ISyE 3133 Project

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Due 05:00 PM 04-21-2021

Introduction

You are hired by a large logistics company as optimization specialist. Your first task is developing an optimization solution for the company's delivery operation. This operation is conducted on a geography that can be represented as a network $G = (N, A)$ where N is a set of cities and A is a set of arcs/roads between cities.

The company has provided you two CSV files that contain coordinate-wise information about G: 1. Nodes.csv, 2. Arcs.csv. The Nodes.csv has three columns, nodes, nodes_x, and nodes_y, that indicate the name of the city, x-coordinate of the city, and y-coordinate of the city, respectively. The Arcs.csv file has two columns, Head and Tail, that indicate the starting city and the ending city, respectively.

On G, the company must deliver all commodities in set K, where each item $k \in K$ must be delivered a quantity, q_k, from its source o_k to sink d_k. The company has provided these information in Commodities.csv. All commodities are unsplittable, i.e. each commodity must be sent on a **path** from its source to sink.

These commodities are delivered by freight trucks. Hence, each arc/road must be assigned enough number of trucks that can carry the total commodity-quantity through the arc/road. Each arc/road $a \in A$ has a cost, m_a, which is a unit cost of a truck moving on a. For the sake of this problem, let the cost m_a be the Euclidean distance from the head to the tail of arc/road $a \in A$.

Your goal is to design a delivery plan that presents:

(1) Which arcs are chosen to compose the delivery path from each commodity's origin to destination (i.e. for a fixed $a \in A$ and $k \in K$, $1 :=$ commodity k is sent through a for delivery, $0 :=$ otherwise)

(2) How many trucks are required on each arc to send all commodities on the constructed delivery paths from (1)

while minimizing the operation cost (i.e. the total cost incurred by assigning number of trucks on arcs).

Formulation (28 points)

Formulate a Mixed Integer Programming model to solve this problem. Submitted model **must** be typed.

In this formulation, assume that the number of trucks must be integer value.

Clearly specify your choice parameters and variables. Explain your choice of the objective function. Explain your constraints.

Implementation - Graph Parameter (20 points)

Note: Your program must read the data from the given files. Hard-coded data will be (severely) penalized.

Lets implement your formulation with Gurobi/Python step-by-step together. First, we will bring data to Python environment from provided CSV files. Create a function "graph_parameter" where its inputs are the 3 CSV files (Nodes.csv, Arcs.csv, Commodities.csv). This function should return a list that contains information as follows:

1. Commodities & Quantity

   (Hint: Try creating a dictionary where its keys are commodities and values are quantity)

2. Nodes

3. Arcs

   (Hint: You may want to search a data-structure *tuplelist* in Gurobi)

4. Sources

   (Hint: Try creating a dictionary where its keys are commodities and values are sources)

5. Sinks

   (Hint: Try creating a dictionary where its keys are commodities and values are sinks)

6. m ($\equiv$ Distance)

   (Hint: Try creating a dictionary where its keys are arcs/roads in tuple-form and values are the Euclidean distance from the head and tail of each arc/road)

In your submission, this script should be named graph_parameter.py.

Explain your data structures. Explain how different parts of your code correspond to different information. I.e. explain which parts of the code gets 3 input-CSV files and collects Commodities, Nodes, Arcs, etc.

Implementation - Gurobi Modeling (15 points)

Note: Your program must read the data from the given files. Hard-coded data will be (severely) penalized.

Now that we have our parameters for the formulation, lets translate our hand-written model into Gurobi/Python and solve for the optimal delivery plan. *Assume for now (except the* Analysis-Truck *section below) that we allow fractional number of trucks for all arcs.* In your submission, this script should be named ModelA.py.

First, type the following code on the very first line of Model.py:

- from graph_parameter import graph_parameter

which will let you call the graph_parameter function from graph_parameter.py. By running the function inside the ModelA.py script, we can now use the parameters from the graph_parameter function to model our problem.

Explain how different parts of your code correspond to different parts of your formulation. More specifically, explain which parts of the code generate the variables, objective function, and each constraint.

Implementation - Solving (4 points)

Note: Your program must read the data from the given files. Hard-coded data will be (severely) penalized.

Assume for now (except the Analysis-Truck *section below) that we allow fractional number of trucks for all arcs.*

Lets finally solve the model. What is the objective value of your solution? What is the running time? What is the optimal solution? More specifically, what is the created path for each commodity? What is the number of trucks required on each arc? In your write up, summarize your solution in a human-readable format, e.g. a table.

Report variables and their value only if they have positive values.

Analysis-Path A (9 points)

Note: Your program must read the data from the given files. Hard-coded data will be (severely) penalized.

Let us run some experiments just so that we learn more about the problem structure. Suppose that you relax the fact that the path construction decision is binary (i.e. no longer $\in \{0,1\}$, but now $\in [0,1]$). Lets solve the model again.

Assume for now (except the Analysis-Truck *section below) that we allow fractional number of trucks for all arcs.*

What is the objective value of your solution now? What is the running time now? Is it longer or shorter? Why do you think this changed caused this observation?

What is the optimal solution now? More specifically, what is the created path for each commodity? Does the path change from previous solution? Does the number of trucks required on each arc change from previous solution?

In your write up, summarize your solution in a human readable format, e.g. a table.

Report variables and their value only if they have positive values.

Below the table, provide your thoughts on these questions in a paragraph.

Analysis-Path B (12 points)

Note: Your program must read the data from the given files. Hard-coded data will be (severely) penalized.

Lets think a little bit more so that we learn every more about the problem structure following what we have seen from the previous section. *Assume for now (except the* Analysis-Truck *section below) that we allow fractional number of trucks for all arcs. For this part, set your path construction decision as binary (i.e. set as* $\in \{0,1\}$*).* Based on your solution, what is the number of trucks on each arc equal to? Using this observation, can we write down the objective function in a different way? If so, what is the resulting formulation? Also, what problem does the resulting formulation resemble?

Provide your thoughts on these questions in a paragraph or 2.

Analysis-Truck (12 points)

Note: Your program must read the data from the given files. Hard-coded data will be (severely) penalized.

Recall that we allowed having fractional number of trucks in our solution. However, in practice, this is impossible (for example, I do not know what 0.3 of a truck would look like). Suppose we now impose the restriction that the number of trucks must be integer values. Lets solve the model with this new restriction. *For this part also, set your path construction decision as binary (i.e. set as* $\in \{0,1\}$*).*

What is the objective value of your solution now? What is the relationship between the value obtained now and the value obtained before? What is the running time now? Is it longer or shorter? Why do you think the new restrictions caused this observation?

What is the optimal solution now? More specifically, what is the created path for each commodity? Does the path change from previous solution? What is the number of trucks required on each arc now?

In your write up, summarize your solution in a human readable format, e.g. a table.

Report variables and their value only if they have positive values.

Below the table, provide your thoughts on these questions in a paragraph.

(Extra Credit) (10 points)

Using package *matplotlib*, or any other graphing package, present 2 figures where the 1st and 2nd figures contain graph with the used arcs when the path decision variables are CONTINUOUS and BINARY, respectively. The truck variables should be integers. What is the main difference that you see between the graphs? What difference does CONTINUOUS vs. BINARY make in your solution? Kindly provide your thoughts on these questions in a paragraph below the figures.