Chapter 1
Proofs of Parser Laws

The approach taken to prove the following parser laws for `parsley` is via equational reasoning on `gigaparsec` semantics, under the assumption that their semantics are equivalent. While there is no formal proof of this equivalence at the present, `gigaparsec` was designed to have semantics equivalent to `parsley`'s.

1.1 Left absorption for `fmap`

```
    f <$> empty
=     { applicative functor law }
    pure f <*> empty
=     { definition of <*> }
    liftA2 ($) (pure f) empty
=     { semantics of liftA2 }
    Parsec $ λst ok err →
      let ok' x st' = (unParsec empty) st' (ok . (x $)) err
      in (unParsec $ pure f) st ok' err
=     { semantics of empty }
    Parsec $ λst ok err →
      let ok' x st' = (unParsec $ raise (`emptyErr` 0)) st' (ok . (x $)) err
      in (unParsec $ pure f) st ok' err
=     { semantics of raise }
    Parsec $ λst ok err →
      let ok' x st' = (unParsec $ Parsec $ λst'' _ bad →
        useHints bad (emptyErr st'' 0) st') st' (ok . (x $)) err
      in (unParsec $ pure f) st ok' err
=     { β-reduction }
    Parsec $ λst ok err →
      let ok' x st' = useHints err (emptyErr st' 0) st'
      in (unParsec $ pure f) st ok' err
=     { semantics of pure }
    Parsec $ λst ok err →
      let ok' x st' = useHints err (emptyErr st' 0) st'
      in (unParsec $ Parsec $ λst'' ok'' _ → ok'' f st'') st ok' err
=     { β-reduction }
    Parsec $ λst ok err →
      let ok' x st' = useHints err (emptyErr st' 0) st'
      in ok' f st
=     { inline ok' }
    Parsec $ λst ok err → useHints err (emptyErr st 0) st
=     { rearrange and α-conversion }
    Parsec $ λst _ bad → useHints bad ((`emptyErr` 0) st) st
=     { fold definition of raise }
    raise (`emptyErr` 0)
=     { fold definition of empty }
    empty
```