

## Chapter 1

# Proofs of Parser Laws

The approach taken to prove the following parser laws for `parsley` is via equational reasoning on `gigaparsec` semantics, under the assumption that their semantics are equivalent. While there is no formal proof of this equivalence at the present, `gigaparsec` was designed to have semantics equivalent to `parsley`'s.

### 1.1 Left absorption for `fmap`

```

f <$> empty
=   { applicative functor law }
    pure f <*> empty
=   { definition of <*> }
    liftA2 ($) (pure f) empty
=   { semantics of liftA2 }
    Parsec $ \st ok err ->
      let ok' x st' = (unParsec empty) st' (ok . (x $)) err
      in (unParsec $ pure f) st ok' err
=   { semantics of empty }
    Parsec $ \st ok err ->
      let ok' x st' = (unParsec $ raise (`emptyErr` 0)) st' (ok . (x $)) err
      in (unParsec $ pure f) st ok' err
=   { semantics of raise }
    Parsec $ \st ok err ->
      let ok' x st' = (unParsec $ Parsec $ \st'' _ bad ->
        useHints bad (emptyErr st'' 0) st') st' (ok . (x $)) err
      in (unParsec $ pure f) st ok' err
=   {  $\beta$ -reduction }
    Parsec $ \st ok err ->
      let ok' x st' = useHints err (emptyErr st' 0) st'
      in (unParsec $ pure f) st ok' err
=   { semantics of pure }
    Parsec $ \st ok err ->
      let ok' x st' = useHints err (emptyErr st' 0) st'
      in (unParsec $ Parsec $ \st'' ok'' _ -> ok'' f st'') st ok' err
=   {  $\beta$ -reduction }
    Parsec $ \st ok err ->
      let ok' x st' = useHints err (emptyErr st' 0) st'

```

```
      in ok' f st
=      { inline ok' }
      Parsec $ \st ok err -> useHints err (emptyErr st 0) st
=      { rearrange and  $\alpha$ -conversion }
      Parsec $ \st _ bad -> useHints bad ((`emptyErr` 0) st) st
=      { fold definition of raise }
      raise (`emptyErr` 0)
=      { fold definition of empty }
      empty
```