# CSC 481: Resolution

## 1- Motivation, the Propositional case


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- We would like to have a way to mechanize this process!

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Also written $A \vee B, \neg A \vdash B$

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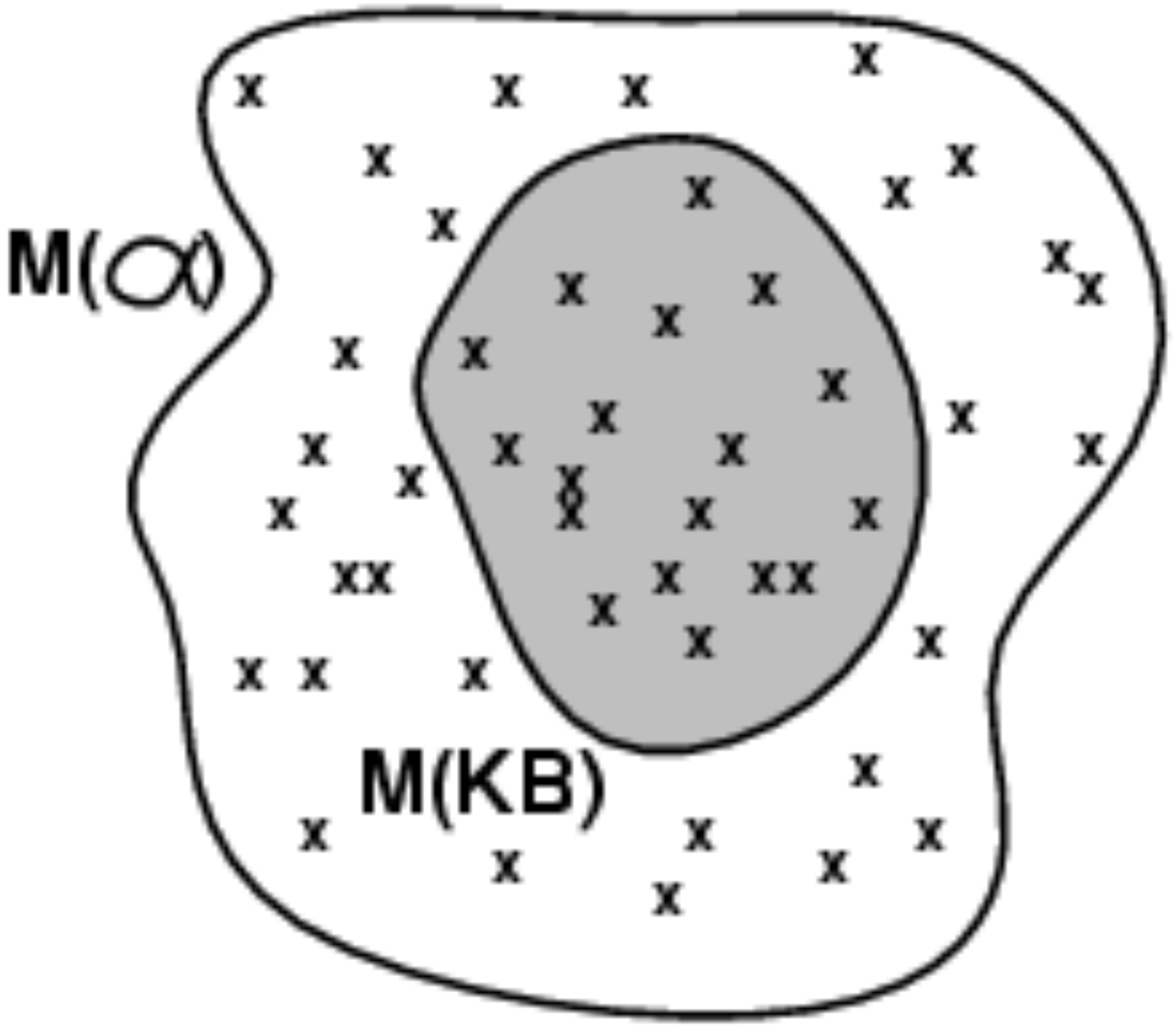


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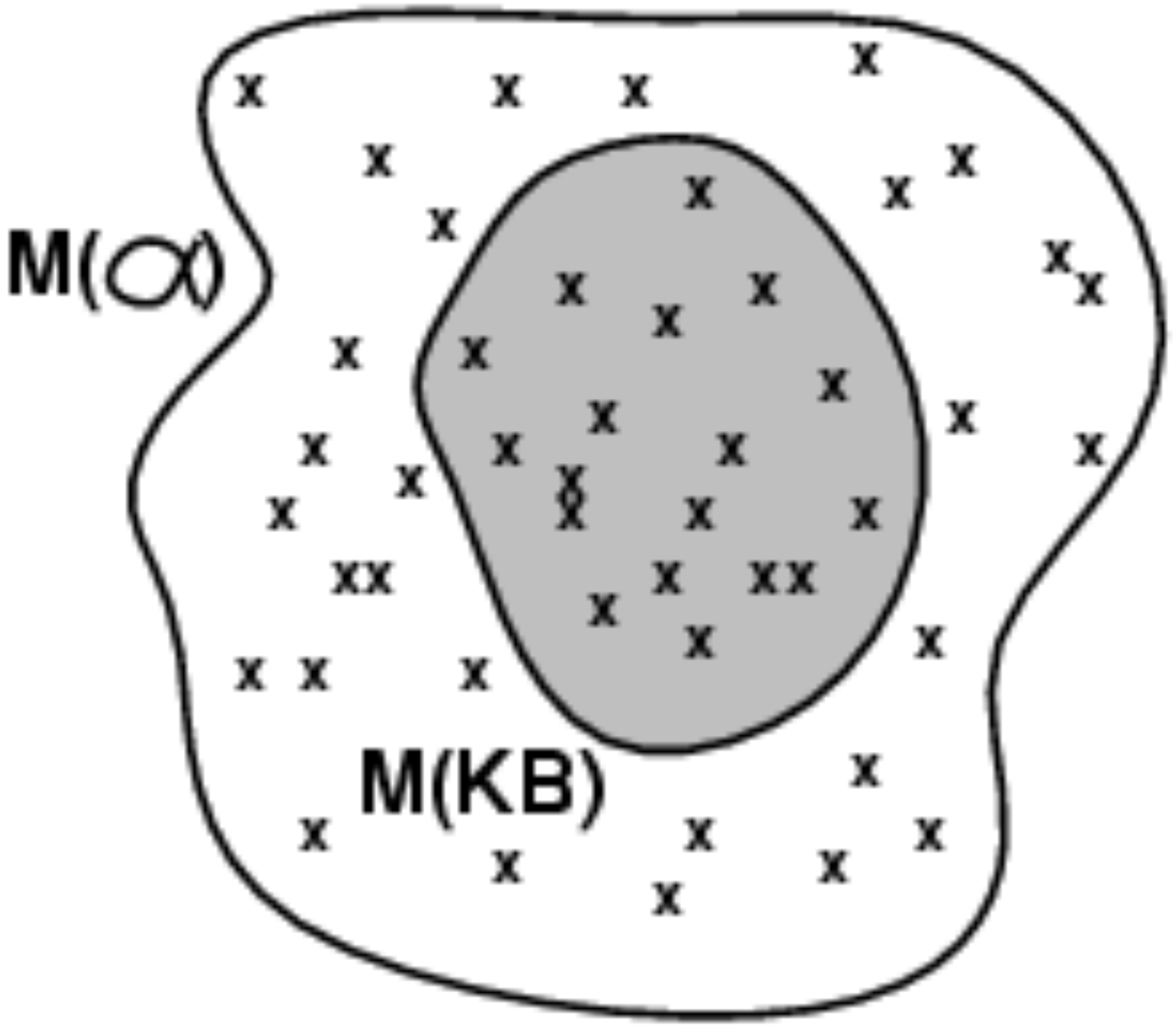


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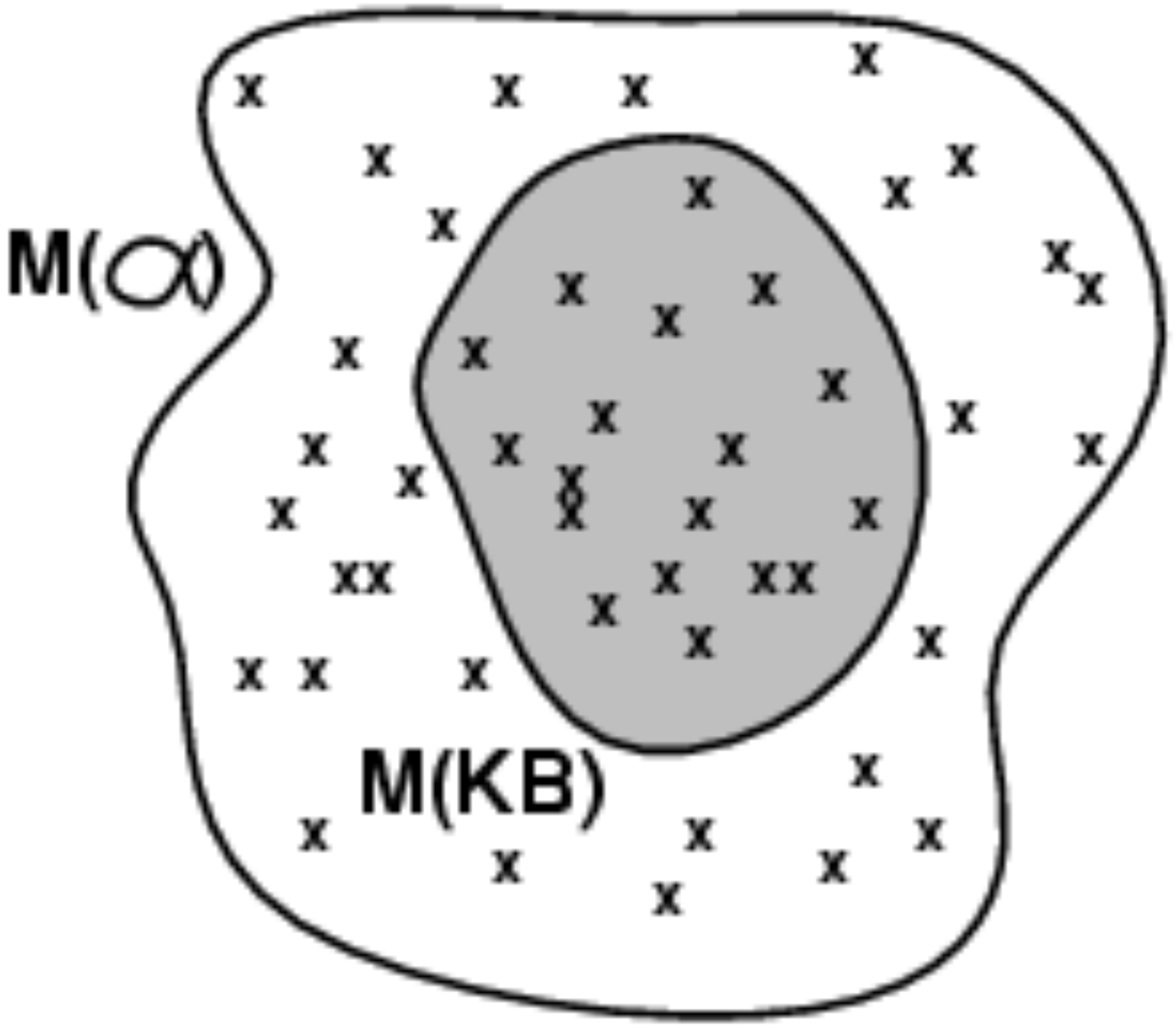


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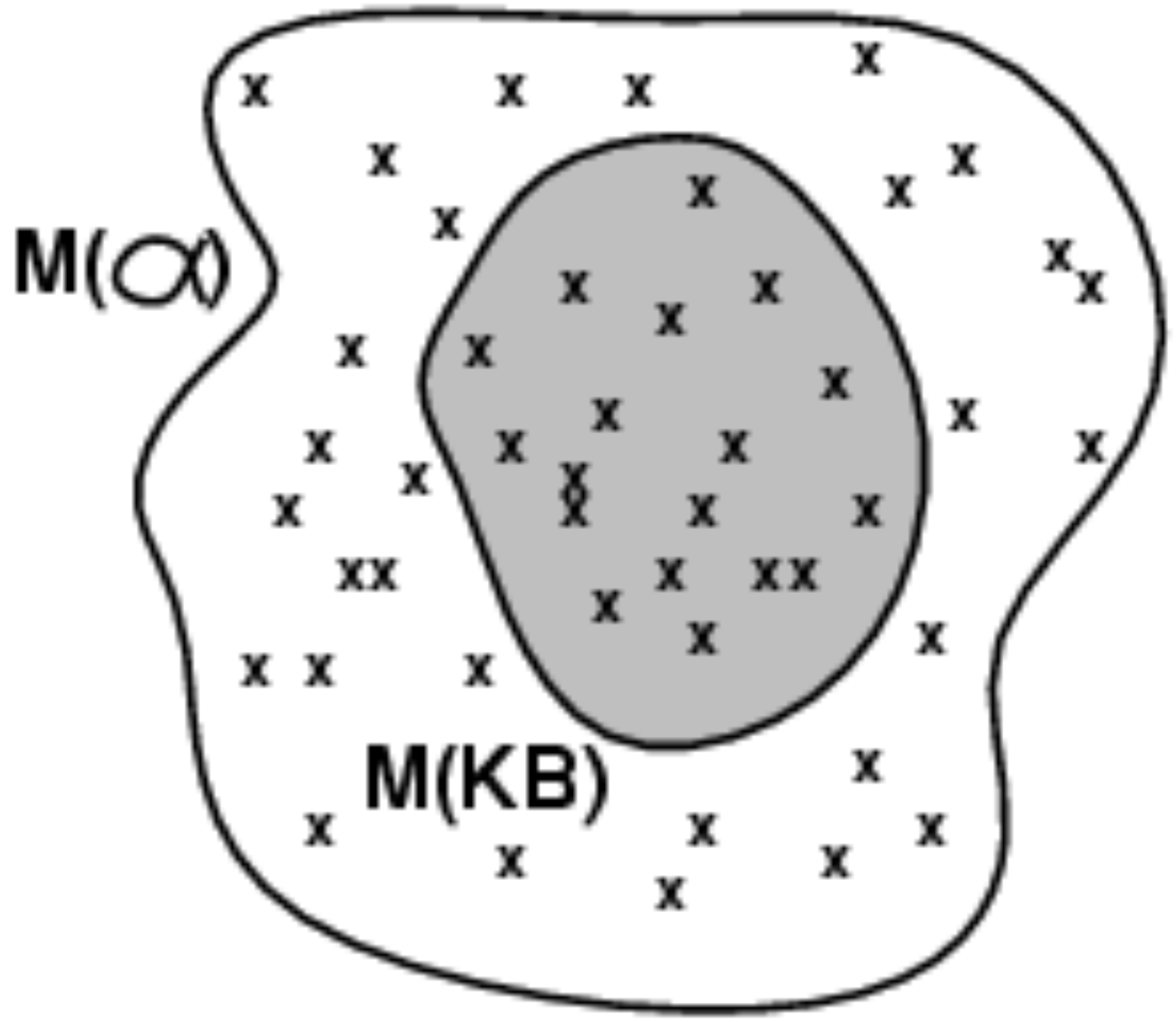


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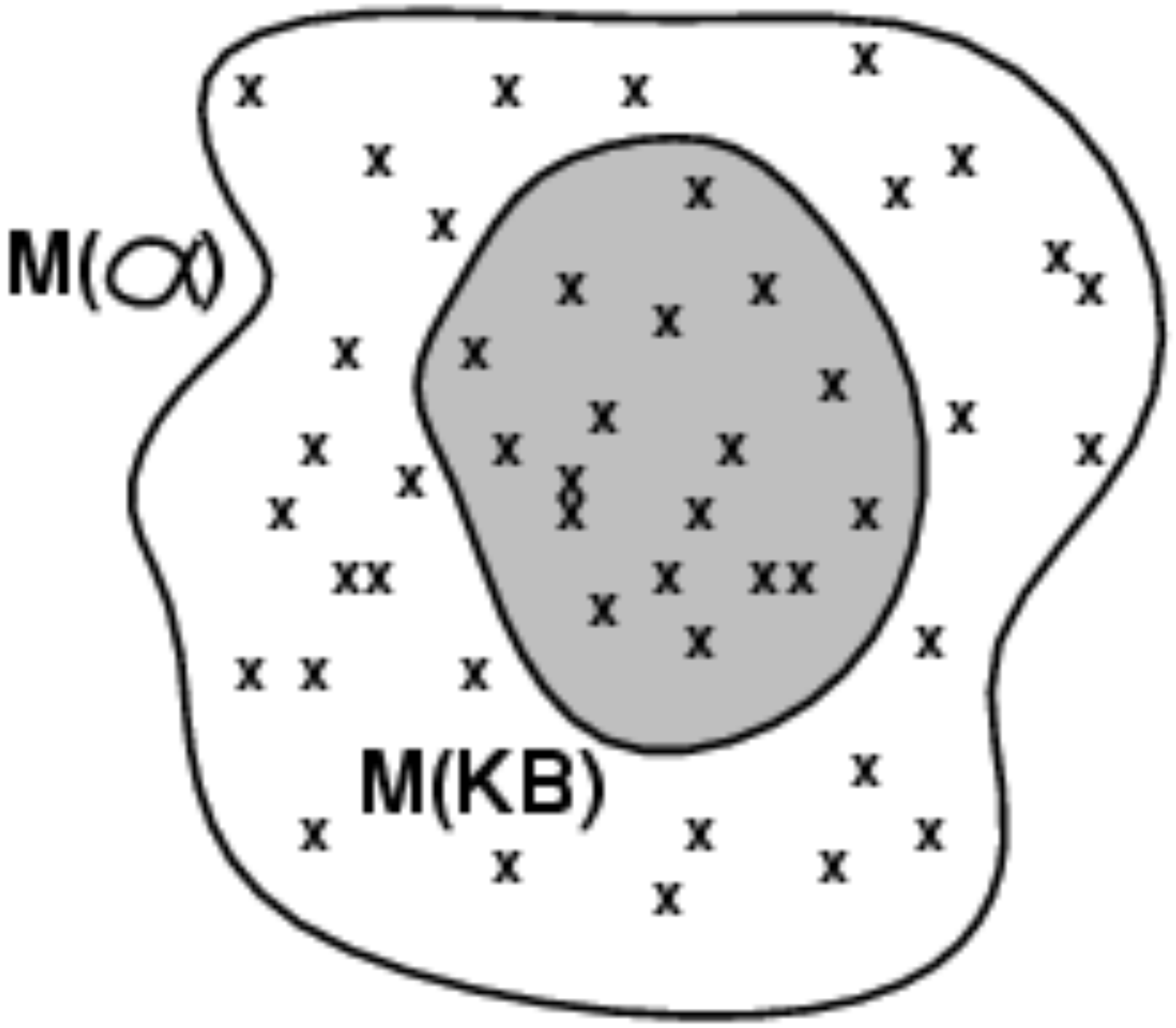


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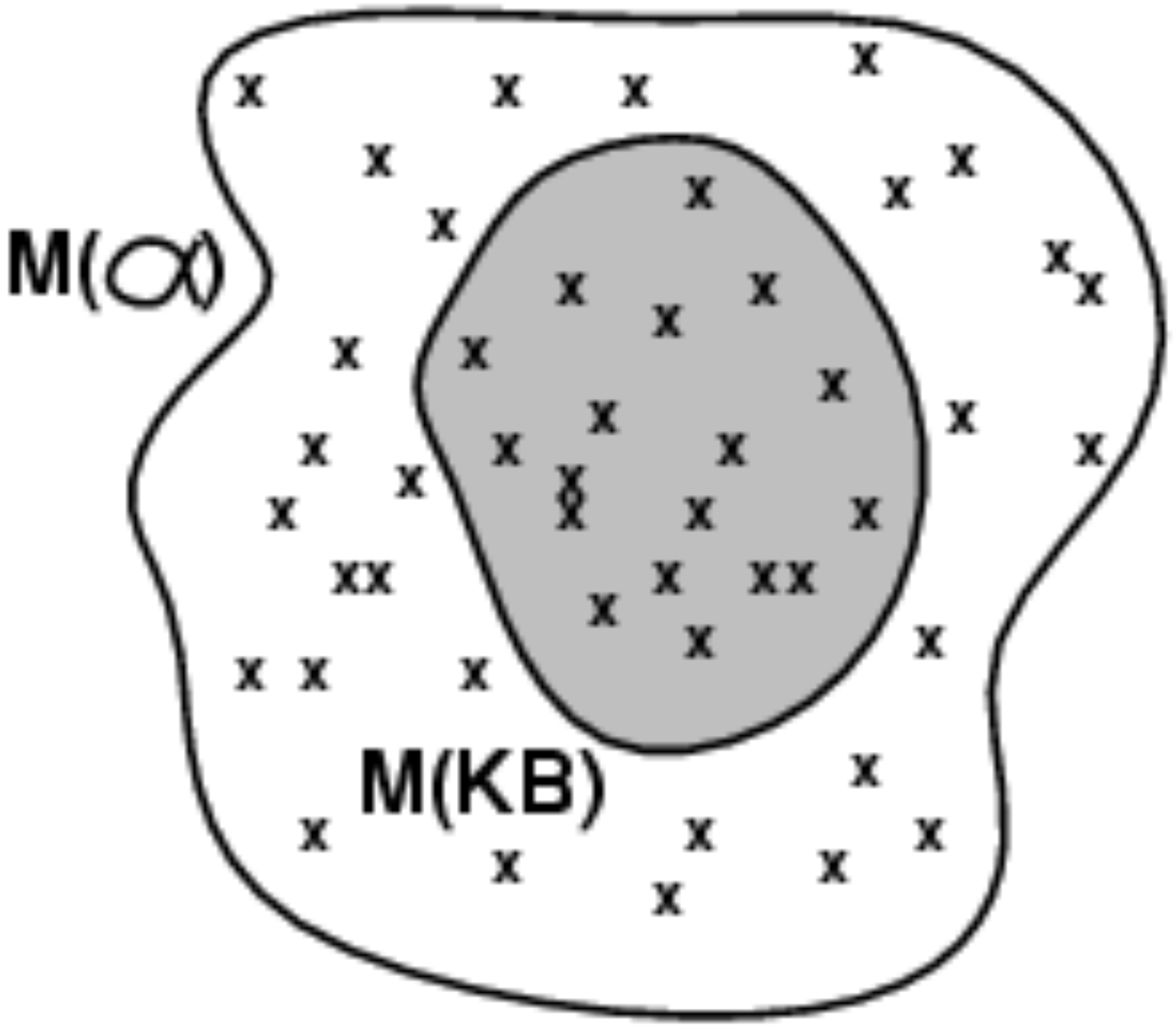


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- But doing it by explicitly enumerating interpretations is infeasible

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In 5 and 6, we abuse notation and interpret KB as the conjunction of all its sentences

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- If whenever $KB \models \alpha$, the algorithm returns “true”, we say that the algorithm is **complete**

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# Resolution in Propositional Logic

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- I will go to the beach or to the movies $A \lor B$
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- Exercise: prove that Modus Ponens is a special case of resolution

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# Discarding Tautologies

Is a tautology

- The resolution rule is applied to all possible pairs of clauses that contain complementary literals. After each application of the resolution rule, the resulting sentence is simplified by removing repeated literals. If the clause contains complementary literals, it is discarded (as a tautology). If not, and if it is not yet present in the clause set *S*, it is added to *S*, and is considered for further resolution inferences.

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$$A \vee B \vee C$$

$$\overline{\phantom{B \vee \neg B \vee C \vee D}}$$

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- The resolution rule is applied to all possible pairs of clauses that contain complementary literals. After each application of the resolution rule, the resulting sentence is simplified by removing repeated literals. If the clause contains complementary literals, it is discarded (as a tautology). If not, and if it is not yet present in the clause set $S$, it is added to $S$, and is considered for further resolution inferences.

Source: https://en.wikipedia.org/wiki/Resolution_(logic)

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# Exercise

- **Convert to CNF** $a \leftrightarrow (b \vee c)$
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# Alternative notation for clauses

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- It is also common to represent clauses as a list
- [¬ A, B, C] is the same as (¬ A v B v C )
- A contradiction is denoted as [ ] (sometimes alto denoted ⊥)

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# Resolution example

- $KB = (A \Leftrightarrow (B \vee C)) \wedge \neg A$
- $\alpha = \neg B$
- Intuiton: A is true if and only if B or C holds. But we know A is false. Therefore, B (and also C) must be false)

# Resolution example

- $KB = (A \Leftrightarrow (B \lor C)) \land \neg A$
- $\alpha = \neg B$
- Converting to CNF (see solution to previous exercise), KB =
    1. [¬ A , B , C]
    2. [¬B , A]
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    3. $[\neg C, A]$
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    5. $[B]$

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- If an inference procedure $I$ allows us to derive the symbol $\alpha$ from the symbols of the sentences $S$ , we write:
  - $S \vdash_I \alpha$
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  - It is however *refutation complete:* if $S \vDash \alpha$ (that is if $S \cup \neg\alpha$ is unsatisfiable), it will terminate and $S \vdash_R \alpha$

# Resolution in First-Order Logic

# Handling variables and quantifiers

# Handling variables and quantifiers

- Strategy: we will again convert the KB to a normal form
  - Similar to CNF, but in addition every variable is universally quantified
  - Note that for any formula $\alpha$, it is true that $\alpha \equiv \forall x \,.\, \alpha$
  - We can then drop all quantifiers
  - Finally, we handle predicates with unification, similar to Prolog
    - Example: P (x,b) unifies with P(a, y) under x/a, y/b

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- Additional steps in converting to CNF:
  - Move $\neg$ inward of quantifiers
    - $\neg \forall x . \alpha \equiv \exists x . \neg \alpha$
    - $\neg \exists x . \alpha \equiv \forall x . \neg \alpha$
  - Eliminate all other existential quantifiers (see Skolemization at the end)
  - Move $\land$ and $\lor$ inside universal quantifiers
    - $\alpha \land \forall x . \beta \equiv \forall x . \alpha \land \beta$
    - $\alpha \lor \forall x . \beta \equiv \forall x . \alpha \lor \beta$

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- Assuming no existential quantifiers for now:
- Intuition: if two clauses have $p$ and $\neg q$, but $p$ unifies with $q$, we can apply resolution after performing the required substitution to both clauses

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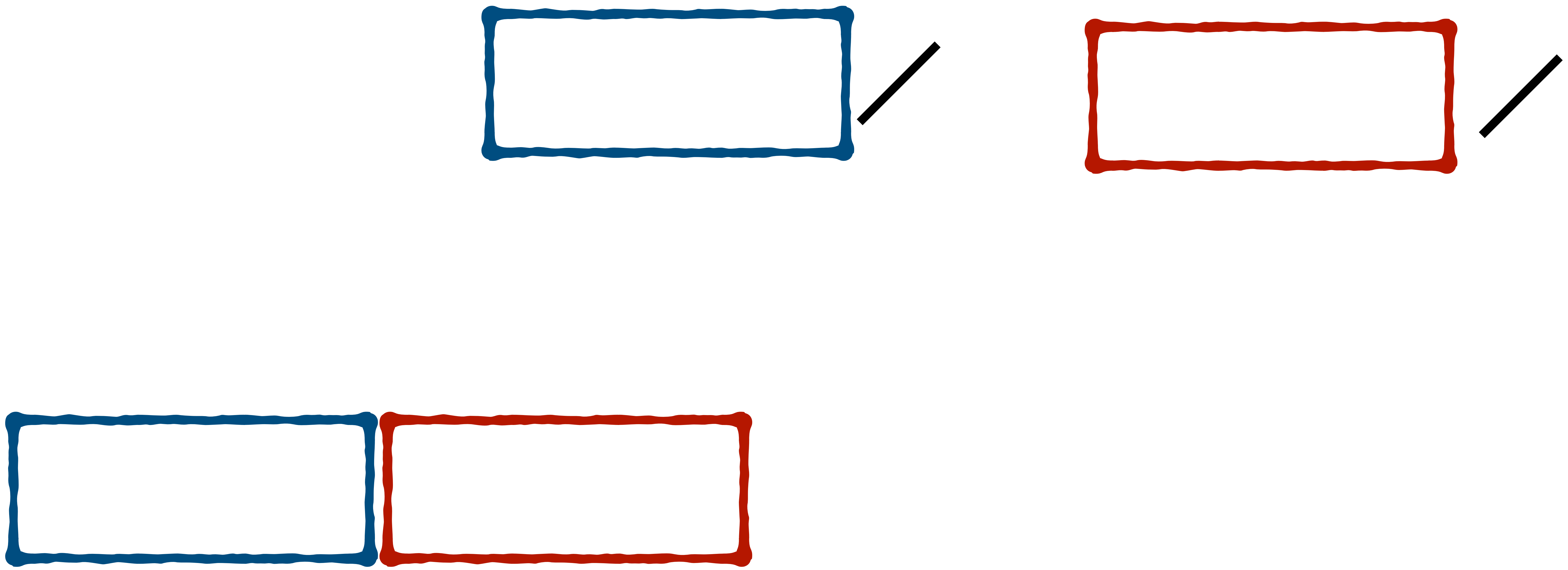
- Assuming no existential quantifiers for now:
- Given two clauses $[a_1, a_2, \ldots, a_n, p]$ and $[b_1, b_2, \ldots, b_m, \neg q]$
  - These can be complex clauses with literals containing predicates, variables etc
- If $p$ unifies with $q$ under some substition $\theta$ we get a new clause:
  - $[c_1, c_2, \ldots, c_n, d_1, d2,...,d_m]$
  - Where $c_i = a_i/\theta$ and $d_j = b_j/\theta$

# Handling variables and quantifiers



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- Example: Consider the clauses:
    - [P(x,a),¬Q(x)]
    - [¬P(b,y), R(b,f(y))]
- The first literal of each clause unifies under x/b, y/a. Applying this to both clauses:
    - [P(b,a),¬Q(b)]
    - [¬P(b,a), R(b,f(a))]
- So we can apply Resolution to eliminate the first literal and get:
    - [¬Q(b), R(b,f(a))]

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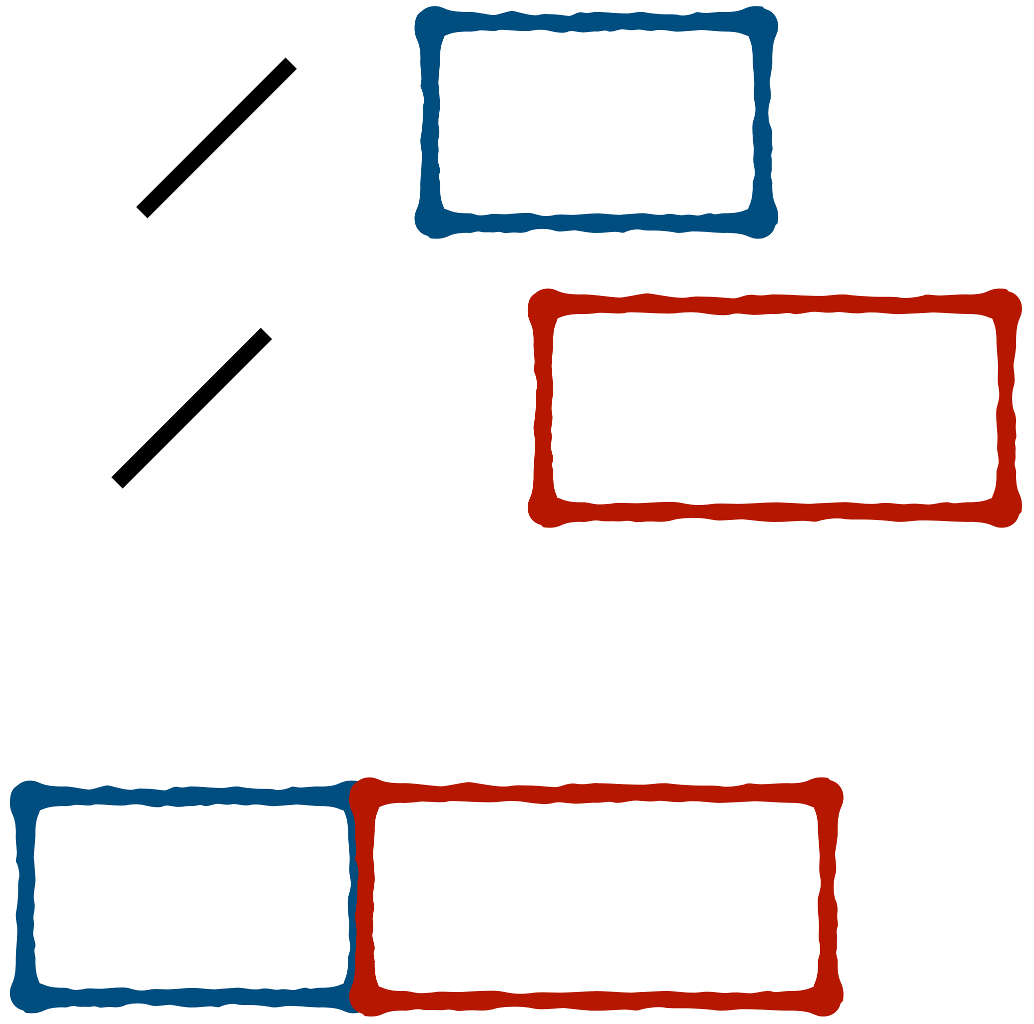

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- $loves(x, mary) \lor happy(mary)$ (implicitly, x is universally quantified)
- $\neg loves(jon, mary)$
- Substitute $x/jon$
- $loves(jon, mary) \lor happy(mary)$
- $\neg loves(jon, mary)$
- We're left with $happy(mary)$

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    - $\exists x \, . \, loves(x, jane)$ becomes $loves(anon_1, jane)$

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  - The sentence then becomes $\forall y \,.\, friends(anon_1, f(y)) \wedge friends(y, f(y))$

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- In full First-Order Logic, it is *sound* but not *complete*, only *refutation complete*

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    - Faster procedures are needed

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- Prolog is built around Horn clauses!
- Less expressive than full FOL, but resolution, if it terminates, uses only a linear number of steps
- Unfortunately, the problem is still undecidable: no sound procedure can be guaranteed to terminate

# Exercise: who killed the cat?

(from Russel & Norvig's “AI - a Modern Approach”, ch. 9)

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7. $\neg cat(x_7) \vee animal(x_7)$

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4. $\neg animal(x_4) \lor loves(Jack, x_4)$
5. $kills(Jack, Tuna) \lor kills(Curiosity, Tuna)$
6. Cat(Tuna)
7. $\neg cat(x_7) \lor animal(x_7)$
8. $\neg kills(Curiosity, Tuna)$

# Task 3 - Proof by resolution

(6,7, x/Tuna)

(3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

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9. $animal(Tuna)$

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(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

# Task 3 - Proof by resolution

9. $animal(Tuna)$

10. $\neg kills(x, Tuna) \lor \neg loves(y, x)$

(6,7, x/Tuna)

(3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

# Task 3 - Proof by resolution

9. $animal(Tuna)$

10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$

11. $kills(Jack, Tuna)$

(6,7, x/Tuna)

(3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

# Task 3 - Proof by resolution

9. $animal(Tuna)$ (6,7, x/Tuna)

10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ (3,9, z/Tuna)

11. $kills(Jack, Tuna)$ (5,8)

12. $\neg loves(y, Jack)$ (10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

# Task 3 - Proof by resolution

| | |
|---|---|
| 9. $animal(Tuna)$ | (6,7, x/Tuna) |
| 10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ | (3,9, z/Tuna) |
| 11. $kills(Jack, Tuna)$ | (5,8) |
| 12. $\neg loves(y, Jack)$ | (10,11, x/Jack) |
| 13. $\neg animal(F(Jack)) \vee loves(G(Jack), Jack)$ | (2,4, $x_2/Jack, x_4/F(Jack)$) |
| | (1,13, $x_1/Jack$ ) |
| | (12,14, $y/G(Jack)$ ) |

# Task 3 - Proof by resolution

| | |
|---|---|
| 9. $animal(Tuna)$ | (6,7, x/Tuna) |
| 10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ | (3,9, z/Tuna) |
| 11. $kills(Jack, Tuna)$ | (5,8) |
| 12. $\neg loves(y, Jack)$ | (10,11, x/Jack) |
| 13. $\neg animal(F(Jack)) \vee loves(G(Jack), Jack)$ | (2,4, $x_2/Jack, x_4/F(Jack)$) |
| 14. $loves(G(Jack), Jack)$ | (1,13, $x_1/Jack$ ) |
| | (12,14, $y/G(Jack)$ ) |

# Task 3 - Proof by resolution

| | |
|---|---|
| 9. $animal(Tuna)$ | (6,7, x/Tuna) |
| 10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ | (3,9, z/Tuna) |
| 11. $kills(Jack, Tuna)$ | (5,8) |
| 12. $\neg loves(y, Jack)$ | (10,11, x/Jack) |
| 13. $\neg animal(F(Jack)) \vee loves(G(Jack), Jack)$ | (2,4, $x_2/Jack, x_4/F(Jack)$) |
| 14. $loves(G(Jack), Jack)$ | (1,13, $x_1/Jack$ ) |
| 15. $\bot$ | (12,14, $y/G(Jack)$ ) |

# Task 3 - Proof by resolution

(6,7, x/Tuna)

(3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution

9. $animal(Tuna)$

(6,7, x/Tuna)

(3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack$, $x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution

9. $animal(Tuna)$ (6,7, x/Tuna)

10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ (3,9, z/Tuna)

(5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution

9. $animal(Tuna)$ (6,7, x/Tuna)

10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ (3,9, z/Tuna)

11. $kills(Jack, Tuna)$ (5,8)

(10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution

9. $animal(Tuna)$ (6,7, x/Tuna)

10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ (3,9, z/Tuna)

11. $kills(Jack, Tuna)$ (5,8)

12. $\neg loves(y, Jack)$ (10,11, x/Jack)

(2,4, $x_2/Jack, x_4/F(Jack)$)

(1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution

| | |
|---|---|
| 9. $animal(Tuna)$ | (6,7, x/Tuna) |
| 10. $\neg kills(x, Tuna) \lor \neg loves(y, x)$ | (3,9, z/Tuna) |
| 11. $kills(Jack, Tuna)$ | (5,8) |
| 12. $\neg loves(y, Jack)$ | (10,11, x/Jack) |
| 13. $\neg animal(F(Jack)) \lor loves(G(Jack), Jack)$ | (2,4, $x_2/Jack, x_4/F(Jack)$) |
| | (1,13, $x_1/Jack$ ) |
| | (12,14, $y/G(Jack)$ ) |

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution

9. $animal(Tuna)$ — (6,7, x/Tuna)

10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ — (3,9, z/Tuna)

11. $kills(Jack, Tuna)$ — (5,8)

12. $\neg loves(y, Jack)$ — (10,11, x/Jack)

13. $\neg animal(F(Jack)) \vee loves(G(Jack), Jack)$ — (2,4, $x_2/Jack, x_4/F(Jack)$)

14. $loves(G(Jack), Jack)$ — (1,13, $x_1/Jack$ )

(12,14, $y/G(Jack)$ )

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution

| | |
|---|---|
| 9. $animal(Tuna)$ | (6,7, x/Tuna) |
| 10. $\neg kills(x, Tuna) \vee \neg loves(y, x)$ | (3,9, z/Tuna) |
| 11. $kills(Jack, Tuna)$ | (5,8) |
| 12. $\neg loves(y, Jack)$ | (10,11, x/Jack) |
| 13. $\neg animal(F(Jack)) \vee loves(G(Jack), Jack)$ | (2,4, $x_2/Jack, x_4/F(Jack)$) |
| 14. $loves(G(Jack), Jack)$ | (1,13, $x_1/Jack$ ) |
| 15. $\perp$ | (12,14, $y/G(Jack)$ ) |

Note:step 13 requires us to rename the variables (x appears on both sides)

# Task 3 - Proof by resolution (alternative visualization from AIMA)

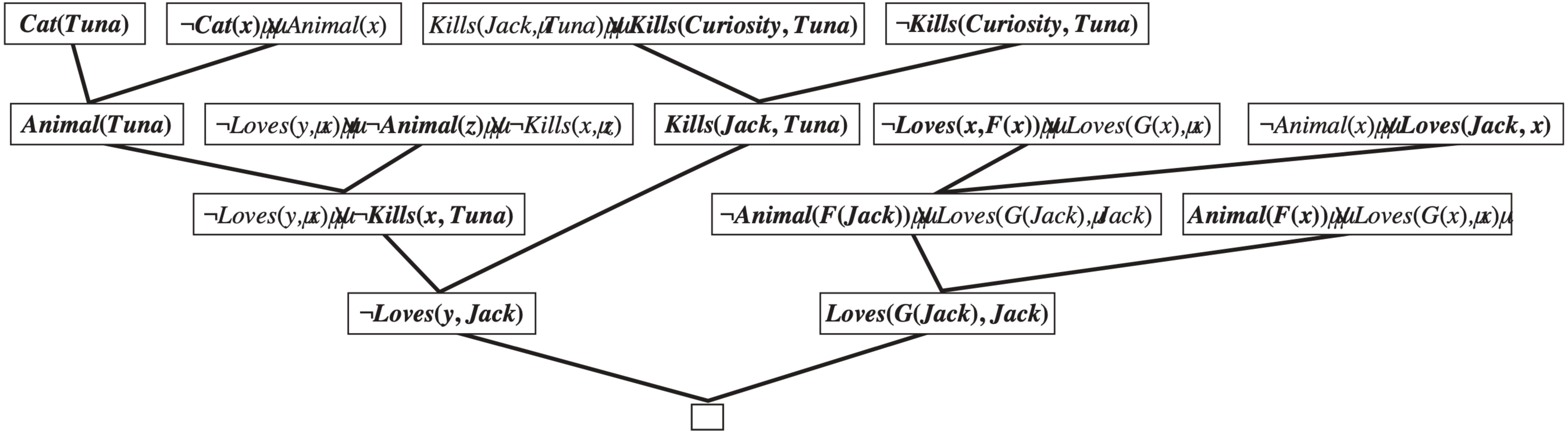


**Figure 9.12** A resolution proof that Curiosity killed the cat. Notice the use of factoring in the derivation of the clause $Loves(G(Jack), Jack)$. Notice also in the upper right, the unification of $Loves(x, F(x))$ and $Loves(Jack, x)$ can only succeed after the variables have been standardized apart.