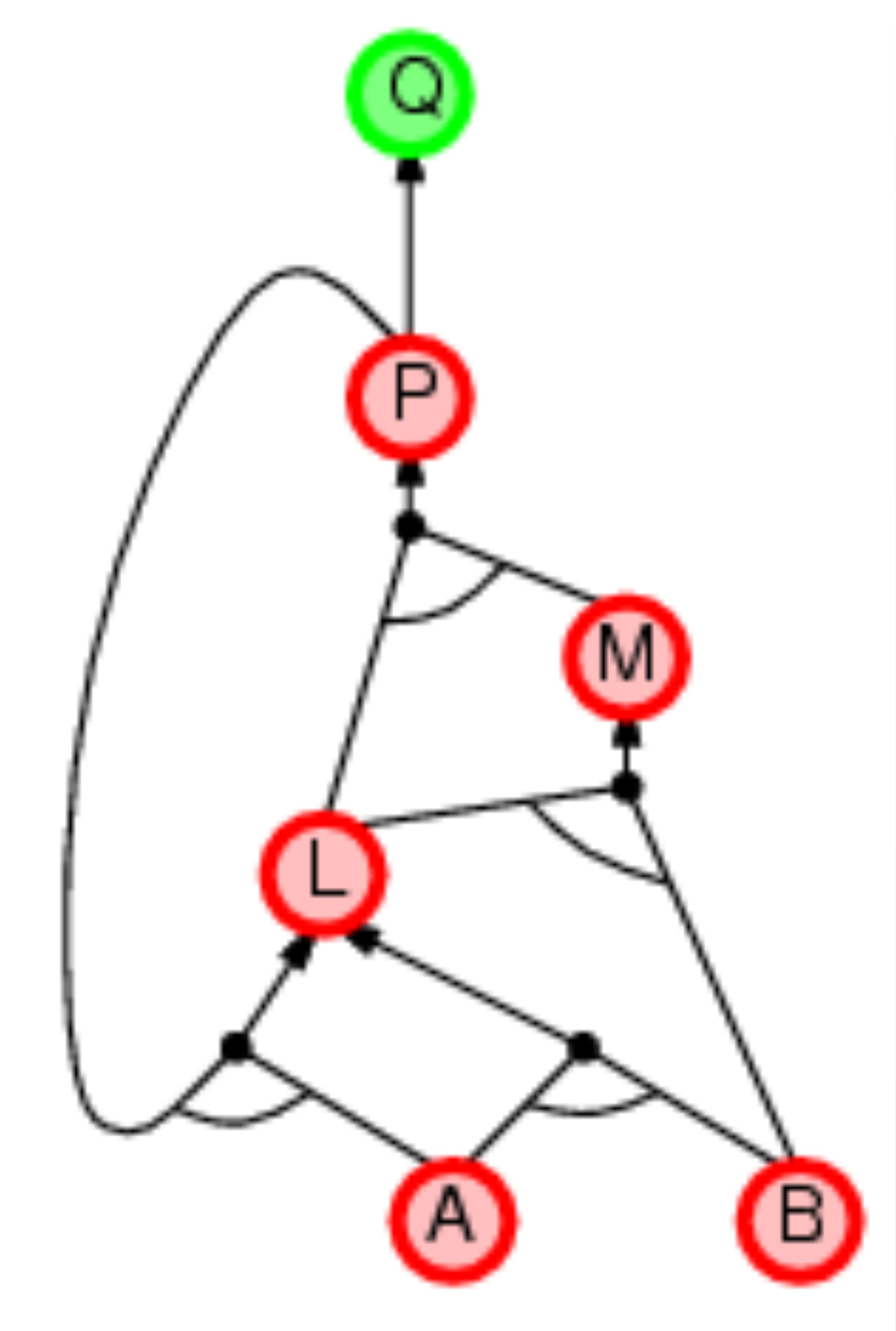

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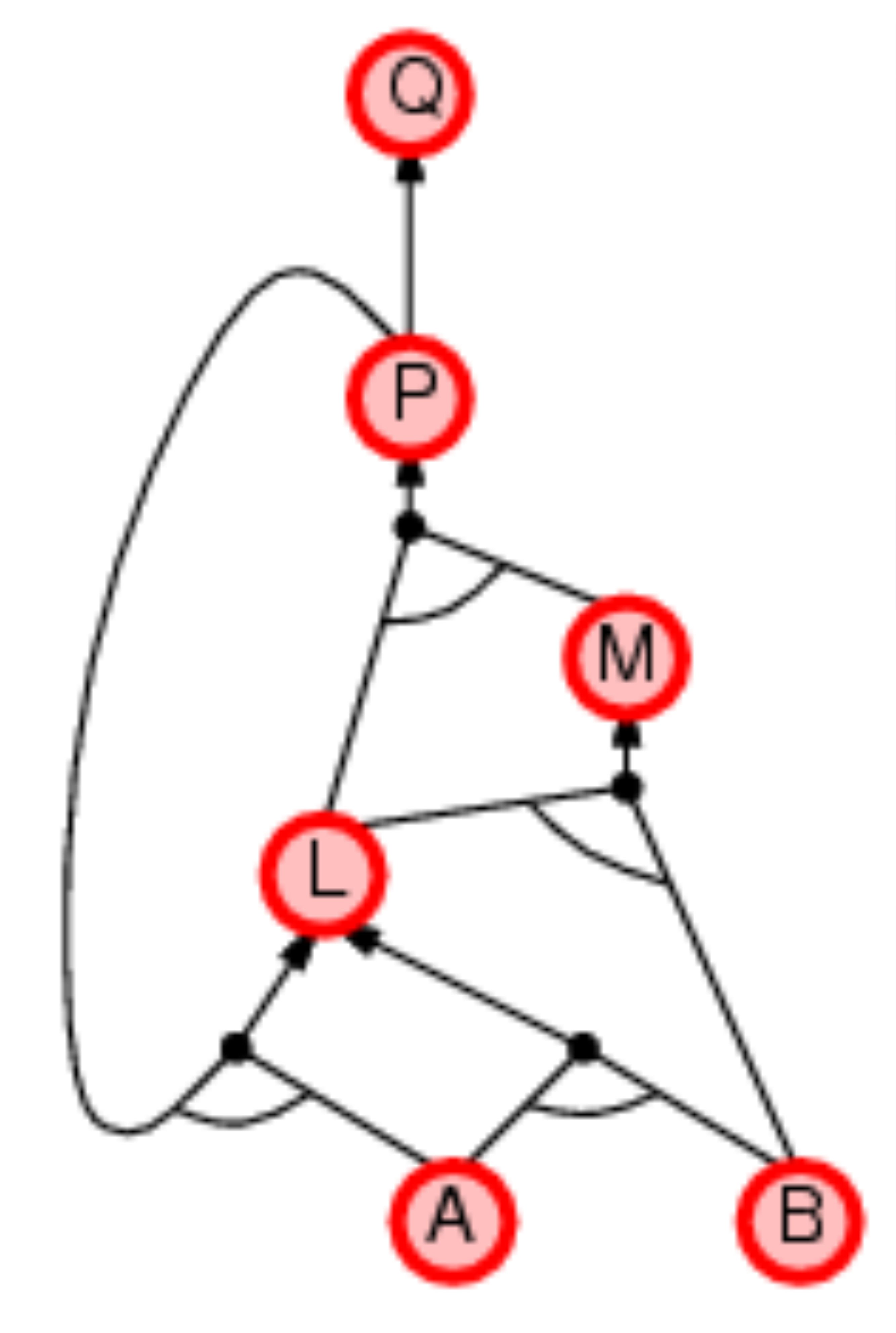

Forward chaining vs Backward chaining

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- Backward Chaining is **goal driven:**
  - We only need to consider clauses that contain one of the current goals as consequences

Pseudocode of naive Forward Chaining

```
1  def naive_FC(KB)
2  #Returns True if KB is unsatisfiable, False otherwise
3      repeat = True
4      while repeat:
5          repeat = False
6          for each clause c1 in KB:
7              for negative literal l1 in c1:
8                  for each clause c2 in KB:
9                      for positive literal l2 in c2:
10                         c1.match = True
11             if all c1.match are true:
12                 new_fact = positive literal in c1
13                 if not_in(new_fact, KB):
14                     KB.append(new_fact)
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So it is O (C+L)

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Even if k is small, the whole algorithm could be O(C+L)^2

Pseudocode: Forward Chaining

- **Forward chaining is sound and complete for Horn KB**
- **This version can be linear in the size of the KB**

```
function PL-FC-ENTAILS?(KB, q) returns true or false
   local variables: count, a table, indexed by clause, initially the number of premises
                    inferred, a table, indexed by symbol, each entry initially false
                    agenda, a list of symbols, initially the symbols known to be true

   while agenda is not empty do
       p ← POP(agenda)
       unless inferred[p] do
           inferred[p] ← true
           for each Horn clause c in whose premise p appears do
               decrement count[c]
               if count[c] = 0 then do
                   if HEAD[c] = q then return true
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   return false
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This in total does some constant amount of work For each clause and *distinct* symbol in the KB, Which is at worst O(C+L)

With a map from each symbol to its occurrences, every premise can be handled in constant time.This effectively ticks every literal in the body of a rule only once, and adds its conclusion only once, so it does O(C+L) work

The amount of work of the inner loop does not depend on the outer loop so the whole algorithm is O(C+L)

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3.B

4.C

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- Prolog uses backward chaining as its backbone!

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  - Example: from
    - $\forall x \,.\, dog(x) \rightarrow mammal(x)$
    - $dog(toto)$
  - We can derive $mammal(toto)$

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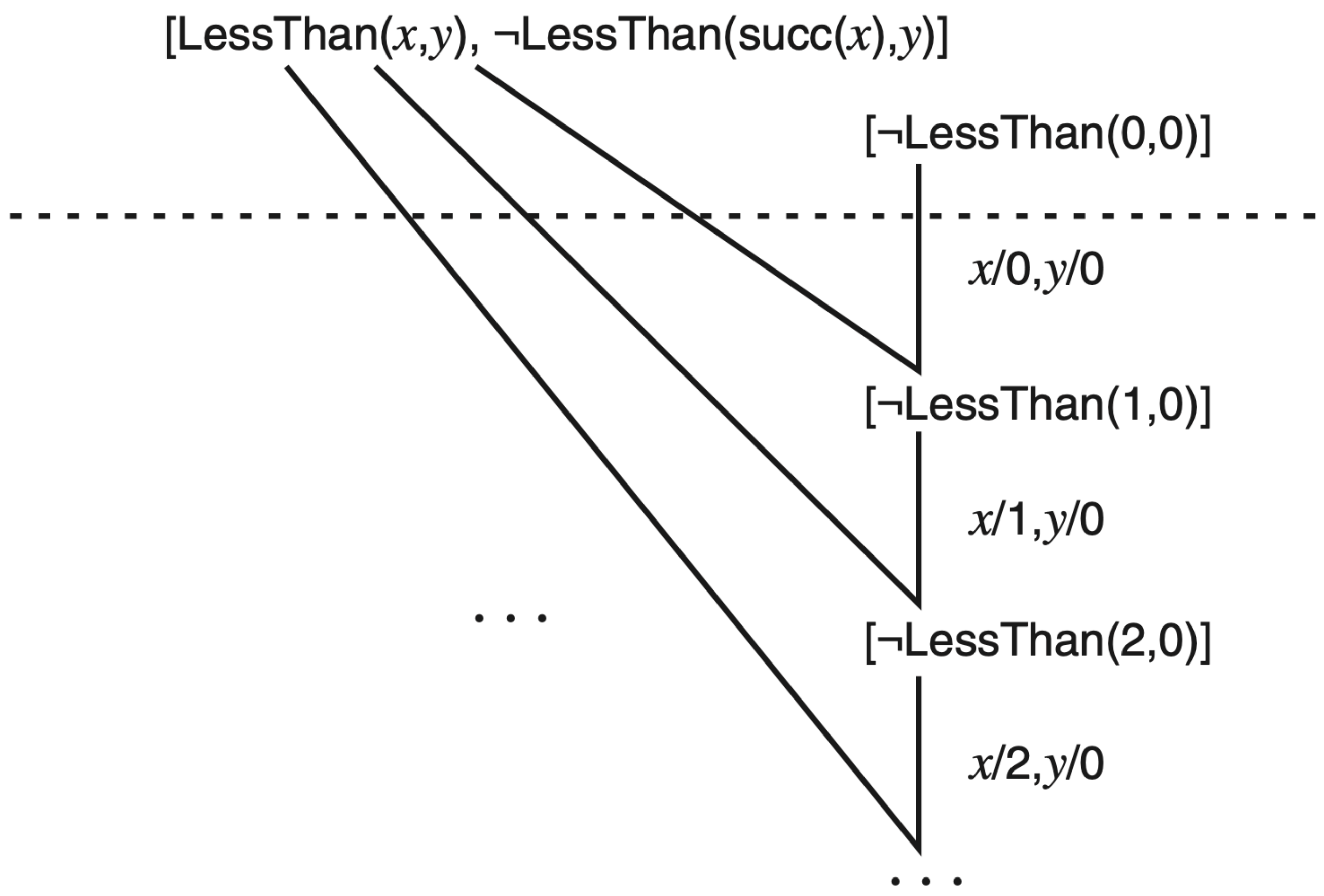
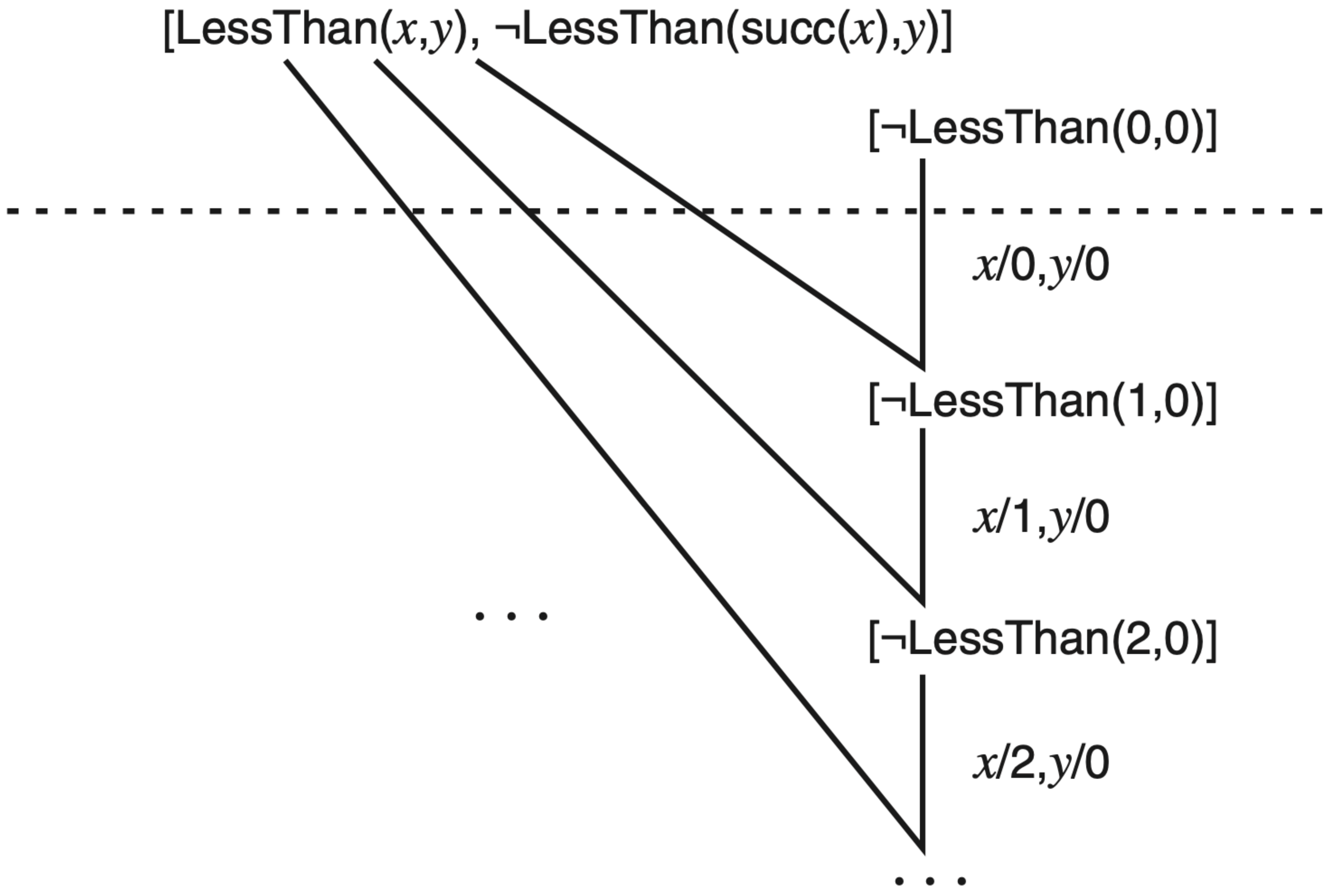


Brachman & Levesque, Fig 4.11

What about FOL?

- However, some statements involving quantifiers may still lead to infinite loops



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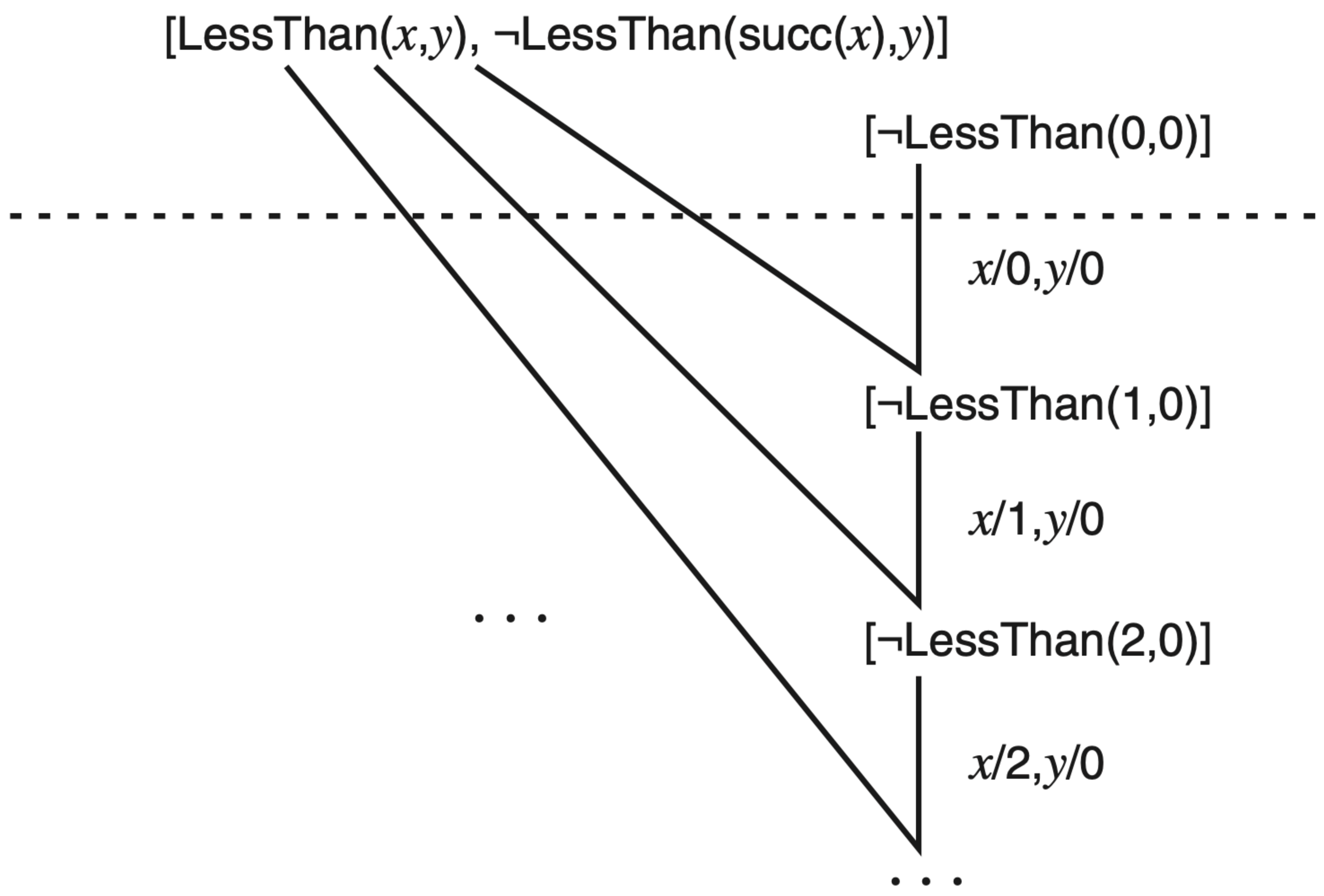
What about FOL?

- However, some statements involving quantifiers may still lead to infinite loops
- The problem of checking if a set of FoL Horn clauses entails an atom is *undecidable*



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- There is no silver bullet to cover all cases: the problem is in general *undecidable*
- But we can try to give as much control to the user as possible in deciding how deduction takes place
  - This is the theme of chapter 6 of the Knowledge Representation and Reasoning book

Lab

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- We will implement forward chaining in Python, both naively and a more efficient version