Formula Sheet

Mean of a sample space

**Def 1.1**

The mean of a sample of n measured responses y1, y2,..., yn is given by

Variance of a sample space

**Def 1.2**

The variance of a sample of measurements y1, y2,..., yn is the sum of the square of the differences between the measurements and their mean, divided by *n* − 1. Symbolically, the sample variance is

Standard Deviation of a sample space

**Def 1.3**

The standard deviation of a sample of measurements is the positive square root of the variance; that is,

Permutation

**Def 2.7**

An ordered arrangement of *r* distinct objects is called a *permutation*. The number of ways of ordering n distinct objects taken *r* at a time will be designated by the symbol

Combination

**Def 2.8**

The number of combinations of *n* objects taken r at a time is the number of subsets, each of size *r*, that can be formed from the n objects. This number will be denoted by or .

Conditional Probability

**Def 2.9**

The conditional probability of an event A, given that an event B has occurred, is equal to

The conditional probability of an event B, given that an event A has occurred, is equal to

Bayes Theorem

**Th 2.9**

**Bayes’ Rule** Assume that {B1, B2,..., Bk} is a partition of *S* such that *P(*Bi*) > 0*, for *i* = 1, 2,..., *k*. Then

Independence of A and B

**Def 2.10**

Two events *A* and *B* are said to be *independent* if any one of the following holds:

Binomial Distribution

**Def 3.7**

A random variable Y is said to have a *binomial* *distribution* based on n trials with success probability *p* if and only if

, *y* = 0, 1, 2,..., *n* and 0 ≤ *p* ≤ 1

Geometric Distribution

**Def 3.8**

A random variable *Y* is said to have a *geometric* *probability* *distribution* if and only if

, *y* = 0, 1, 2, 3..., *n* and 0 ≤ *p* ≤ 1

Hypergeometric Distribution

**Def 3.8**

A random variable *Y* is said to have a *hypergeometric* *probability* *distribution* if and only if

Poisson Distribution

**Def 3.11**

A random variable *Y* is said to have a *Poisson* *probability* *distribution* if and only if

, *y* = 0, 1, 2,..., λ > 0

Negative Binomial Distribution

**Def 3.9**

A random variable *Y* is said to have a *negative* *binomial* *probability* *distribution* if and only if

Tchebysheff’s Theorem

**Th 3.11**

**Tchebysheff’s Theorem** Let *Y* be a random variable with mean *μ* and finite variance *σ2*. Then, for any constant k > 0,

*P(|Y − µ| < kσ) ≥ 1 −* or *P(|Y − µ| ≥ kσ) ≤*

Probability Distribution for a Continuous Random Variable

**Definitions 4.1 – 4.4**

**4.1**

Let *Y* denote any random variable. The *distribution function* of *Y*, denoted by *F(y)*, is such that

*F(y)* = *P(Y ≤ y)* for −∞ < y < ∞.

**4.2**

A random variable *Y* with distribution function *F*(y) is said to be *continuous* if *F*(y) is continuous, for −∞ < y < ∞.

**4.3**

Let *F(y)* be the distribution function for a continuous random variable *Y*. Then *f* (y), given by

wherever the derivative exists, is called the *probability density function* for the random variable *Y.*

It follows from Definitions 4.2 and 4.3 that F(y) can be written as

**4.4**

Let denote any random variable. If the th *quantile* of , denoted by , is the smallest value such that ≥ . If is continuous, is the smallest value such that ) = ) = p. Some prefer to call the 100th *percentile* of.

**Theorems 4.1 – 4.3**

**4.1**

**Properties of a Distribution Function** If is a distribution function, then

1. is a nondecreasing function of y. [If is a nondecreasing function of . [If and are any values such that , then (

**4.2**

**Properties of a Density Function** If is a density function for a continuous random variable, then

1. for all y, .

**4.3**

If the random variable has density function and , then the probability that falls in the interval is

Expected Value for a Continuous Random Variable Y

**Definitions**

**4.5**

The expected value of a continuous random variable *Y* is

provided that the integral exists.

**Theorems 4.4 – 4.5**

**4.4**

Let be a function of ; then the expected value of is given by

provided that the integral exists.

**4.5**

Let be a constant and let be functions of a continuous random variable Then the following results hold:

Uniform Probability Distribution

**Definitions 4.6 – 4.7**

**4.6**

If a random variableis said to have a continuous *uniform probability distribution* on the interval (, ) if and only if the density function of is

**4.7**

The constants that determine the specific form of a density function are called *parameters* of the density function.

**Theorems**

**4.6**

If and is a random variable uniformly distributed on the interval(, ), then

and .

Other Expected Values

**Definitions 4.13 – 4.14**

**4.13**

If Y is a continuous random variable, then the kth moment about the origin is

given by

*­­­­­ k = E( ), k = 1, 2,....*

The kth moment about the mean, or the kth central moment, is given by

*k = E[(Y - ], k = 1, 2,….*

**4.14**

If Y is a continuous random variable, then the moment-generating function of Y is given by

*m(t) = E().*

The moment-generating function is said to exist if there exists a constant b > 0 such that m(t) is finite for |t| ≤ b.

**Theorems**

**4.12**

Let Y be a random variable with density function f (y) and g(Y) be a function of Y . Then the moment-generating function for g(Y) is

*E[] = .*

Bivariate and Multivariate Probability Distributions

**Definitions 5.1 – 5.3**

**5.1**

Let Y1 and Y2 be discrete random variables. The joint (or bivariate) probability function for Y1 and Y2 is given by

**5.2**

For any random variables and , the joint (bivariate) distribution function is

**5.3**

Let and be continuous random variables with joint distribution function If there exists a nonnegative function , such that

for all −∞ < < ∞, −∞ < < ∞, then and are said to be jointly continuous random variables. The function *f (, )* is called the joint probability density function.

**Theorems 5.1 – 5.5**

**5.1**

If Y1 and Y2 are discrete random variables with joint probability function p(y1, y2), then

1. for all
2. , where the sum is over all values that are assigned nonzero probabilities.

**5.2 pt. 1**

If and are random variables with joint distribution function , then

1. .
2. .
3. If and then

((((.

**5.2 pt. 1**

If and are jointly continuous random variables with a joint density function given by (, then

1. ( for all .
2. .

**5.4**

If and are discrete random variables with joint probability function ( and marginal probability functions( and (, respectively, then and are independent if and only if

*( = ((*

for all pairs of real numbers ().

If and are continuous random variables with joint density function ( and marginal density functions( and (, respectively, then and are independent if and only if

*( = ((*

for all pairs of real numbers ().

**5.5**

Let and have a joint density *(* that is positive if and only if *a* ≤ ≤ *b* and *c* ≤ ≤*d*, for constants a, b, c, and d; and *(* = 0 otherwise. Then and are independent random variables if and only if

*( = g((*

where *g(*is a nonnegative function of alone and *h()* is a nonnegative function of alone.

Bivariate and Multivariate Probability Distributions

**Definitions 5.4 – 5.8**

**5.4**

**a** Let and be jointly discrete random variables with probability function (. Then the *marginal probability functions* of and , respectively, are given by

**b** Let and be jointly continuous random variables with joint density function (. Then the *marginal density functions* of and , respectively, are given by

and .

**5.5**

If and are jointly discrete random variables with joint probability function and marginal probability functions and , respectively, then the *conditional discrete probability function* of given is

provided that .

**5.6**

If and are jointly continuous random variables with jointly density function , then the *conditional distribution function* of given is

.

**5.7**

Let and be jointly continuous random variables with joint density and marginal densities and , respectively. For any such that , the conditional density of given

= is given by

and, for any such that , the conditional density of given is given by

**5.8**

Let have a distribution function , have a distribution function , and and have joint distribution function . Then and are said to be *independent* if and only if

for every pair of real numbers (, ).

If and are not independent, they are said to be *dependent*.