Preamble

In astrophysical environments, most plasmas are maid of protons and electrons. With few alpha particles in cosmic rays, or few heavy ions in ionospheres or lower layers of the magnetosphere/exosphere, non-hydrogen plasmas are very rare. In this manuscript, I hence only consider hydrogen plasmas. Consequently, the mass ratio denoted by μ is equal to 1836, and the charge of a proton is $+e$.

I note ω_{Ps} the angular plasma frequency[1] of the s specie (depending on the mass of the s specie) and Ω_s the gyrofrequency of the particles of s specie. This angular frequency is algebraic ; it is hence negative for electrons.
To characterize the temperature effects, I also introduce the thermal speed of each s specie

$$v_{Ts} = \left(\frac{k_B T_s}{m_s}\right)^{1/2} \tag{1}$$

as well as the sound speed

$$c_s = \left(\frac{\gamma p}{\rho}\right)^{1/2} \tag{2}$$

where p is the total kinetic pressure (including the contribution of both protons and electrons), ρ the mass density and γ the adiabatic index. As plasmas are always quasi-neutral and considering the large μ value, then $\rho \simeq m_p n$. Morever, the total pressure value depends on the scaling between protons temperature and electrons temperature. Hence, for $T_e \gg T_p$, one has $c_s = v_s = (\gamma k_B T_e/m_p)^{1/2}$, that is the ion acoustic speed. It is also convenient to introduce the β parameter, the ratio between total kinetic pressure and magnetic pressure :

$$\beta = \frac{2}{\gamma}\frac{c_s^2}{v_A^2} \tag{3}$$

where v_A is the Alfvén speed :

$$v_A^2 = \frac{B^2}{\mu_0 n m} \tag{4}$$

To study the polarization of a mode, the static magnetic field (when it exists) is in the z direction. The wave number $\mathbf{k}$ is in the xz plane and makes an angle Θ with the z axis. $\widehat{\mathbf{k}}$ is the unit vector in the $\mathbf{k}$ direction and $\widehat{\mathbf{t}}$ is the unit vector normal to $\widehat{\mathbf{k}}$ also in the xz plane (see Fig. 1).

We recall in the appendix **??** the definition of the relative dielectric tensor $\boldsymbol{\varepsilon}$ which connects the displacement vector $\mathbf{D} = \varepsilon_0 \boldsymbol{\varepsilon}.\mathbf{E}$ to the electric field vector $\mathbf{E}$. We also show that Maxwell's equations in Fourier space provide the eigen mode dispersion relation with $N = kc/\omega$:

[1] one also speak about plasma frequency ... but we must not forget the factor 2 π which exists between the two.

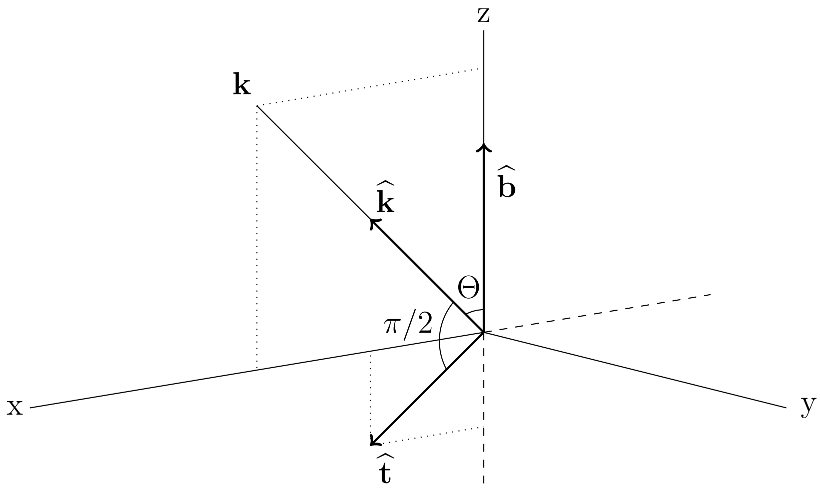

Figure 1: Definition of the vectors $\widehat{\mathbf{b}}$, $\widehat{\mathbf{k}}$ and $\widehat{\mathbf{t}}$ defining the field aligned frame.

$$(\mathbf{NN} - N^2\mathbf{1} + \boldsymbol{\varepsilon}) \,.\, \mathbf{E}(\mathbf{k}, \omega) = 0 \tag{5}$$

The dispersion matrice is the term in front of $\mathbf{E}$. The couples (ω, k) for which the determinant is zero are the ones defining the eigen modes. This determinant is noted $D(\omega, k)$[2].

We introduce the decomposition of ω in its real (ω_r) and imaginary $(\imath\gamma)$ parts. The above determinant can also be split in its real and imaginary components

$$D(\omega_r, \gamma, \mathbf{k}) = D_r(\omega_r, \gamma, \mathbf{k}) + \imath D_i(\omega_r, \gamma, \mathbf{k}) \tag{6}$$

Using a Taylor expansion of $D(\omega_r, \gamma, \mathbf{k})$ around $\gamma = 0$ for the modes weakly damped or amplified, one obtains

$$D(\omega_r, \gamma, \mathbf{k}) = D_r(\omega_r, 0, \mathbf{k}) + \imath\gamma \left.\frac{\partial D_r(\omega_r, \gamma, \mathbf{k})}{\partial \omega}\right|_{\gamma=0} + \imath D_i(\omega_r, 0, \mathbf{k}) = 0 \tag{7}$$

Then the dispersion relation and growth rate[3] of the mode are given by

$$D_r(\omega_r, 0, \mathbf{k}) = 0 \qquad\qquad \gamma(\omega, \mathbf{k}) = -\frac{D_i(\omega_r, 0, \mathbf{k})}{\partial D_r(\omega_r, \gamma, \mathbf{k})/\partial\omega|_{\gamma=0}} \tag{8}$$

[2]the determinant D should not be confused with the electric displacement vector $\mathbf{D}$

[3]it is clear that the determinant $D(\omega_r, \gamma, \mathbf{k})$ should have an imaginary part in order for the associated mode to be amplified or damped (see the associated discussion in appendix A).