Simulation tools in plasma physics & astrophysics

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What is the minimum physics?

- \bullet What is the plasma β value ?
- How collisional is the plasma?
- What is the geometry/size of the system?
- What are the temporal scales?
- Do we have kinetic effects?
- Do we have quantum effects?
- Are radiative effects important?

For LULI2000, (shot on Cu foil in 2017 w. J. Fuchs)

$$n_e = 10^{26} \ \mathrm{m}^{-3}$$
, $T_e = T_i = 10^7 \ \mathrm{K}$, $A = 63$, $Z = 29$ $B_{\mathrm{ext}} = 50 \ \mathrm{T}$

What is the plasma β value ?

- Magnetic field produced by external coils, up to 50 T
- → produced on few ns [see e.g. Albertazzi et al., 2013]
- Lasers on solid foil target
- \rightarrow with ns lasers : 100 eV
- \rightarrow with ps lasers : up to 15 MeV (with TNSA)
- Lasers on a gas
- → heating with inverse Bremsstrahlung
- **②** $P_m = 10^9$ Pa, $P_{i+} = 5 \times 10^8$ Pa, $P_{e^-} = 2 \times 10^{10}$ Pa
- $\ensuremath{f \oslash}\ eta\gg 1$ leaded by $e^ (eta_{e^-}\sim 15,\ eta_{i^+}\sim 0.5)$

How collisional is the plasma?

• Number of e^- in Debye sphere : $n_e \lambda_D^3 \sim 6 \times 10^3$ \rightarrow Coulomb logarithm : $\ln \Lambda = \ln \left(\frac{\lambda_D}{\lambda_L} \right) = \ln (12\pi n_e \lambda_D^3) \sim 7$

Lorentz model :
$$au_e = \left[\frac{1}{(4\pi\varepsilon_0)^2} \frac{n_e e^4 \ln \Lambda}{m_e^{3/2} T_e^{3/2}} \right]^{-1} = 1 \ \mu \text{s}$$

Lorentz model :
$$\tau_i = \left[\frac{1}{(4\pi\varepsilon_0)^2} \frac{n_i Z^2 e^4 \ln\Lambda}{m_i^{3/2} T_i^{3/2}}\right]^{-1} = 1 \text{ s}$$

The plasma is collisionless (or weakly collisional)

What is the geometry/size of the system?

- The systems are (of course) 3D and bounded
- ightarrow system size \sim 1 mm
- ightarrow ion inertial length \sim 30 μ m
- ightarrow ion Larmor radius \sim 30 μ m

- lacktriangle meso-scale system w. $L/\rho_i \sim L/d_i \sim 10^3$
- ◆ but wave-length spectra reduced to 3 decades...

What are the temporal scales?

- Gyro-frequency : $\Omega_{i^+}^{-1}=$ 0.4 ns.rad $^{-1}$ (0.1 ps.rad $^{-1}$ for electrons...)
- \bullet Plasma frequency : $\omega_{P,i^+}^{-1}=$ 0.1 ps.rad $^{-1}$ (1 fs.rad $^{-1}$ for electrons !)
- ightarrow hence $\omega_{P,i^+}/\Omega_{i^+}\sim 10^3$ (like in solar wind, earth magnetosphere)
- Alfvén time $\tau_A = 1.6$ ns
- ightarrow w. Alfvén velocity \sim 600 km.s $^{-1}$ on $L\sim$ 1 mm

- May allow most ion cyclotron effects
- → but not the development of a turbulent cascad

Do we have kinetic effects?

- The plasma is weakly collisional
- → not at thermal equilibrium
- → distribution functions can departe from Maxwellian
- Shocks, current sheets, plasma instabilities
- → wave-paticle interactions (Landau & cyclotron)
- → finite Larmor radius effects
- → can eventually break the Alfven theorem

Kinetic effects can be important, at least for ions

Do we have quantum effects?

How Landau length compare to de Broglie length :

$$\lambda_L = \frac{Z_i}{12\pi n_e \lambda_{De}} = 10^{-11} \mathrm{m}$$

$$\lambda_B = \frac{\hbar}{\sqrt{m_e k_B T_e}} = 2 \times 10^{-11} \text{m}$$

- Quantum effects are not at play...
- \rightarrow essentially because the plasma is collisionless!

Are these regimes radiative or not?

• Ratio between plasma kinetic energy and radiative energy :

$$\frac{e}{E_r} = \frac{P/(\gamma - 1)}{(4\sigma/c)T^4} \ll 1$$
 : Mihalas number

• Ratio between plasma kinetic flux and radiative flux :

$$\frac{\phi_e}{F_r} = \frac{ve}{\sigma T^4} ve = \frac{v}{c} \frac{e}{E_r} \sim 1$$
 : Boltzmann number

- radiative effects can oftenly be omitted
- \rightarrow but the plasma is not a black body and eventually optically thick

The EM part (curl) of a plasma code

- Maxwell-Faraday : $\partial_t \mathbf{B} = -\nabla \times \mathbf{E}$
- → this equation is always solved except for ES codes
- \rightarrow eventually in a different form like $\partial_t \mathbf{B} + \nabla \cdot (\mathbf{U}\mathbf{B} \mathbf{B}\mathbf{U}) = 0$

f A Might eventually need a correction to ensure a divergence-free f B field

- Maxwell-Ampère : $\partial_t \mathbf{E} = c^2 \nabla \times \mathbf{B} \varepsilon_0^{-1} \mathbf{J}$
- \rightarrow for modes with $\omega/k \ll c$: Darwin approximation
- → neglect the transverse component of the displacement current

▲ The longitudinal component of the displacement current is still here... hopefully!

The EM part (divergence) of a plasma code

Both of these equations appear as "initial conditions":

- Maxwell-Thomson : $\nabla . \mathbf{B} = 0$
- → such error can increase in time
- \rightarrow wrong topology of the B-field lines \Rightarrow orthogonal plasma transport
- \rightarrow "constrained transport methods" : special discretization of the B field
- \rightarrow "Hodge projection" in Fourier space (∇ domain decomposition)
- Maxwell-Gauss : $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$
- \rightarrow with a Boris correction $\mathbf{E}^{\star} = \mathbf{E} + \nabla \phi$ so that $\nabla \cdot \mathbf{E}^{\star} = \rho/\varepsilon_0$
- \rightarrow solve Poisson $-\Delta \phi = \nabla \cdot \mathbf{E} \rho/\varepsilon_0$ (Marder method w. diffusion eq.)
- → "structure preserving discretization" (Esirkepov method) charge and current densities not deposited in the same way

Fluid codes: Maxwellian plasma?

• Hypothesis and approximations :

$$\partial_t n + \nabla \cdot (n\mathbf{V}) = 0 \tag{1}$$

$$nm(\partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V}) = nq(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - \nabla \cdot \mathbf{P} + \eta \mathbf{J}$$
 (2)

- One needs a closure equation
- \rightarrow generally on a scalar pressure, isothermal (isotropic) or adiabatic
- ullet Then the species are only described by the first 3 moments, n, V & P
- lacklose It does not mean that the plasma is maxwellian (that is eventually collisional), but it means that higher moments (\mathbf{Q} , \mathbf{R} ...) are not needed to describe the physical evolution of the system

Fluid codes: Darwin approximation

- Any charge separation would need to solve a Maxwell-Gauss equation
- \rightarrow hence, a local density defined on a whole cell is not enough
- No way to solve any charge separations with a fluid formalism

- We then always have the Darwin approximation for fluid formalisms
- → but some electrostatic modes still exist... like eg Ion Acoustic Waves
- \rightarrow the compressional character of the plasma is still handled!

Fluid codes then always need an Ohm's law

Fluid codes: Ohm's law

ullet While ${f E}$ field is needed in Maxwell-Faraday, one needs an Ohm's law ullet it has to be the one associated to the massless electrons

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{1}{en} (\mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{P}_e) - \frac{m}{e} d_t \mathbf{V} + \frac{m}{e} d_t \left(\frac{\mathbf{J}}{ne} \right) + \eta \mathbf{J} - \eta' \Delta \mathbf{J}$$

- The electron fluid velocity results from the ion velocity and total current
- The pressure is the electron one... hence needing a closure equation
- Dissipative terms can be physical and/or numerical (for stability)

Fluid codes: general features

- The MHD fluid can be relativistics
- The ηJ term is associated to e^-/p^+ collisions
- p^+/p^+ collisions conserve momentum and energy for ellastic collisions \rightarrow nothing to add in the fluid equations...
- The dissipative term can be anomalous
- → "contain" various physical process like EM fluctuations...
- No characteristic length, only the Alfvén velocity
- → this can complicate the comparison between codes

Fluid codes: bi-fluid and Hall MHD

- Bi-fluid codes have then to be "hybrid"
- \rightarrow needs a closure equation for the "second" fluid (that is the e^-)
- Ø bi-fluid codes w. different closure for the 2 fluids (& current density)
- Another approach is the Hall-MHD
- \rightarrow the single MHD fluid can then let some current develop
- \rightarrow this current results from the slip between p^+ and e^-
- \rightarrow the Hall-MHD equations are then parabolic (and not anymore hyerbolic)
- electron MHD (e^- inertia-less, see Kingsep et al., 1987)
- → total current given by electron velocity in non-moving background ions

Fluid codes: single fluid (MHD)

- MHD is a single fluid: summation over all species.
- ightarrow then, the ${f E}$ field disappears from momentum equation by quasi-neutrality

f A The closure is for this single fluid... no way to discriminate $p^+ \& e^-$

- ullet Ideal MHD for $\mathbf{E} = -\mathbf{V} imes \mathbf{B}$
- \rightarrow the plasma is collisionless
- ightarrow low k and ω values, that is $kL \ll 1$ and $\omega au \ll 1$
- \rightarrow Alfvén theorem : plasma and B field are frozen together

Fluid codes: algorithms

- MHD codes are oftenly written in a conservative form
- → use of finite volume methods on a structured grid
- \rightarrow Godounov formulations (*Godunov*, 1959) are then very popular
- → TVD schemes (with flux limiters) for discontinuities
- \rightarrow for such explicit schemes, one needs to satisfly the CFL conditions
- With strong gradients ⇒ that is converging characeristics
- → eventual use of Lagrangian approach on an unstructured moving mesh
- → closure equations might not be local

Fluid codes: brief overview of few (french) MHD cdes

- RAMSES: Romain Teyssier
- \rightarrow cartesian AMR grid, include self-gravity and cooling (eventually particles)
- Heracles: Edouard Audit
- \rightarrow cartesian/cylindrical/spherical fixed grid for hydro, MHD, hydro-rad, self-gravitating flows
- Gorgon : Andrea Ciardi
- → cartesian MHD including vaccuum
- FCI2: Alain Grisollet
- → hydro-rad with laser energy deposition using a Lagrangian approach

Vlasov codes: governing plasma equation

- The Vlasov equation can be written for any Hamiltonian system
- \rightarrow start with the Boltzmann equation & neglect the collision operator
- Each species are then described by a Vlasov equation

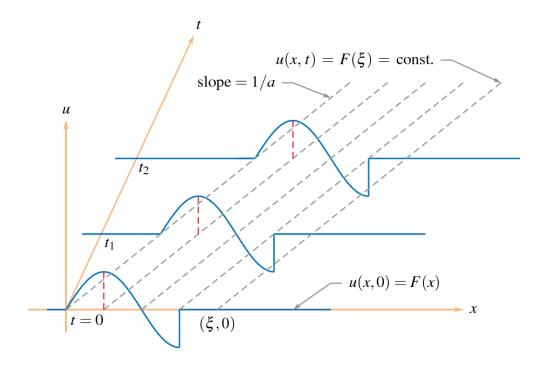
$$\partial_t f_s + \mathbf{v} \cdot \partial_{\mathbf{r}} f + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) \cdot \partial_{\mathbf{v}} f_s = 0$$

- The distribution function should then be noiseless.
- \rightarrow the tail of the distribution is described as well as the core
- The plasma-ElectroMagnetic field system can be closed in 2 ways
- → Vlasov-Poisson for electrostatic problems
- → Vlasov-Maxwell for electromagnetic problems

Vlasov codes: conservative form

• The Vlasov equation is fundamentally a hyperbolic conservative equation

$$\to \partial_t u + a \nabla u = 0$$



ightarrow Finite volume techniques should then be well suited

Vlasov codes: use of characteristics?

- Fillamentation where characteristics are converging in phase space
- \rightarrow needs some numerical cautions : Euler solvers can then be challenged
- Lagrangian approach
- ightarrow a sample of the distribution function is followed in the same way as a macro-particle
- semi-Lagrangian approach
- ightarrow a new sampling on a uniform Eulerian grid is performed
- The Liouville's theorem imply mass, momentum, energy... conservation

Vlasov codes: numerical cost

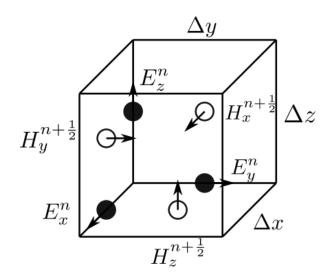
- $f_s(\mathbf{r}, \mathbf{v}, t)$ depends on 7 unknowns that have to be discretized
- Simulation (Eulerian) of the Earth magnetosphere: 10¹⁸ cells...
- \rightarrow 8 ExaBytes of RAM memory! out of reach, even in far future...
- \rightarrow now, we can go up to 10^{12} sample points

PIC: [Daughton et al., 2011], Vlasov: [Palmroth et al., 2017]

- Different solutions are possible :
- \rightarrow use a sparse grid representation
- ightarrow drop the azimuthal velocity dimension (gyrokinetic approach) for gyrotrop distributions
- → reduce the grid resolution in less important areas (of phase space)
- \rightarrow pruning the phase-space : remove grid elements in low density regions
- \rightarrow adapt the coordinate system like v_{\parallel} , v_{\perp} , v_{ϕ} & lower resolution of v_{ϕ}

Vlasov codes: FDTD algorithms

- The Finite-Difference Time Domain (FDTD) approach has became a standard in plasma EM simulation
- \rightarrow Maxwell equations staggered in space with Yee lattice [Yee, 1966]
- \rightarrow the scheme is time centered with a leap-frog technique [Verlet, 1967]



 The Vlasov equation is Lorentz-invariant, provided the velocity is correctly handled Vlasov codes: brief overview of few (non-french) Vlasov

- Vlasiator : Mina Palmroth
- ightarrow collisionless hybrid-Vlasov code essentially for magnetosphere & space weather
- Impacta: Robert Kingham
- \rightarrow 2D, implicit, with a linearized version

PIC codes: macro-particles

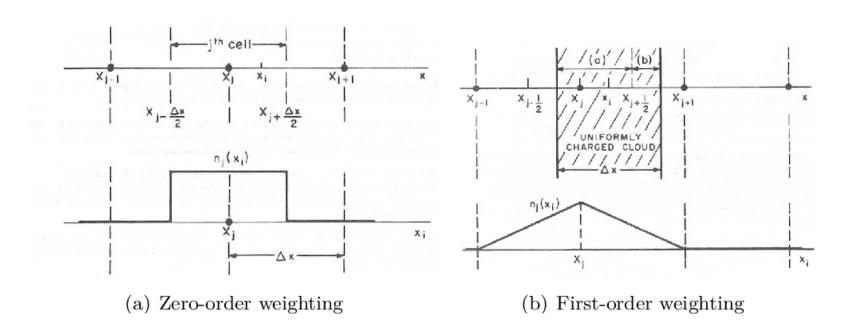
- Instead of using a distribution function, one can use a piece of it
- → a macro-particle is a "sample" of the distribution function
- The equations of motion of a macro-particle are simply :

$$d_t \mathbf{r} = \mathbf{v}$$
 , $d_t \mathbf{v} = q/m(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

- \rightarrow characteristics of the Vasov equation for Liouville's theorem
- The Klimontovitch equation on $f^K(\mathbf{r}, \mathbf{v}, t) = \sum_{i=1}^N \delta(\mathbf{r} \mathbf{r}_i(t)) \delta(\mathbf{v} \mathbf{v}_i(t))$ can not be handled as is, because of the δ 's
- \rightarrow as usually done in statistical physics, $f(\mathbf{r}, \mathbf{v}, t) = \langle f^K(\mathbf{r}, \mathbf{v}, t) \rangle_{\text{ensemble}}$
- ightarrow but how to deposit the δ values on the grid ?
- ightarrow the magnetic & (Debye shielded) electric field are calculated on a grid

PIC codes: shape factor (assignment function)

• Assignment function is generally a b-spline function of order 1, 2 or 3



 \rightarrow support of an assignment function has to be larger than the grid size

PIC codes: shape factor (assignment function)

- The shape factor is the local value of the assignment function
- \rightarrow hence the charge density and momentum,

$$\rho(\mathbf{r}) = q \sum_{i=1}^{N} w_i S(\mathbf{r} - \mathbf{r}_i)$$
 , $\rho(\mathbf{r}) \mathbf{U}(\mathbf{r}) = \sum_{i=1}^{N} w_i \mathbf{v}_i S(\mathbf{r} - \mathbf{r}_i)$

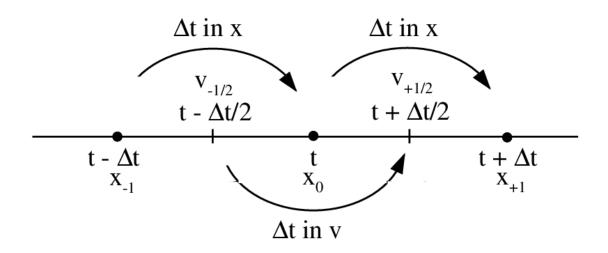
- Physical and numerical involvements
- \rightarrow The larger the b-spline order, the better the code stability
- \rightarrow The narowwer the b-spline, the deeper the density gradients
- ullet Large dispariries in the w_i 's values should allow to investigate large density gradients

PIC codes: particle management

- ullet In a given cell, the number of particles N_G is oftenly of the order of 100
- \rightarrow not enough to properly describe the tails of the distribution function
- → need to consider ghost cells to correctly build the fluid moments
- → this can strongly burden the code efficiency
- For 3D system, the # of macro-particle is dramatically RAM consuming
- \rightarrow 80% of CPU time to push [Boris, 1973] & deposit particles on the grid
- Coulomb collisions with other particles using Monte-Carlo methods [Takizuka & Abe, 1984]
- \rightarrow then consider that total CPU time is multiplied by a factor 2
- For statistical reasons, need to split or merge particles when needed
- \rightarrow some techniques exist, but can be specific [Smets et al., 2021] or time consuming [Gosnokov, 2021]

PIC codes: algorithms

- FDTD (Finite Difference Time Domain) methods are widely used
- → method of second order in both space and time
- \rightarrow space centering oftenly resulting from the use of a [Yee, 1966] lattice
- → time centering results from the use of leap-frog schemes [Verlet, 1967]



PIC codes: full-PIC & hybrid-PIC

- Full-PIC codes manage both p^+ and e^- as paticles
- → resolve the charge separation with Maxwell-Gauss equation
- \rightarrow generally dedicated to problems where the e^- play a central role
- \rightarrow typical scales (& normalization) are then ω_{Pe}^{-1} , λ_{De} , and need c/V_A
- → can easily be relativistic at a modest cost
- ightarrow one generally cheat with mass ratio m_p/m_e and/or c/v_A
- → well-fitted to study pair-plasmas
- ullet Hybrid-PIC codes manage p^+ as particles and e^- as a massless fluid
- \rightarrow hence, this approximation means $m_e/m_p \rightarrow 0$ (opposite to full-PIC)
- \rightarrow one then needs an Ohm's law and a closure equation for the electrons
- ightarrow typical scales (& normalization) are then Ω_p^{-1} , l_p , and $c/V_A
 ightarrow\infty$
- \rightarrow none of electron scales can be handled... unless some efforts [Sladkov et al., 2021]

PIC codes: brief overview of (french) PIC codes

- Smilei: M. Greck
- \rightarrow field ionization, binary collisions and impact ionization, QED processes, such as high-energy photon emission and its back-reaction on the electron dynamics, as well as pair production through the Breit-Wheeler process
- Zeltron : B. Cerutti
- ightarrow general relativity & radiation reaction force including synchrotron and inverse Compton back-reaction force
- PHARE: N. Aunai
- \rightarrow Hybrid-PIC, still in development, using AMR (SAMRAI) and many more nice upcoming features !

Radiative part of a (fluid) plasma code

- ullet The radiative transfer equation should be solved for the specific intensity $I({f r},t,{f n},
 u)$
- → This equation contain absorption, emmisivity and diffusion coefficients
- ullet One can wether deal with the moment of order 0, 1 & 2 of this equation \to 3 equations on the radiative energy density, radiative energy flux & radiative pressure tensor

[Mihalas & Mihalas, 1984]

- As for plasma fluids, the system needs to be closed
- \rightarrow M1 approximation [Levermore, 1984] : $\mathbb{P}_0 = \frac{1}{2}E_0[(1-f)\mathbb{I} + (3-f)\mathbf{n}_0\mathbf{n}_0]$
- \rightarrow Flux-Limited-Diffusion [Alme & Wilson, 1973] : $\mathbf{F}_0 = -K \nabla E_0$
- $oldsymbol{ol}oldsymbol{oldsymbol{oldsymbol{ol{ol}}}}}}}}}}}}}}}}}}}}}}}}$

Units and free parameters

- "Modern" codes are always unit-less (?)
- → but scales discrepencies of physical origin are unavoidable
- → No spatial characteristic length scale in MHD
- \rightarrow length scales can be the inertial length or the Debye length $(p^+ \text{ or } e^-)$
- ullet Vaschy-Buckingham theorem : N variables & 4 units
- \rightarrow need N-4 dimensionless parameters for characterizing a simulation
- MKS system was adopted in commerce and ingeneering in 1889, extended to MKSA in 1901 and published in 1960
- \rightarrow consider that CGS-Gaussian is not the only possibility to include electromagnetism in CGS
- \rightarrow in CGS, pressure in Barye, viscosity in Poise and wavelength in Kayser, electric charge in Franklin...

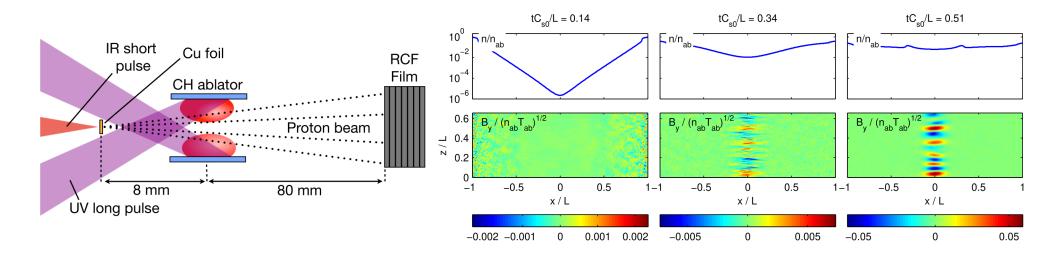
Engineering of plasmas codes

- domain decomposition
- run on HPC and exascale ready... (?)
- open sources or at least open to the community
- using a versioning system as GIT
- including unitary, functional & non-regressive tests
- generally working together with continuous integration
- written in a "long living" language, dedicated to 21st century challenges
- with "clean code" practices & design-patterns
- efficient for present (CPU) and future (GPU ?) architectures

Weibel instability

- Mediated by temperature anisotropy or multiple counterstreaming beams
- → ablation density feeded at open boundary conditions
- → characteristic wavelength of the order of ion inertial length

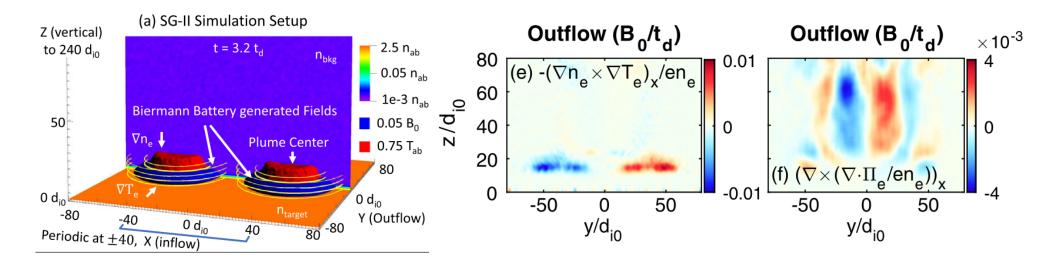
[Fox et al., 2013]



Biermann-battery effects

- Growth of seedless (clockwise) magnetic field, $\partial_t \mathbf{B} = -(en_e)^{-1} \nabla n_e \times \nabla T_e$ \rightarrow can generate magnetic field up to 100th of Teslas in HEDP
- ullet Density and temperature gradient (for e^-) feeded by ad-hoc operator
- → modify the compression of the CS, and the reconnected flux pattern

[Matteucci et al., 2018]

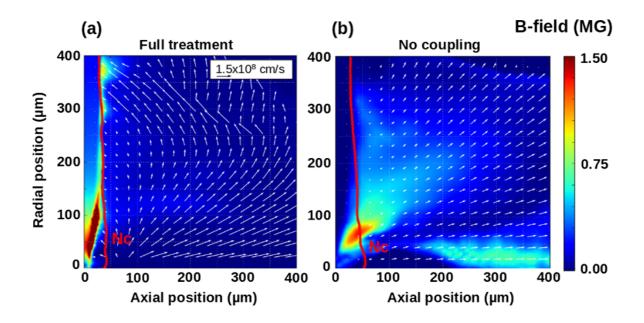


▼ The Biermann-battery effect contributes to the reconnected flux generation

Nernst & Righi-Leduc effects

- ullet For e^- heat transport, Spitzer-Härm breaks down : $\lambda_{\mathsf{mfp}} > T_e/\nabla T_e$
- → non-classical heat flow modifies the Nernst advection effect
- ightarrow B-field advection & compression in denser region : $\mathbf{q}_e = -\kappa \mathbf{B} imes \mathbf{\nabla} T_e$
- → Nernst (radial) advection underestimated by [Braginskii, 1965]
- \rightarrow q_e needs to be non-local so that $U_{Nernst} = (\gamma 1)q_{NL}/p_e$ is realistic

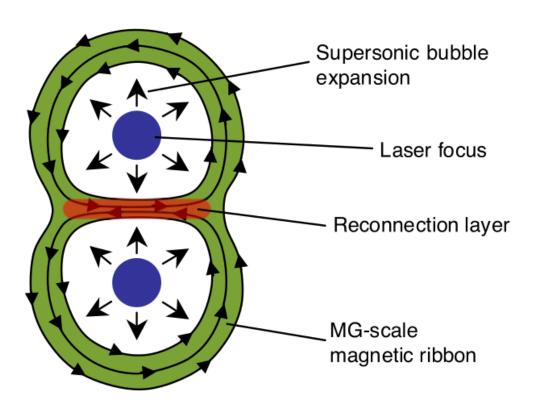
[Lancia et al., 2014]

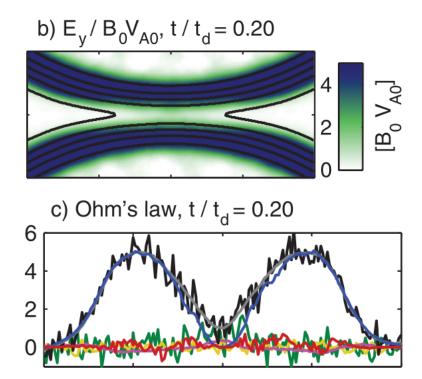


Magnetic reconnection

Two opposing magneic bubbles squeeze together and then reconnect
 → strong inflow can drive "flux-pileup" reconnection

[Fox et al., 2011]





Nernst effects for reconnection

- Electron Ohm's law results from 1st order moment of Vlasov-FP
- → B-field frozen in the hot collisionless electrons
- \rightarrow it is not associated to current (driven by the cooler electrons)
- → importance of Nernst advection compared to Hall effect in reconnection

[Joglekar et al., 2014] with Vlasov code, [Kingham et al., 2004]

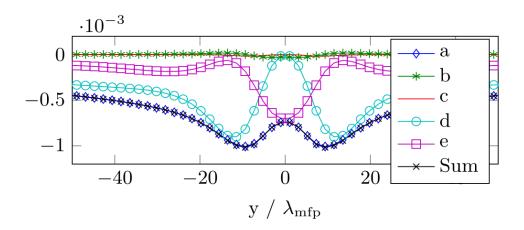
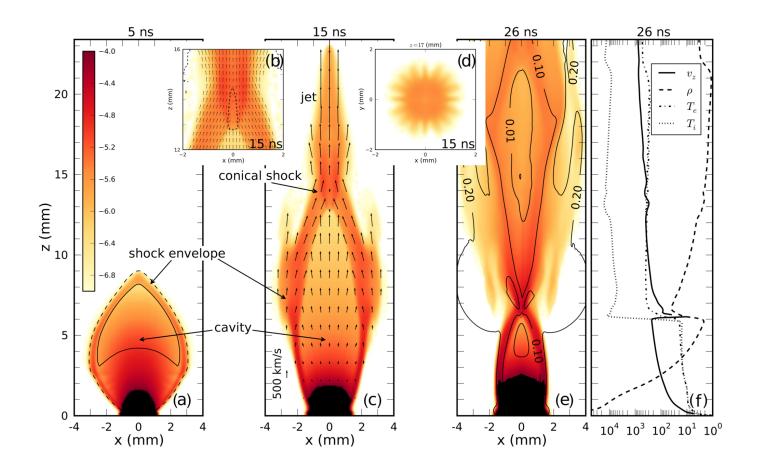


FIG. 2 (color online). Illustration of the contribution of the different components of Ohm's law in Eq. (2) taken from the simulation at a time $t = 11000\tau_n$. (a) E_z calculated from the code, (b) $\bar{\eta}j_z$, (c) $[\mathbf{j} \times \mathbf{B}]_z$, (d) $[\mathbf{v}_T \times \mathbf{B}]_z$, (e) $[(\nabla \cdot \langle \mathbf{v} \mathbf{v} v^3 \rangle)/(2\langle v^3 \rangle)]_z$. (f) Sum of all contributions (b)–(e).

Collimation of plasma jets

- recollimation of wide-angle winds from stars and discs
- \rightarrow differential rotation of the poloidal B field $\Rightarrow B_{\theta}$ collimate the jet

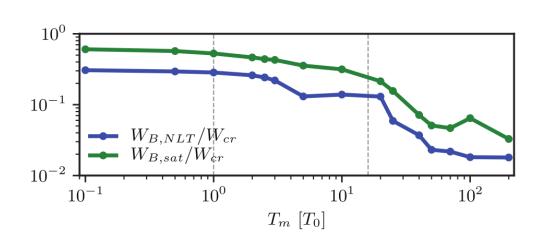
[Ciardi et al., 2013]

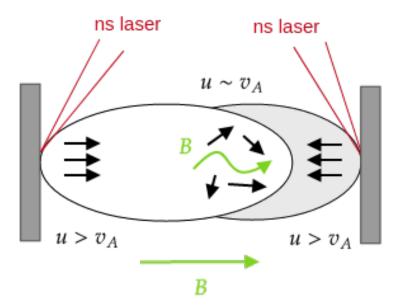


Beam-plasma instability

- [Bell, 2004] instability (NR) can drive large B fluctuations
- \rightarrow decrease of B field saturation level with temperature
- → the growth rate of the instability can increase with collisions

[Marret et al., 2021]





Concluding remarks

- Wide range of laser-based lab. experiments with pros. & cons. :
- These experiments are reproducible
- They are far less expensive than in-situ or remote measurements
- They are faster to achieve (less than a PhD thesis duration)
- They are less risky than space mission with satellites & probes

But...

- Not that easy to rescale with appropriate unitless numbers
- They are quite hard to investigate with dedicated diagnostics
- Need numerical simulations to disentangle the results
- ightarrow The astrophysical community needs to think about the growing opportunities of laser-based laboratory experiments
- \rightarrow keep in mind that 10^6 CPU hours means 4 tons of CO₂!