## Chapter 1

# **Removing Left-Recursion**

*Left-recursive* grammar rules are a common pattern to represent left-associativity. Take for example the following definition of a left-associative addition operator:

$$\langle expr \rangle ::= \langle expr \rangle '+' \langle term \rangle \mid \langle term \rangle$$

Since the first production of  $\langle expr \rangle$  is itself, this rule is said to be left-recursive. This poses a problem for recursive-descent parsers, such as those that parsley produces: it will try to parse  $\langle expr \rangle$  by first trying to parse  $\langle expr \rangle$ , and so on, resulting in an unproductive infinite loop.

Although it is possible to address the issue by transforming the grammar with algorithms such as Paull's algorithm [Moore 2000], in the context of parser combinators this is considered an anti-pattern by Willis and Wu [2021]. They argue that this transformation obscures the original intent of the grammar, and exposes lower-level implementation details when this can be abstracted behind a combinator. Instead, they propose that the idiomatic method to handle left-recursion in parser combinators is to use the chain family of combinators [Fokker 1995]. These combinators encapsulate the behaviour of right-associating left-recursive rules and correcting the result back to a left-associative form.

Left-recursion often comes as a nasty surprise for novice users naïvely translating BNF grammars into parser implementations — this issue is not unique to parser combinators, but also extends to many popular parser generators that use recursive-descent. Thus, it would be beneficial to provide a linting rule for parsley that can warn users when parsers are left-recursive. In fact, the next major release of parsley 5.0 will introduce a detectDivergence combinator, which performs *dynamic* analysis to detect unproductive looping at runtime. Therefore, parsley-garnish could complement this functionality with an auto-fix rule to refactor left-recursive parsers to use parsley's idiomatic chain combinators.

**Running example** The following left-recursive parser and its transformation into a non-left-recursive form will be used as an example for this chapter:

```
lazy val example: Parsley[String] = (example, string("a")).zipped(_ + _) | string("b")
```

The example parser intends to express the following simple grammar expressed using left-recursion. The goal is to refactor example so that it retains the intended semantics, but is transformed into a parser that parsley can handle correctly.

$$\langle example \rangle ::= \langle example \rangle$$
 "a" | "b"

## 1.1 The Left-Recursion Factoring Transformation

parsley-garnish bases its left-recursion factoring transformation on the work of Baars and Swierstra [2004], adapted to fit the PEG semantics of parsley. At a high-level, the transformation involves "unfolding" each non-terminal production into three parts:

- results: The semantic actions of the parser, if it can derive the empty string. Conceptually, this has type Option[A] where A is the type of the result.
- nonLeftRec: The non-left-recursive part of the parser that does not derive the empty string. This will have some type Parsley[A].
- leftRec: The left-recursive call, which in the general left-recursive case, corresponds to a repeated postfix operator of type Parsley[A => A]. This is a function which requires the semantics of the left-recursive non-terminal argument.

This transformation is applied in-order to each parser in the source file, replacing the original parser with its factored form if it was left-recursive. An unfolded parser is recombined using chain.postfix: this combinator

encapsulates the general form of left-associative parsing, and most other iterative combinators can be derived from it [Willis 2024].

```
def transform(
  results: Option[A],
  nonLeftRec: Parsley[A],
  leftRec: Parsley[A => A]
): Parsley[A] = {
  val result: Parsley[A] = results match {
    case None => empty
    case Some(x) => pure(x)
  }
  // The type signature of postfix in Parsley:
  // def postfix[A](p: Parsley[A])(op: => Parsley[A => A]): Parsley[A]
  chain.postfix(nonLeftRec | results)(leftRec)
}
```

## 1.2 Necessary Infrastructure

The comparatively simple linting rules discussed in the previous ?? were implemented by directly inspecting the generic Scala AST provided by Scalafix. However, even though parsley programs are written in Scala, it is important to remember that parsley is a DSL borrowing Scala as a host language. Domain-specific transformations like left-recursion factoring are therefore naturally defined as transformations on the parsley AST, at a higher level of abstraction than the generic Scala AST. Thus, this section discusses the extra infrastructure used to support the left-recursion factoring transformation:

- Firstly, §1.2.1 motivates the idea of using an intermediate AST representation for parsers, distinct from the general-purpose Scala AST.
- Following this, §1.2.2 shows how the AST of a Scala source file is converted into this intermediate representation.
- Finally, §1.2.3 discusses how the intermediate AST is converted back into Scala code so that it can be applied as a Scalafix patch.

#### 1.2.1 An Intermediate AST

The transformations described by Baars and Swierstra [2004] require an explicit representation of the grammar and production rules so that they can be inspected and manipulated before generating code. They achieve this by representing parsers as a deep-embedded datatype in the form of an intermediate AST, in a similar manner to parsley.

Since parsley-garnish is a linter, by nature, it has access to an explicit grammar representation in the form of the full scala.meta.Tree ast of the source program. However, this datatype represents general-purpose abstract Scala syntax, rather than the abstract syntax of a specialised parser combinator DSL. This makes it not well-suited for performing domain-specific operations over the AST.

Take for example the task of combining two AST nodes Term.Name("p") and Term.Name("q"), representing named parsers p and q, with the combinator <\*> (pronunced "ap"). This operation can be concisely expressed with Scalameta quasiquotes, rather than manually writing out the full explicit AST:

```
q"p <*> q" ==
Term.ApplyInfix(
    Term.Name("p"),
    Term.Name("<*>"),
```

```
Type.ArgClause(Nil),
Term.ArgClause(List(Term.Name("q")), None)
)
```

However, the operation of inspecting the individual parsers p and q is not as straightforward. Although quasiquotes can be used as extractor patterns in pattern matching, this usage is discouraged due to limitations in their design that makes it easy to accidentally introduce match errors<sup>1</sup>. Thus, extracting the parsers necessitates a long-winded pattern match like so:

```
val ap = SymbolMatcher.normalized("parsley.Parsley.`<*>`")

def deconstructAp(parser: Term) = parser match {
   case Term.ApplyInfix(p, ap(_), _, Term.ArgClause(List(q), _)) => (p, q)
}
```

This involves dealing with abstract general-purpose syntax constructs like Term.ApplyInfix, which are low-level details not relevant to the task of manipulating parsers. Although this is not an issue for simple one-off transformations, for more specialised transformations like left-recursion factoring, it would be desirable to abstract away from these low-level syntactic details. This motivates the need for an higher-level, intermediate AST representation that is more specialised to the domain of parser combinators.

#### The Parser ADT

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parsley-garnish therefore takes a similar approach as Baars and Swierstra [2004] and parsley itself, building an intermediate AST as a deep-embedded parser combinator tree. Fig. 1.1 shows how this is implemented as a Parser algebraic data type (ADT). All Parser types represent parsley combinators, with the sole exception of NonTerminal to represent references to named parsers.

```
trait Parser
case class NonTerminal(ref: Symbol) extends Parser
case class Pure(x: Term) extends Parser
case object Empty extends Parser
case class <*>(p: Parser, q: Parser) extends Parser
case class <|>(p: Parser, q: Parser) extends Parser
case class Str(s: String) extends Parser
case class Chr(c: Char) extends Parser
```

Fig. 1.1: A subset of the Parser ADT, representing the core combinators in parsley-garnish.

**Deconstructing parsers** Scala allows users to define symbolic class names (as evidenced by the definitions of <\*> and <|> in fig. 1.1), and provides syntactic sugar to pattern match on these constructors using infix notation. This results in a very natural and readable pattern matching syntax:

```
def deconstructAp(parser: Parser) = parser match {
  case p <*> q => (p, q)
}
```

**Constructing parsers** Defining infix operators as extension methods on the Parser trait provides a similar syntactic sugar for constructing parsers:

```
def |(q: Parser) = <|>(p, q)
  def map(f: Term) = FMap(p, f)
}
extension (ps: List[Parser]) {
  def zipped(f: Term) = Zipped(f, ps)
}
```

This makes the syntax for writing Parser terms feel natural and similar to writing parsley code. For example, notice how constructing the *code* representation of the example parser resembles how the original parser itself would be written:

```
val EXAMPLE = NonTerminal(Sym(Term.Name("example").symbol))

// val example: Parsley[String] = (example, string("a")).zipped( _ + _ ) | string("b")
    val example: Parser = List(EXAMPLE, Str("a")).zipped(q"_ + _") | Str("b")
```

#### 1.2.2 Lifting to the Intermediate Parser AST

Converting the raw Scala AST to this intermediate parser combinator AST requires the following basic operations:

- 1. Identifying all named parsers defined in the source program these correspond to non-terminal symbols in the grammar.
- 2. Lifting the definition each parser into the intermediate AST, i.e. a Parser object.
- 3. Collecting these into a map to represent the high-level grammar the unique symbol of each named parser is mapped to its corresponding Parser object, along with extra metadata required for the transformation.

Most importantly, this metadata includes a reference to a parser's original node in the Scala AST, so lint diagnostics or code rewrites can be applied to the correct location in the source file:

```
case class ParserDefn(name: Term.Name, parser: Parser, tpe: Type.Name, originalTree: Term)
```

#### **Identifying Named Parsers**

Finding AST nodes corresponding to the definition sites of named parsers involves pattern matching on val, var, and def definitions with a type inferred to be some Parsley[\_]. This type information is accessed by querying the Scalafix semantic API for the node's symbol information. Consider the labelled AST structure of the example parser:

Note that the decltpe field refers to the *syntax* of the explicit type annotation, not the *semantic* information the variable's inferred type. Therefore, this field will not always be present, so in the general case, the type must be queried via a symbol information lookup:

```
exampleTree match {
  case Defn.Val(_, List(Pat.Var(varName)), _, body) =>
    println(s"qualified symbol = ${varName.symbol}")
    // Query the symbol information of the variable name, and get its type signature
    varName.symbol.info.get.signature match {
      // Scalameta treats this as a zero-arg method, so the relevant part is its return type
      case MethodSignature(_, _, returnType) =>
        println(s"type = $returnType")
        println(s"structure of type object = ${returnType.structure}")
    }
}
// qualified symbol = path/to/package/ObjectName.example.
// type = Parsley[String]
// structure of type object = TypeRef(
// NoType,
// Symbol("parsley/Parsley#"),
// List(TypeRef(NoType, Symbol("scala/Predef.String#"), List()))
// )
```

Seeing that the type of this AST node is Parsley[String], parsley-garnish can then proceed to convert the rhs term into a Parser ADT object. The map entry uses the fully qualified symbol for example as the key, and the lifted Parser object as the value.

#### Converting Scalameta Terms to the Parser ADT

Having identified the AST nodes which represent parsers, they need to be transformed into the appropriate Parser representation. This involves pattern matching on the scala.meta.Term to determine which parser combinator it represents, and then constructing the appropriate Parser instance.

Each Parser defines a partial function fromTerm to instantiate a parser from the appropriate scala.meta.Term. These fromTerm methods perform the menial work of pattern matching on the low-level syntactic constructs of the Scala AST. All fromTerm methods are combined to define the toParser extension method on scala.meta.Term — this is where AST nodes are lifted to their corresponding Parser representation.

The pattern matching example from §1.2.1 makes a reappearance in the definition of <\*>.fromTerm, where the arguments to the <\*> combinator are instead recursively lifted to Parser objects:

```
// Type signatures in Parsley:
// p: Parsley[A => B], q: =>Parsley[A], p <*> q: Parsley[B]
case class <*>(p: Parser, q: Parser) extends Parser
object <*> {
    // Match the specific symbol for parsley's <*> combinator
    val matcher = SymbolMatcher.normalized("parsley.Parsley.`<*>`")

def fromTerm: PartialFunction[Term, <*>] = {
    // Pattern match succeeds only if the term has the structure 'p <*> q'
    case Term.ApplyInfix(p, matcher(_), _, Term.ArgClause(List(q), _)) =>
        p.toParser <*> q.toParser
}
```

Where a combinator takes a non-parser argument, this is treated as a black box and kept as a raw AST node of type scala.meta.Term:

```
// x: A, pure(x): Parsley[A]
case class Pure(x: Term) extends Parser
```

```
object Pure {
  val matcher = SymbolMatcher.normalized("parsley.ParsleyImpl.pure")

def fromTerm: PartialFunction[Term, Pure] = {
    // expr is an opaque AST node that can't be further inspected
    case Term.Apply(matcher(_), Term.ArgClause(List(expr), _)) => Pure(expr)
  }
}
```

#### **Building the Grammar Map**

The overall process of converting the source file AST to a high-level map of the grammar can therefore be expressed as a single traversal over the AST:

```
// Encapsulate all valid pattern matches into a single extractor object
object VariableDecl {
  def unapply(tree: Tree): ParserDefn = tree match {
    // isParsleyType uses symbol info to check if variable type is Parsley[_]
    case Defn.Val(_, List(Pat.Var(varName)), _, body) if isParsleyType(varName) =>
      // If the pattern match is successful, convert the definition body to a Parser
      // Collect metadata and bundle into a parser definition object
      ParserDefn(
       name = varName,
        parser = body.toParser,
       tpe = getParsleyType(varName),
       originalTree = body
    // ... similar cases for Defn.Var and Defn.Def
  }
}
type Grammar = Map[Symbol, ParserDefn]
val grammar: Grammar = doc.tree.collect {
  // Every AST node that satisfies the pattern match is added to the map
  case VariableDecl(parserDef) => parserDefn.name.symbol -> parserDef
}.toMap
```

#### 1.2.3 Lowering Back to the Scalameta AST

After all necessary transformations have been applied to parser terms, the final step is to convert them back to a textual representation to be applied as a Scalafix patch. Parsers can be lowered back to scala.meta.Term nodes by the inverse of the original fromTerm transformation. The Parser trait defines this transformation as the method term, using quasiquotes to simplify the construction of the scala.meta.Term nodes. For example:

```
case class Zipped(func: Function, parsers: List[Parser]) extends Parser {
  val term: Term = q"(..${parsers.map(_.term)}).zipped(${func.term})"
}
```

The Parser trait can then define a toString method based on Scalameta's AST pretty-printer on the term field. This "stringified" term is therefore guaranteed to be syntactically well-formed and is the appropriate type expected by Scalafix's Patch methods.

## 1.3 Implementing the Left-Recursion Transformation

With the Parser AST defined, it is now possible to implement the left-recursion factoring transformation in a more high-level manner. Using the grammar map collected previously, the overall transformation can be summarised as follows (in Scala-like pseudocode for brevity):

```
val transformedGrammar = grammar.to(mutable.Map)
for (currentNT <- grammar.keys) {
    // Unfold each non-terminal parser into its three parts
    val (results, nonLeftRec, leftRec) = unfold(transformedGrammar, currentNT)
    // Update the grammar with the recombined postfix parser if it was left-recursive
    transformedGrammar(currentNT).parser = transform(results, nonLeftRec, leftRec)
}
// Generate rewrite patches for each parser definition
transformedGrammar.mapValues { defn =>
    Patch.replaceTree(defn.originalTree, defn.parser.toString)
}
```

The unfolding transformation is defined as follows, which is dynamically dispatched to the appropriate combinator's unfold method:

```
def unfold(env: Grammar, currentNT: Symbol): UnfoldedParser =
  env(currentNT).parser.unfold(currentNT, env, visited = Set.empty)
```

#### 1.3.1 Unfolding the Core Combinators

The most important core combinators in the left-recursion transformation are NonTerminal, Pure, Empty, <|>, and <\*>. Character combinators such as Str (string), Chr (char), and Item (item), although technically handled by parsley-garnish as primitives, are grouped together as they behave in a simple and uniform manner. The rest of the combinators in parsley are defined in terms of the primitive combinators, and are desugared into the core combinators during unfolding.

**Non-terminals** Non-terminals are the primary base case of the recursive unfolding transformation. There are three cases to consider when a non-terminal symbol is encountered:

- This non-terminal is the same as the current non-terminal being analysed, which means it may be in a left-recursive position. Recall that the leftRec result corresponds to a function, where the call to the left-recursive non-terminal is stripped off. Therefore in this case, the left-recursive call is the identity function which just returns its argument, i.e. the non-terminal itself. If the non-terminal is not in left-recursive position, this doesn't matter as the leftRec result will be subsumed by the nonLeftRec result in the final recombination.
- The non-terminal refers to a different non-terminal, but has already been visited in the current unfolding process. This means it shouldn't be recursively visited again, as it has already been factored out. It is not left-recursive and the nonLeftRec part is just a reference to this non-terminal's name.
- The non-terminal refers to a different non-terminal that has not been visited yet. In this case, the non-terminal is recursively unfolded, and the visited set is updated to include this non-terminal.

The three cases can be implemented as so:

```
case class NonTerminal(ref: Symbol) extends Parser {
  def unfold(currentNT: Symbol, env: Grammar) =
    if (ref == currentNT) (None, Empty, Pure(q"identity"))
    else if (visited.contains(ref)) (None, NonTerminal(ref), Empty)
```

```
else env(ref).parser.unfold(currentNT, env, visited + ref)
}
```

**Base cases** The base cases are mostly straightforward. The unfolded result portion can be reasoned with by the combinators' correspondence to PEG expressions [Eichenroth, Rein, and Hirschfeld 2022], as shown in table 1.1. If the combinator can succeed by parsing the empty string, result is a Some value of its semantic action, otherwise it is None. The other two portions are also easy: none are left-recursive, and if the combinator consumes input on success then nonLeftRec is just the combinator itself. Thus, the base cases can be summed up as follows:

Combinator	Can succeed without consuming input?
pure(x)	Yes, semantic action is x.
empty	No, combinator fails immediately regardless of input.
string(s)	Functionally no. Can theoretically be given the empty string "", but this is illegal in
	parsley and triggers a runtime exception.
char(c)	No, always consumes the given character.
item	No, always consumes any single character.

Table 1.1: Semantic action (result) portion behaviour for each base case.

Choice (<|>) This recursive case is also relatively simple: each branch is unfolded, then the nonLeftRec and leftRec portions are recombined with the | combinator. The result portion is that of the left branch if it can derive the empty string, otherwise it is the right branch. This behaviour differs slightly from the original presentation of the transformation by Baars and Swierstra [2004], where | is based on CFG semantics. The consequence of this is that choice is unbiased in the original transformation, triggering an ambiguity error if both branches can succeed without consuming input. However, since parsley uses PEG semantics, this ambiguity is not possible – the choice operation represented by the | combinator is *left-biased*, so if the left parser succeeds the right parser is not attempted.

```
case class <|>(p: Parser, q: Parser) extends Parser {
  def unfold() = {
    val (pe, pn, pl) = p.unfold
    val (qe, qn, ql) = q.unfold
        (pe.orElse(qe), pn | qn, pl | ql)
    }
}
```

**Ap (<\*>)** This is an important case, as parsley-garnish chooses to use <\*> as the primitive combinator for composing parsers. It is also the most complex:

- p <\*> q can succeed without consuming input only if *both* p and q can do so. Parser combinators abide by the law that pure(f) <\*> pure(x) = pure(f(x)), so the resulting semantic action is that of p applied as a function to the semantic action of q.
- In the nonLeftRec and leftRec portions, if p can succeed without consuming input, then q must be subjected to the semantic value corresponding to p [Swierstra and Duponcheel 1996]. This is achieved using the result shown in eq. (1.1), which is a special case of grammar right-factoring in reverse.

- The leftRec portion requires re-associating semantic actions back to the left, which requires using the flip higher-order function. Thus, the left branch of the RHS of eq. (1.1) is reformulated as p.map(flip) <\*> a.
- Since leftRec is a parser of a function, the right branch also needs adjusting: the function composition
  operation must be used in order to apply the function to q with the <\*> combinator.

$$(p <|> pure f) <*> q \Rightarrow (p <*> q) <|> (f <$> q)$$
 (1.1)

Given the above observations, the implementation of the <\*> combinator's unfolding is as follows:

```
case class <*>(p: Parser, q: Parser) extends Parser {
  def unfold() = {
    val (pe, pn, pl) = p.unfold
    val (qe, qn, ql) = q.unfold
    val result = if (pe.isDefined && qe.isDefined) Some(q"${pe.get}(${qe.get})") else None
    val nonLefts = {
      val lnl = pn <*> q
     val rnl = pe.map(q"f => qn.map(f)").getOrElse(Empty) // f <$> qn
     lnl | rnl
    val lefts = {
      val llr = pl.map(q"flip") <*> q
     val rlr = pe.map(q"$ql.map(compose)").getOrElse(Empty) // f.map(compose) <$> ql
     llr | rlr
    }
    (result, nonLefts, lefts)
  }
}
```

### 1.3.2 Composite Combinators

The remaining combinators are defined in terms of the core combinators, and are unfolded by recursively unfolding their constituent parts. For example, the map combinator is defined as p.map(f) = pure(f) <\*> p, so its unfolding is simply implemented as:

```
case class FMap(p: Parser, f: Term) extends Parser {
  def unfold() = (Pure(f) <*> p).unfold
}
```

Further high-level combinators are defined in a similar manner. The improved sequencing combinators lift and zipped, as well as bridge constructors using the *Parser Bridges* design pattern [Willis and Wu 2022], represent a further generalisation of the map combinator:

```
liftN(f, p1, ..., pN) // explicit lift syntax, where f is a function of arity N
f.lift(p1, ..., pN) // implicit lift syntax
(p1, ..., pN).zipped(f) // zipped syntax, improves Scala's ability to infer types
F(p1, ..., pN) // parser bridge pattern, abstracts the construction of f behind bridge F
// All are equivalent to the following desugared form,
// which is idiomatic in Haskell, but not in Scala for performance reasons
= pure(f.curried) <*> p1 <*> ... <*> pN
```

These are handled in parsley-garnish as the LiftLike family of combinators, which converts them into the desugared form in terms of core combinators pure and <\*>, allowing the unfolding transformation to be applied:

```
trait LiftLike extends Parser {
   // foldLeft builds a parser of the form 'pure(f.curried) <*> p1 <*> ... <*> pN'
   def unfold() = parsers.foldLeft(Pure(q"$func.curried"))(_ <*> _).unfold
}
```

#### 1.3.3 Defining Utility Functions

In various places within the unfolding transformations, three higher-order functions were utilised which may not be provided by the Scala standard library:

- The flip function reverses the order of arguments applied to a function. This isn't defined in the standard library, so it must be defined manually.
- Function composition is defined in the standard library, but a more versatile curried version is required by the transformation, so it is also defined manually.
- The identity function identity[A]: A => A is defined in the standard library.

Therefore, parsley-garnish will insert the following definitions into the source file as a patch:

```
def flip[A, B, C](f: A => B => C)(x: B)(y: A): C = f(y)(x)
def compose[A, B, C](f: B => C)(g: A => B)(x: A): C = f(g(x))
```

This brings these higher-order functions into scope, allowing the transformed code to make use of it.

#### Success...?

With the unfolding transformations defined for all core combinators, the left-recursion factoring transformation is now complete. Running the transformation on the example parser yields the output in fig. 1.2.

```
def flip[A, B, C](f: A => B => C)(x: B)(y: A): C = f(y)(x)
def compose[A, B, C](f: B => C)(g: A => B)(x: A): C = f(g(x))

lazy val example: Parsley[String] = chain.postfix(
  empty | (empty.map((_ + _).curried) | empty <*> example) <*> string("a")
  | string("b") | empty
)(
  (empty.map(flip) <*> example | pure(identity).map(compose((_ + _).curried)))
  .map(flip) <*> string("a")
  | empty | empty
)
```

Fig. 1.2: The initial attempt at factoring out left-recursion from the example parser.

This is... disappointing, to say the least. There are many things wrong with the transformed output:

- The parser is horrendously complex and unreadable, its intent entirely obfuscated in a sea of combinators. It's especially frustrating that there are so many empty combinators, when p | empty and empty | p are both actually just equivalent to p.
- Having to define the flip and compose functions is not ideal, but inlining them as lambdas would make
  the code even worse.

• Even worse, the parser does not even typecheck – unlike classical Hindley-Milner-based type systems, Scala only has *local* type inference [Cremet et al. 2006]. As a result, the compiler is unable to correctly infer correct types for flip and also asks for explicit type annotations in the lambda (\_ + \_).curried.

The result is discouraging especially because it is not impossible to factor out the left-recursion in a nice manner: a hand-written equivalent using postfix would resemble the concisely defined parser in fig. 1.3. There is still hope, though – if the empty combinators can be removed and something is done about the higher-order functions, perhaps fig. 1.2 could be salvaged into something that looks more like the human-written version.

```
lazy val example: Parsley[String] = chain.postfix(string("b"))(string("a").as(_ + "a"))
```

Fig. 1.3: An idiomatic way to express the example parser using chain.postfix.