CSC 481: Resolution

1- Motivation, the Propositional case

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- We would like to have a way to mechanize this process!

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Also written $A \vee B, \neg A \vdash B$

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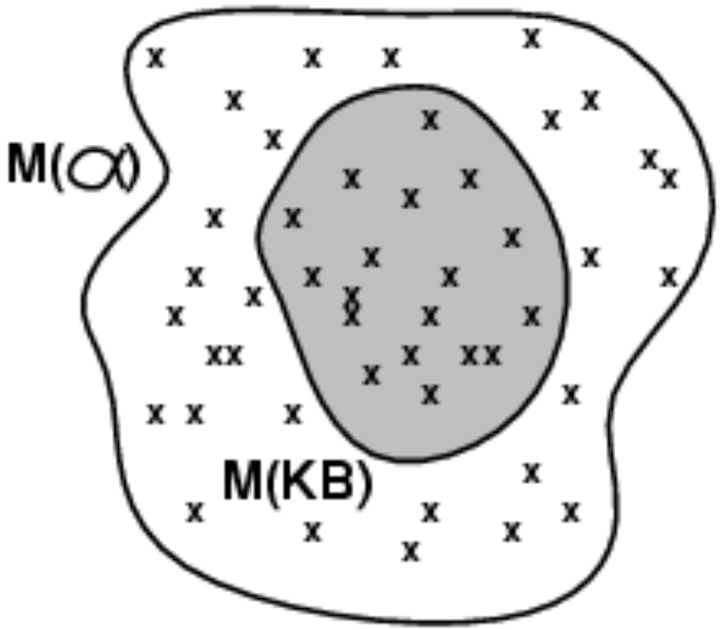
Remember entailment



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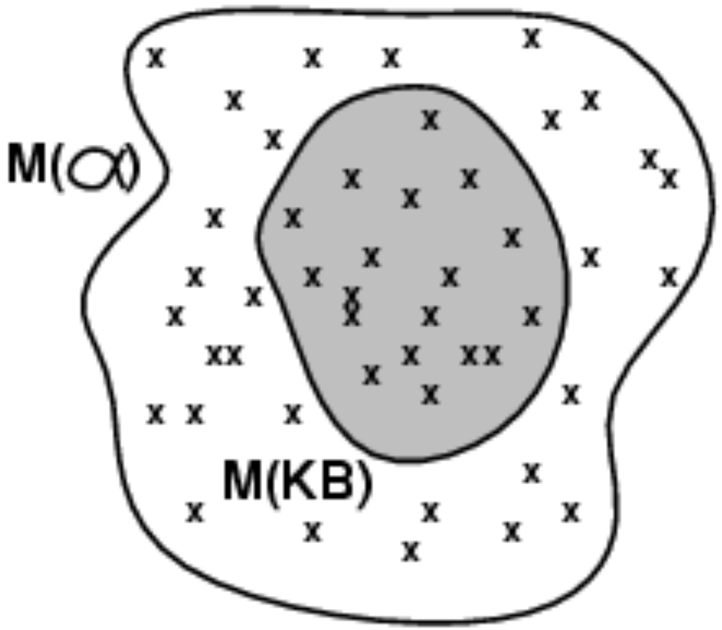

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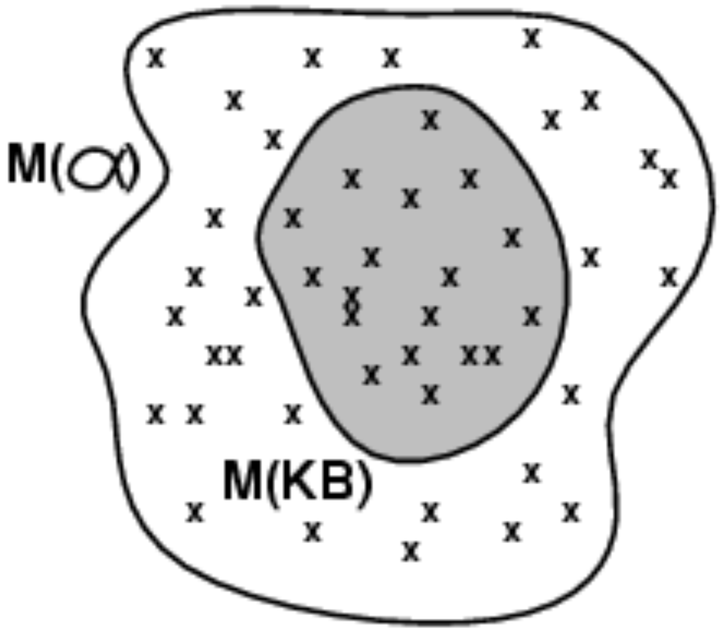

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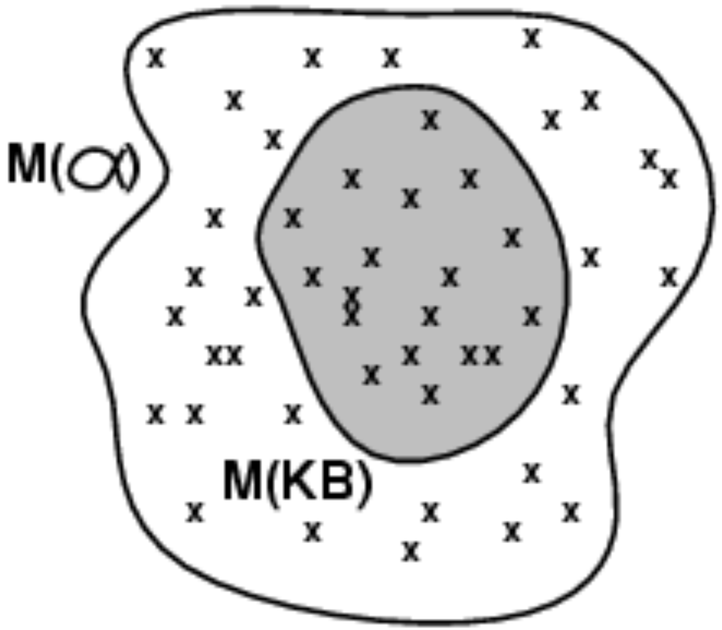

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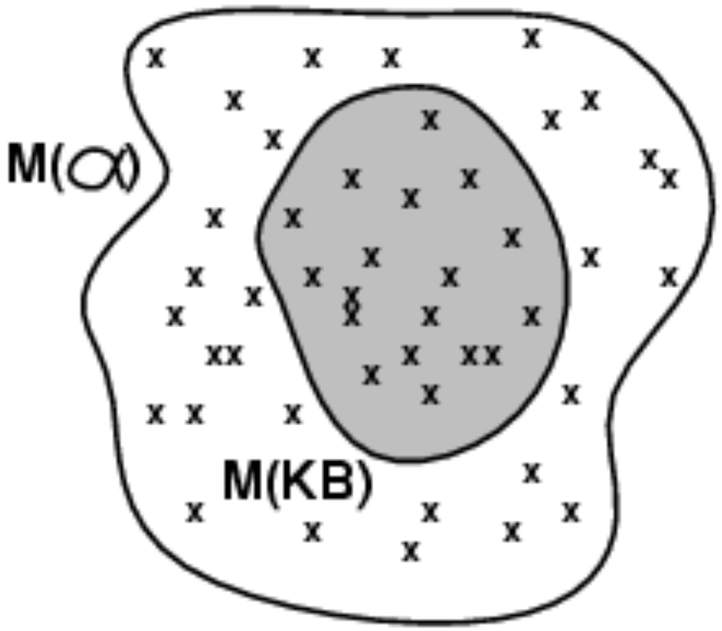

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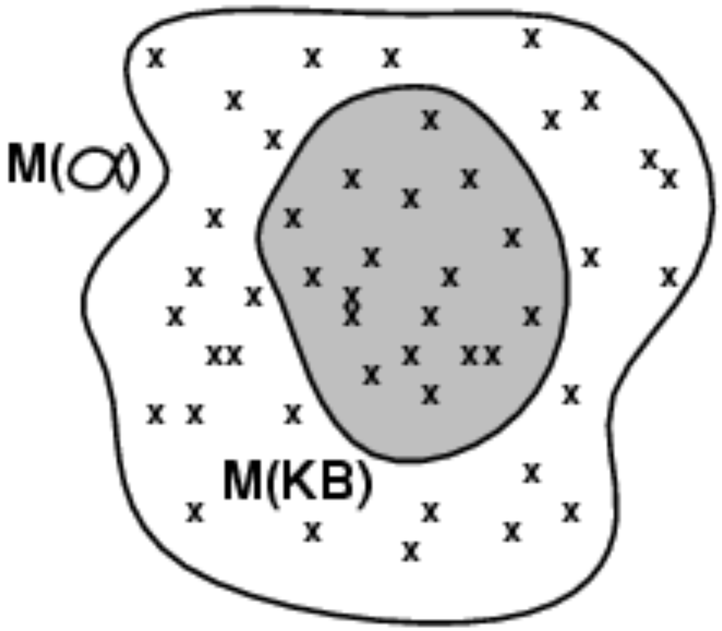

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- If whenever $KB \models \alpha$, the algorithm returns “true”, we say that the algorithm is **complete**

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Resolution in Propositional Logic

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- Exercise: prove that Modus Ponens is a special case of resolution

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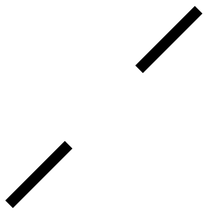

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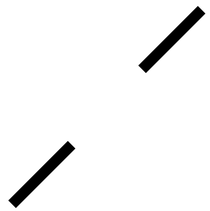

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Source: https://en.wikipedia.org/wiki/Resolution_(logic)

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Source: https://en.wikipedia.org/wiki/Resolution_(logic)

Discarding Tautologies

$$A \vee B \vee C$$

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Exercise

- **Convert to CNF** $a \leftrightarrow (b \vee c)$
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Alternative notation for clauses

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- It is also common to represent clauses as a list
- [¬ A, B, C] is the same as (¬ A v B v C)
- A contradiction is denoted as [ ] (sometimes alto denoted ⊥)

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Resolution example

- $KB = (A \Leftrightarrow (B \vee C)) \wedge \neg A$
- $\alpha = \neg B$
- Intuiton: A is true if and only if B or C holds. But we know A is false. Therefore, B (and also C) must be false)

Resolution example

- $KB = (A \Leftrightarrow (B \vee C)) \wedge \neg A$
- $\alpha = \neg B$
- Converting to CNF (see solution to previous exercise), KB =
    1. $[\neg A , B , C]$
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    3. $[\neg C , A]$
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- If an inference procedure I allows us to derive the symbol α from the symbols of the sentences S , we write:
  - $S \vdash_I \alpha$
- If S entails α (that is, every interpretation that satisfies S also satisfies α), we write:
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  - It is however *refutation complete:* if $S \models \alpha$ (that is if $S \cup \neg\alpha$ is unsatisfiable), it will terminate and $S \vdash_R \alpha$

Resolution in First-Order Logic

Handling variables and quantifiers

Handling variables and quantifiers

- Strategy: we will again convert the KB to a normal form
  - Similar to CNF, but in addition every variable is universally quantified
  - Note that for any formula α, it is true that $\alpha \equiv \forall x \,.\, \alpha$
  - We can then drop all quantifiers
  - Finally, we handle predicates with unification, similar to Prolog
    - Example: P (x,b) unifies with P(a, y) under x/a, y/b

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- Additional steps in converting to CNF:
  - Move ¬ inward of quantifiers
    - $\neg \forall x \,.\, \alpha \equiv \exists x \,.\, \neg \alpha$
    - $\neg \exists x \,.\, \alpha \equiv \forall x \,.\, \neg \alpha$
  - Eliminate all other existential quantifiers (see Skolemization at the end)
  - Move $\wedge$ and $\vee$ inside universal quantifiers
    - $\alpha \wedge \forall x \,.\, \beta \equiv \forall x \,.\, \alpha \wedge \beta$
    - $\alpha \vee \forall x \,.\, \beta \equiv \forall x \,.\, \alpha \vee \beta$

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- Assuming no existential quantifiers for now:
- Intuition: if two clauses have p and $\neg q$, but p unifies with q, we can apply resolution after performing the required substitution to both clauses

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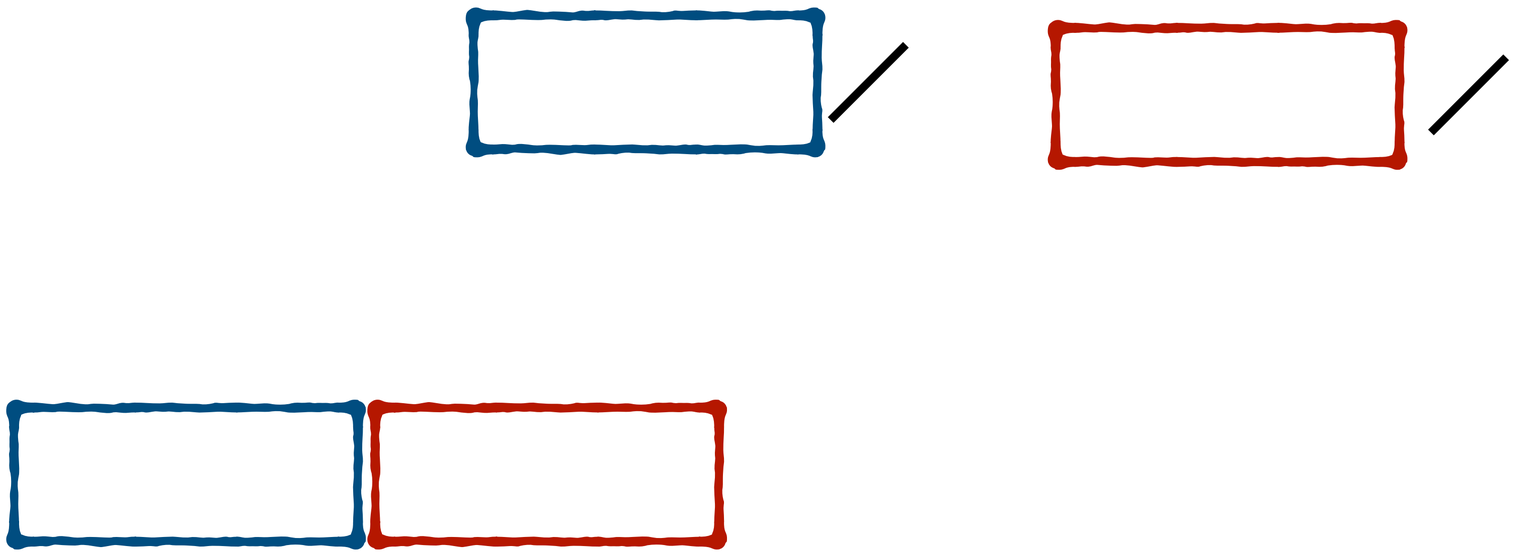
- Given two clauses $[a_1, a_2, \ldots, a_n, p]$ and $[b_1, b_2, \ldots, b_m, \neg q]$
  - These can be complex clauses with literals containing predicates, variables etc
- If p unifies with q under some substition θ we get a new clause:
  - $[c_1, c_2, \ldots, c_n, d_1, d2,...,d_m]$
  - Where $c_i = a_i/\theta$ and $d_j = b_j/\theta$

Handling variables and quantifiers



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- Example: Consider the clauses:
  - [P(x,a),¬Q(x)]
  - [¬P(b,y), R(b,f(y))]
- The first literal of each clause unifies under x/b, y/a. Applying this to both clauses:
  - [P(b,a),¬Q(b)]
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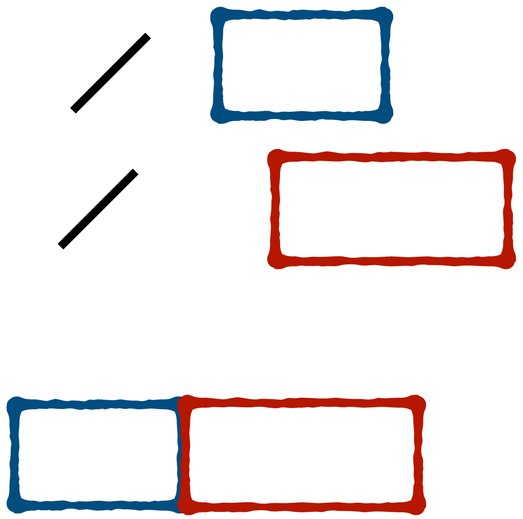

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- $loves(x, mary) \vee happy(mary)$ (implicitly, x is universally quantified)
- $\neg loves(jon, mary)$
- Substitute x/jon
- $loves(jon, mary) \vee happy(mary)$
- $\neg loves(jon, mary)$
- We're left with $happy(mary)$

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  - The sentence then becomes $\forall y \, . friends(anon_1, f(y)) \wedge friends(y, f(y))$

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- Prolog is built around Horn clauses!
- Less expressive than full FOL, but resolution, if it terminates, uses only a linear number of steps
- Unfortunately, the problem is still undecidable: no sound procedure can be guaranteed to terminate

Exercise: who killed the cat?
(from Russel & Norvig's "AI - a Modern Approach", ch. 9)

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Task 4: Use resolution to answer the question “Did Curiosity kill the cat?”

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7. $\neg cat(x_7) \vee animal(x_7)$

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Task 3 - Proof by resolution

(6,7, x/Tuna)

(3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack$, $x_4/F(Jack)$)

(1,13, $x_1/Jack$)

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11. $kills(Jack, Tuna)$ (5,8)

12. $\neg loves(y, Jack)$ (10,11, x/Jack)

(2,4, $x_2/Jack$, $x_4/F(Jack)$)

(1,13, $x_1/Jack$)

(12,14, $y/G(Jack)$)

Task 3 - Proof by resolution

9. $animal(Tuna)$ (6,7, x/Tuna)

10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ (3,9, z/Tuna)

11. $kills(Jack, Tuna)$ (5,8)

12. $\neg loves(y, Jack)$ (10,11, x/Jack)

13. $\neg animal(F(Jack)) \vee loves(G(Jack), Jack)$ (2,4, $x_2/Jack$, $x_4/F(Jack)$)

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15. $\perp$ (12,14, $y/G(Jack)$)

Task 3 - Proof by resolution

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(3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack$, $x_4/F(Jack)$)

(1,13, $x_1/Jack$)

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Note:step 13 requires us to rename the variables (x appears on both sides)

Task 3 - Proof by resolution

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(3,9, z/Tuna)

(5,8)

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Note:step 13 requires us to rename the variables (x appears on both sides)

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14. $loves(G(Jack), Jack)$ (1,13, $x_1/Jack$)

15. $\perp$ (12,14, $y/G(Jack)$)

Note:step 13 requires us to rename the variables (x appears on both sides)

Task 3 - Proof by resolution (alternative visualization from AIMA)

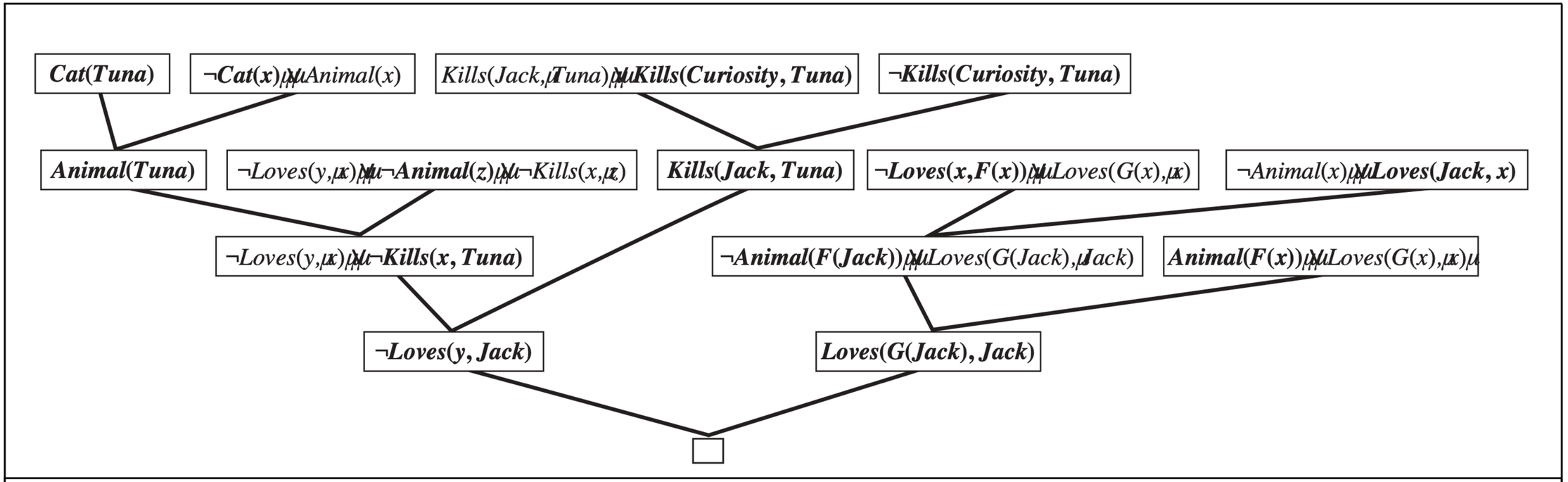

Figure 9.12 A resolution proof that Curiosity killed the cat. Notice the use of factoring in the derivation of the clause $Loves(G(Jack), Jack)$. Notice also in the upper right, the unification of $Loves(x, F(x))$ and $Loves(Jack, x)$ can only succeed after the variables have been standardized apart.