# Contents

# — 1 —

# Beam-plasma instabilities

Beam-plasma instability is one of the best prototype of microscopic (or kinetic) instability, *i.e.* which origin depends on the distribution function. In order to feed the electric and possibly magnetic fluctuations for the modes that may exist, a source of free energy is needed. For this type of instability, it is the bulk flow energy of the beam.

## 1.1 Electrostatic modes

The electrostatic modes are the easiest to derive from an analytical point of view, because in addition to the Vlasov equation, the Maxwell-Gauss equation is the only equation needed because of the electrostatic nature of the instability. Thus, for a magnetized plasma and for the $s$ specie,

$$\frac{\partial f_s}{\partial t} + \mathbf{v}.\frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}).\frac{\partial f_s}{\partial \mathbf{v}} = 0 \tag{1.1}$$

The Maxwell-Gauss equation is written

$$\boldsymbol{\nabla}.\mathbf{E} = \frac{1}{\varepsilon_0}\sum_s n_s q_s \tag{1.2}$$

where, moreover, the electrostatic field derives from a scalar potential

$$\mathbf{E} = -\boldsymbol{\nabla}\phi \tag{1.3}$$

By omitting the $s$ index, Eq. (1.1) linearized at first order gives

$$\frac{\partial f_1}{\partial t} + \mathbf{v}.\frac{\partial f_1}{\partial \mathbf{r}} + \frac{q}{m}(\mathbf{v} \times \mathbf{B}_0).\frac{\partial f_1}{\partial \mathbf{v}} + \frac{q}{m}\mathbf{E}_1.\frac{\partial f_0}{\partial \mathbf{v}} = 0 \tag{1.4}$$

The sum of the 3 first terms is equal to $d_t f_1$, provided that we are along the trajectory

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \tag{1.5}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{v} \times \mathbf{B}_0) \tag{1.6}$$

*i.e.* along the unperturbed orbit of a particle in the $\mathbf{B}_0$ field. Along this orbit,

$$\frac{df_1}{dt} = -\frac{q}{m}\mathbf{E}_1.\frac{\partial f_0}{\partial \mathbf{v}} \tag{1.7}$$

We are looking for an electric field of the form $\mathbf{E}_1(\mathbf{r},t) = \mathtt{E}_1 e^{\imath(\mathbf{k}.\mathbf{r}-\omega t)}$. By integrating Eq. (1.7), one gets $f_1$ :

$$f_1 = -\frac{q}{m}\int_{-\infty}^{t} e^{\imath(\mathbf{k}.\mathbf{r}'-\omega t')}\mathtt{E}_1 \cdot \frac{\partial f_0}{\partial \mathbf{v}}\, dt' \tag{1.8}$$

where $\mathbf{r}'(t')$ and $\mathbf{v}'(t')$ are the position and velocity of the particle arriving at $\mathbf{r}$ at the velocity $\mathbf{v}$ at time $t$, along the unperturbed orbit.
By noting $\varphi$ the phase at the origin of the particle at time $t$ and $\tau = t' - t$, $\mathbf{v}'(\tau)$ is written

$$\begin{aligned} v'_x &= -v_\perp \sin(\Omega_c\tau + \varphi) && (1.9)\\ v'_y &= +v_\perp \cos(\Omega_c\tau + \varphi) && (1.10)\\ v'_z &= +v_\parallel && (1.11)\end{aligned}$$

for electrons[1]. One gets $\mathbf{r}'(t')$ by integration

$$\begin{aligned} x' &= x + v_\perp/\Omega_c[\cos(\Omega_c\tau + \varphi) - \cos\varphi] && (1.12)\\ y' &= y + v_\perp/\Omega_c[\sin(\Omega_c\tau + \varphi) - \sin\varphi] && (1.13)\\ z' &= z + v_\parallel\tau && (1.14)\end{aligned}$$

We are looking for a solution of the form

$$f_1(\mathbf{r},\mathbf{v},t) = f_1(\mathbf{k},\mathbf{v},\omega)e^{\imath(\mathbf{k}.\mathbf{r}-\omega t)} \tag{1.15}$$

We only focus on parallel modes for which $\mathbf{k} = k_\parallel \hat{\mathbf{z}}$ (we will write $k$ instead of $k_\parallel$ by simplification). So, $\mathbf{k}.\mathbf{r} = kv_\parallel t$ and, for the unperturbed orbit, the cyclotron motion has no effect.

**Remark 1.** *This work is necessary. For non-electrostatic modes, the wave number $\mathbf{k}$ has a perpendicular component. Its scalar product with $\mathbf{r}'(t')$ then involves the cyclotron motion of the particle. The integral over $\tau$ which results from it is then a little more difficult because it involves Bessel functions. The most classical example is surely that of Bernstein's modes, whether associated to electrons or protons.*

Eq. (1.8) can then be written

$$f_1(k,\mathbf{v},\omega) = -\frac{q}{m}\int_{-\infty}^{0} e^{\imath(kv_\parallel-\omega)\tau}\, d\tau \mathtt{E}_1 \cdot \frac{\partial f_0}{\partial \mathbf{v}} \tag{1.16}$$

Moreover, in Fourier space, we have for an electrostatic mode $\mathtt{E}_1 = \imath\mathbf{k}\phi_1$ polarized in the direction of $\mathbf{B}_0$ *i.e.* along $z$. Then the integral can be written

[1]The calculation indeed shows that the modes we are interested in are electronic. For a mode associated with protons, it would be necessary to change the direction of rotation around the field $\mathbf{B}_0$, that is considering a positive $\Omega_c$.

$$f_1(k,\mathbf{v},\omega) = -\frac{q}{m}\frac{1}{kv_\parallel - \omega}k\phi_1\frac{\partial f_0}{\partial v_\parallel} \tag{1.17}$$

This form of $f_1(k,\mathbf{v},\omega)$ does not involve the static magnetic field $\mathbf{B}_0$. We therefore suspect that the form of this instability is the same as that which can exist in unmagnetized plasma. It can be checked ; in “ field-free ”, the differential equation which governs $f_1$ no longer involves a derivative with respect to the velocity. We can then obtain the form of $f_1$ by simply using a Fourier transform, without any need of the characteristic method. The form of the Vlasov equation is simplified by losing the magnetic term

$$\frac{\partial f_1}{\partial t} + \mathbf{v}.\frac{\partial f_1}{\partial \mathbf{r}} + \frac{q}{m}\mathbf{E}_1.\frac{\partial f_0}{\partial \mathbf{v}} = 0 \tag{1.18}$$

One can get in the Fourier space to obtain $f_1$

$$-\imath(\omega - \mathbf{k}.\mathbf{v})f_1(\mathbf{k},\mathbf{v},\omega) = \frac{q}{m}\phi_1(\mathbf{k},\omega)\imath\mathbf{k}.\frac{\partial f_0}{\partial \mathbf{v}} \tag{1.19}$$

Eq. (1.17) and (1.19) are clearly the same. This is a fairly general result ; the dynamics of a plasma in the parallel direction is quite often the same as it would be in the same unmagnetized system.

By integration over velocity space, we get the first order density fluctuation

$$n_1(k,\omega) = \frac{q}{m}\phi_1(k,\omega)\int_{\mathbb{R}^3}\frac{d\mathbf{v}}{kv_\parallel - \omega}k\frac{\partial f_0}{\partial v_\parallel} \tag{1.20}$$

Writing Eq. (1.2) in the Fourier space, one gets

$$k^2\phi_1 = \frac{1}{\varepsilon_0}\sum_s q_s n_{1s} \tag{1.21}$$

With these two equations, we can write the dispersion relation of the electrostatic modes :

$$1 - \sum_s \frac{q_s^2}{m_s\varepsilon_0 k^2}\int_{\mathbb{R}^3}\frac{d\mathbf{v}}{kv_\parallel - \omega}k\frac{\partial f_0}{\partial v_\parallel} = 0 \tag{1.22}$$

As often, we assume a Maxwell-Boltzmann distribution, with a bulk flow directed along the magnetic field, *i.e.* in the same direction as the wave number $\mathbf{k}$. Being only interested in the electrostatic modes, everything happens in the $z$ direction. As this distribution function is separable, we can write

$$f_0(v_\parallel,\mathbf{v}_\perp) = n_0 F_{0\parallel}(v_\parallel)F_{0\perp}(\mathbf{v}_\perp) \tag{1.23}$$

The normalization conditions is

$$\int_{\mathbb{R}} F_{0\parallel}(v_\parallel)\,dv_\parallel = \int_{\mathbb{R}^2} F_{0\perp}(\mathbf{v}_\perp)\,d\mathbf{v}_\perp = 1 \tag{1.24}$$

In Eq. (1.22), we simplify the integral by calculating it on the two directions of $\mathbf{v}_\perp$. Thus,

$$1 - \sum_s \frac{n_{0s} q_s^2}{m_s \varepsilon_0 k} \int_{\mathbb{R}} \frac{dv_\parallel}{k v_\parallel - \omega} \frac{\partial F_{0\parallel}}{\partial v_\parallel} = 0 \tag{1.25}$$

To continue this computation, one needs to give the form of the zeroth order distribution function in the parallel direction in order to be able to clarify the integral. For a Maxwell-Boltzmann distribution with a bulk-flow velocity $v_0$ in the parallel direction,

$$F_{0\parallel}(v_\parallel) F_{0\perp}(v_\perp) = \left[\frac{\alpha_\parallel}{\pi}\right]^{1/2} e^{-\alpha_\parallel (v_\parallel - v_0)^2} \left[\frac{\alpha_\perp}{\pi}\right] e^{-\alpha_\perp v_\perp^2} \tag{1.26}$$

with

$$\alpha_\parallel = \frac{m}{2 k_B T_\parallel} \ , \ \alpha_\perp = \frac{m}{2 k_B T_\perp} \tag{1.27}$$

One then gets

$$\frac{\partial F_{0\parallel}}{\partial v_\parallel} = -2\alpha_\parallel (v_\parallel - v_0) F_{0\parallel} \tag{1.28}$$

so the dispersion relation writes

$$1 - \sum_s \frac{\omega_{Ps}^2}{k} \int_{\mathbb{R}} \frac{dv_\parallel}{k v_\parallel - \omega} [-2\alpha_\parallel (v_\parallel - v_0) F_{0\parallel}] = 0 \tag{1.29}$$

We note $\mathcal{I}$ the integral of Eq. (1.29). By setting $u^2 = \alpha_\parallel (v_\parallel - v_0)^2$, $\mathcal{I}$ can be rewritten

$$\mathcal{I} = -2\alpha_\parallel \left[\frac{1}{\pi}\right]^{1/2} \int_{\mathbb{R}} \frac{du}{ku + \alpha_\parallel^{1/2}(k v_0 - \omega)} u e^{-u^2} \tag{1.30}$$

Using the notation

$$\zeta = \left[\frac{m}{2 k_B T_\parallel}\right]^{1/2} \left(\frac{\omega}{k} - v_0\right) \tag{1.31}$$

then

$$\mathcal{I} = -\frac{\alpha_\parallel}{k} \left[\frac{1}{\pi}\right]^{1/2} \int_{\mathbb{R}} \frac{2u e^{-u^2} \, du}{u - \zeta} \tag{1.32}$$

This form makes it possible to naturally introduce the Fried & Conte function [2]. This function appears naturally in the dispersion relation of the eigenmodes in a hot plasmas with a Maxwell-Boltzmann distribution. The Fried & Conte function is defined by

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{\mathbb{R}} \frac{e^{-u^2}}{u - \zeta} \, du \tag{1.33}$$

One recognize the Hilbert transform of the Gauss function. Deriving under the sum sign, one gets

[2] That one is also called the plasma dispersion function

$$Z'(\zeta) = \frac{1}{\sqrt{\pi}} \int_{\mathbb{R}} \frac{e^{-u^2}}{(u-\zeta)^2}\, du \tag{1.34}$$

so that an integration by part gives

$$Z'(\zeta) = -\frac{1}{\sqrt{\pi}} \int_{\mathbb{R}} \frac{2ue^{-u^2}}{u-\zeta}\, du \tag{1.35}$$

hence giving the relation

$$Z'(\zeta) = -2[1 + \zeta Z(\zeta)] \tag{1.36}$$

Then,

$$\mathcal{I} = \frac{\alpha_\parallel}{k} Z'(\zeta) \tag{1.37}$$

so that the disperison relation of this mode is

$$1 - \sum_s \frac{\omega_{Ps}^2}{k^2} \alpha_\parallel Z'(\zeta_s) = 0 \tag{1.38}$$

which can be simply written

$$1 + \sum_s \frac{2}{k^2 \lambda_{Ds\parallel}^2} [1 + \zeta_s Z(\zeta_s)] = 0 \tag{1.39}$$

where $\lambda_{Ds\parallel}$ is the Debye length for the $s$ specie along the DC magnetic field, and

$$\zeta_s = \left[\frac{m}{2k_B T_{s\parallel}}\right]^{1/2} \left(\frac{\omega}{k} - v_0\right) \tag{1.40}$$

Introducing the thermel speed $v_{Ts}$ of the $s$ specie, one gets

$$\zeta_s = \frac{1}{\sqrt{2} v_{Ts\parallel}} \left(\frac{\omega}{k} - v_0\right) \tag{1.41}$$

In the next section, we will discuss the consequences of the growth of the Langmuir modes in type-3 solar radio bursts.

## 1.2 Type-3 radio bursts

The sun is a tremendous particle accelerator. “ Solar flares ” are manifested by the appearance of intense flashes near the surface of the sun. These are events in which broad spectrum electromagnetic emissions occur, often accompanied by “ Coronal Mass Ejection ”, *i.e.* violent and massive ejections of energetic particles. The most consensual scenario is one in which magnetic arches having their 2 feet in the chromosphere swell and then open by magnetic reconnection. They thus release the stored magnetic energy in kinetic energy of the particles, which are accelerated and heated. The released energy can reach $10^{25}$ J in order to accelerate up to $10^{36}$ electrons. The interaction of these particles with the chromospheric plasma then produces intense electromagnetic radiation, from the radio domain to $\gamma$ rays. However, they are classified by their emission maximum in the 100-800 pm band, *i.e.* in the X band.

During a solar flare, the energetic electrons that are produced are non-thermal, and have a power law distribution of their speed with a negative slope. By time-of-flight effect, the fastest electrons overtake the slower. Thus, at a finite distance, this population of fast electrons will produce a velocity distribution with a positive slope. Such a distribution is unstable and will produce Langmuir waves by Čerenkov effect. There is therefore a minimum distance necessary for this electron distribution to be unstable. The local electron density makes it possible to calculate the plasma frequency at which the first emissions occur. Then, during the transport of these structures, the regions visited being less and less dense, the plasma frequency at which the emissions occur is increasingly low. We thus observe a drift in the frequency of radio waves measured on the ground over time. An example is given in Fig. 1.1. The paper by [Ginzburg and Zhelezniakov, 1958] is the founding paper for the theory of type-3 radio bursts.

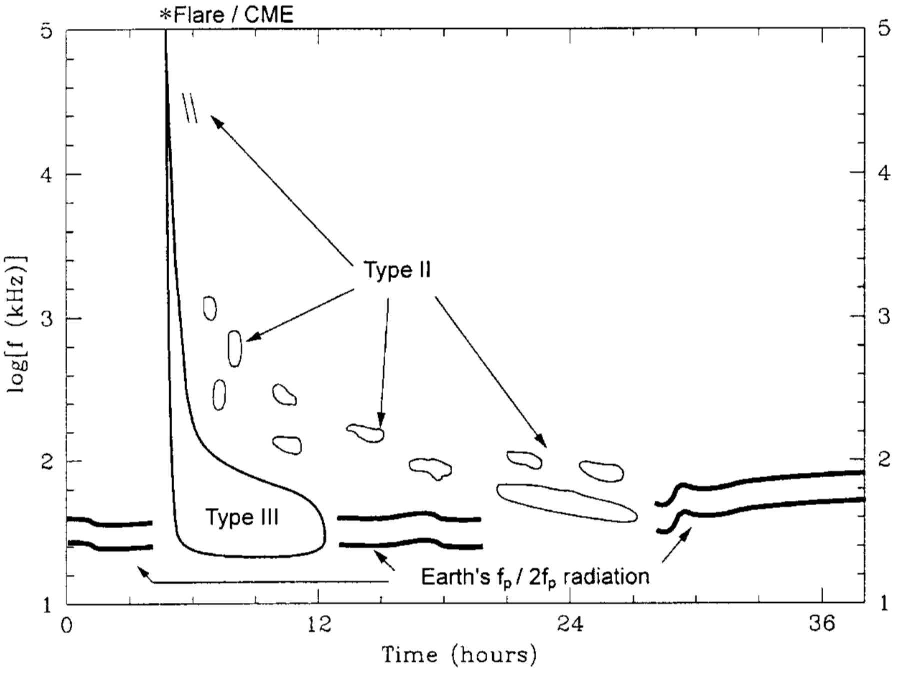


Figure 1.1: Schematic of a dynamical spectra for a type 3 radio burst (see [Cairns and Robinson, 1999])

The essential mechanism foreseen is still relevant today. The beam-plasma electrostatic instability produces Langmuir waves. A conversion mechanism, which remains debated, then ensures the conversion of these electrostatic waves into electromagnetic modes, at the same frequency (fundamental) or at a double frequency (second harmonic). There are different *scenari* for this :

— Langmuir waves scatter protons, thus modifying their distribution function. The mechanism described in the previous section is purely linear, but the amplitude of Langmuir waves will be limited within the framework of the quasi-linear theory, which jointly describes the evolution of particles and wave energy. These scattered protons will in turn be able to induce the emission of electromagnetic waves at the fundamental frequency.

— By a process similar to Brillouin scattering, the Langmuir wave can decrease producing a scattered Langmuir wave as well as an ion acoustic wave. By non-linear coupling, this ion acoustic wave can again couple with the mother Langmuir wave to produce a fundamental electromagnetic wave.

— By the same Brillouin scattering process, the scattered Langmuir wave can also couple with the mother wave to produce the 2$^{\text{nd}}$ harmonic of an electromagnetic wave.

— In the previous mechanism, the scattered Langmuir wave can also result, not from Brillouin scattering, but from the scattering of ions by the mother Langmuir wave. The same coalescence mechanism as previously then also produces the 2$^{\text{nd}}$ harmonic of an electromagnetic wave.

To close this brief presentation of type-3 radio bursts[3], we must also underline the remarkable persistence of these electron beams. Indeed, within the framework of the quasi-linear theory, the diffusion of electrons by Langmuir waves should lead to the disappearance of this beam after the electrons have traveled a few meters ! However, the observations show that these beams can exist on distances of the order of the astronomical unit. Let's quote in particular the numerical work of [Takakura & Shibahashi, 1976], showing that Langmuir waves can be continuously produced at the nose of the beam, and reabsorbed at its end, which has been proposed analytically by [Zhelenyakov & Zaitsev, 1970]. An alternative process would be the existence of local density fluctuations, by which the associated Langmuir frequency could no longer resonate with the electron beam (see [Smith and Sime, 1979]).

## 1.3 Electromagnetic modes

The calculations as presented below are very similar to those published by [Gary and Feldman, 1978]. The Vlasov equation for the specie $s$ is,

$$\frac{\partial f_s}{\partial t} + \mathbf{v}.\frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s}[\mathbf{E} + \mathbf{v} \times \mathbf{B}].\frac{\partial f_s}{\partial \mathbf{v}} = 0 \tag{1.42}$$

Since these are electromagnetic modes, we also need the Maxwell's equations (the “ curl one ”). By forgetting Maxwell-Thomson and Maxwell-Gauss, which are implicit in the other 2 equations, the Maxwell-Ampère equation is

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{1.43}$$

and the Maxwell-Faraday equation is

[3]For a recent review on the physics of type-3 solar bursts, you can read the paper by [Reid and Ratcliffe, 2014].

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.44}$$

**Exercize 1.** *From the zeroth order Vlasov equation, show that a stationary and homogeneous distribution in a magnetic field is independent of the gyrophase $\varphi$.*

By omitting the $s$ index, the first order linearization of Eq. (1.42) gives

$$\frac{\partial f_1}{\partial t} + \mathbf{v}.\frac{\partial f_1}{\partial \mathbf{r}} + \frac{q}{m}\mathbf{E}_1.\frac{\partial f_0}{\partial \mathbf{v}} + \frac{q}{m}(\mathbf{v} \times \mathbf{B}_1).\frac{\partial f_0}{\partial \mathbf{v}} + \frac{q}{m}(\mathbf{v} \times \mathbf{B}_0).\frac{\partial f_1}{\partial \mathbf{v}} = 0 \tag{1.45}$$

The sum of the first 2 terms and the last is equal to $d_t f_1$, provided that we are along the trajectory

$$\frac{d\mathbf{x}}{dt} = \mathbf{v} \tag{1.46}$$

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{v} \times \mathbf{B}_0) \tag{1.47}$$

*i.e.* along the unperturbed orbit of a particle in the $\mathbf{B}_0$ field. Along this orbit, we therefore have

$$\frac{df_1}{dt} + \frac{q}{m}(\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1).\frac{\partial f_0}{\partial \mathbf{v}} = 0 \tag{1.48}$$

You will notice how elegant it is[4] : it allows to transform a partial differential equation into an ordinary differential equation ... the second class of equations being much simpler to solve than the first one.

In Eq. (1.48), we can make $\mathbf{B}_1$ disappear thanks to its relation to $\mathbf{E}_1$ via Eq. (1.44). Its linearization in Fourier space gives $\omega \mathbf{B}_1 = \mathbf{k} \times \mathbf{E}_1$. We can then rewrite the term $\mathbf{v} \times \mathbf{B}_1$ as a function of $\mathbf{E}_1$. By developing the double cross product,

$$\mathbf{v} \times \mathbf{B}_1 = \left(\frac{\mathbf{v}.\,\mathbf{E}_1}{\omega}\right)\mathbf{k} - \left(\frac{\mathbf{k}\,.\mathbf{v}}{\omega}\right)\mathbf{E}_1 \tag{1.49}$$

so Eq. (1.48) can be written

$$\frac{df_1}{dt} + \frac{q}{m}\mathbf{E}_1 \,.\left[\mathbf{1} + \frac{\mathbf{v}\mathbf{k}}{\omega} - \left(\frac{\mathbf{k}\,.\mathbf{v}}{\omega}\right)\mathbf{1}\right].\,\frac{\partial f_0}{\partial \mathbf{v}} f_0 = 0 \tag{1.50}$$

We are still looking for an electric field of the form $\mathbf{E}_1 = \mathrm{E}_1 e^{\imath(\mathbf{k}.\mathbf{r}-\omega t)}$. By integrating Eq. (1.50), we get $f_1$:

$$f_1 = -\frac{q}{m}\int_{-\infty}^{t} e^{\imath(\mathbf{k}.\mathbf{r}'-\omega t')}\mathrm{E}_1 \,.\left[\mathbf{1}\left(1 - \frac{\mathbf{v}'.\,\mathbf{k}}{\omega}\right) + \frac{\mathbf{v}'\mathbf{k}}{\omega}\right].\,\frac{\partial f_0}{\partial \mathbf{v}}\,dt' \tag{1.51}$$

where $\mathbf{r}'(t')$ and $\mathbf{v}'(t')$ are the position and velocity of the particle arriving at $\mathbf{r}$ at the speed $\mathbf{v}$ at time $t$, along an unperturbed orbit.

[4]this method is called “ the method of characteristics ”.

We are only interested in parallel modes. We therefore have $\mathbf{k} = k_{\parallel}\hat{\mathbf{z}}$ which simplifies the calculations. But the essential source of simplification, compared to the classical calculation of the dielectric tensor in hot plasma, is the fact that the unperturbed orbit of the paticle is much simpler. We have $\mathbf{v}'(t')|_z = v_{\parallel}$ [5] and therefore $\mathbf{r}'(t')|_z = v_{\parallel}(t'-t)+z$. We can therefore rewrite the exponential argument in Eq. (1.51) by introducing $\tau = t' - t$

$$e^{\imath(\mathbf{k}.\mathbf{r}'-\omega t')} = e^{\imath(kv_{\parallel}-\omega)\tau}e^{\imath(kz-\omega t)} \tag{1.52}$$

Looking for a solution of the form

$$f_1(\mathbf{r},\mathbf{v},t) = f_1(k,\mathbf{v},\omega)e^{\imath(kz-\omega t)} \tag{1.53}$$

Eq. (1.51) can then be written

$$f_1(k,\mathbf{v},\omega) = -\frac{q}{m}\int_{-\infty}^{0} e^{\imath(kv_{\parallel}-\omega)\tau}\, d\tau \mathtt{E}_1 \cdot \left[\mathbf{1}\left(1-\frac{v_{\parallel}k}{\omega}\right)+\frac{\mathbf{v}'\mathbf{k}}{\omega}\right]\cdot\frac{\partial f_0}{\partial \mathbf{v}} \tag{1.54}$$

Calling $\mathtt{T}$ the double scalar product,

$$\mathtt{T} = \left(1-\frac{v_{\parallel}k}{\omega}\right)\left(\mathtt{E}_1 \cdot \frac{\partial f_0}{\partial \mathbf{v}}\right) + \mathtt{E}_1.\mathbf{v}'\frac{k}{\omega}\frac{\partial f_0}{\partial v_{\parallel}} \tag{1.55}$$

We then make a hypothesis, or rather an important choice : we are only interested in purely electromagnetic modes (*i.e.* no electrostatic contributions, $\mathbf{k}.\mathbf{E}_1 = 0$). The electric field is consequently polarized in the plane normal to the wave vector $\mathbf{k}$, *i.e.* in the $(x, y)$ plan.

The second parenthesis of the first term of Eq. (1.55) can be rewritten, with a cartesian decomposition of the two vectors. For this, we use

$$\begin{aligned} v'_x &= v_{\perp}\cos\varphi && (1.56)\\ v'_y &= v_{\perp}\sin\varphi && (1.57)\end{aligned}$$

where $\varphi(t')$ is a function of $t'$. One then gets the derivatives

$$\begin{aligned} dv_{\perp} &= \cos\varphi\, dv'_x + \sin\varphi\, dv'_y && (1.58)\\ -v_{\perp}\, d\varphi &= \sin\varphi\, dv'_x - \cos\varphi\, dv'_x && (1.59)\end{aligned}$$

so we can write the derivatives of $f_0$ in cartesian coordinates

$$\begin{aligned} \frac{\partial f_0}{\partial v_x} &= \cos\varphi\,\frac{\partial f_0}{\partial v_{\perp}} - \frac{1}{v_{\perp}}\sin\varphi\,\frac{\partial f_0}{\partial \varphi} && (1.60)\\ \frac{\partial f_0}{\partial v_y} &= \sin\varphi\,\frac{\partial f_0}{\partial v_{\perp}} + \frac{1}{v_{\perp}}\cos\varphi\,\frac{\partial f_0}{\partial \varphi} && (1.61)\end{aligned}$$

[5] The value of the parallel speed, in field-free, is constant.

from which we deduce (the last term of these two equations being null)

$$\mathtt{E}_1 \,. \frac{\partial f_0}{\partial \mathbf{v}} = \frac{\mathtt{E}_1 \,.\mathbf{v}'}{v_\perp} \frac{\partial f_0}{\partial v_\perp} \tag{1.62}$$

We can then rewrite Eq. (1.54) in the form

$$f_1(k, \mathbf{v}, \omega) = -\frac{q}{m\omega} \int_{-\infty}^{0} e^{\imath(kv_\parallel - \omega)\tau} \, d\tau (\mathtt{E}_1 \,.\mathbf{v}') \left[ k \frac{\partial f_0}{\partial v_\parallel} + \frac{\omega - kv_\parallel}{v_\perp} \frac{\partial f_0}{\partial v_\perp} \right] \tag{1.63}$$

To carry the integral, it is necessary to clarify the shape of the orbit. By using the gyrofrequency, we can rewrite Eq. (1.46) and (1.47)

$$\frac{d\mathbf{x}'}{dt'} = \mathbf{v}' \tag{1.64}$$

$$\frac{d\mathbf{v}'}{dt'} = \Omega(\mathbf{v}' \times \widehat{\mathbf{z}}) \tag{1.65}$$

The resolution is direct in the complex plane by choosing the new unit vectors in the perpendicular plane

$$\widehat{\mathbf{e}}_\pm = \frac{1}{\sqrt{2}}(\widehat{\mathbf{x}} \pm \imath \widehat{\mathbf{y}}) \tag{1.66}$$

We then define the two velocity vectors

$$v'_\pm = \frac{1}{\sqrt{2}}(v'_x \pm \imath v'_y) \tag{1.67}$$

which allows to express $\mathbf{v}'_\perp(\tau) = v'_+ \widehat{\mathbf{e}}_- + v'_- \widehat{\mathbf{e}}_+$. The solution of the unperturbed orbit is then

$$v'_\parallel(\tau) = v_\parallel \tag{1.68}$$

$$v'_+(\tau) = v_+ e^{-\imath\Omega\tau} \tag{1.69}$$

$$v'_-(\tau) = v_- e^{+\imath\Omega\tau} \tag{1.70}$$

**Remark 2.** *Note that the vectors of the new basis being complex, they have an unusual scalar product, i.e.* $\hat{\mathbf{e}}_+ . \,\hat{\mathbf{e}}_- = 1$ *and* $\hat{\mathbf{e}}_+ . \,\hat{\mathbf{e}}_+ = \hat{\mathbf{e}}_- . \,\hat{\mathbf{e}}_- = 0$.

One then deduce

$$\mathtt{E}_1 \,.\mathbf{v}' = E_- v_+ e^{-\imath\Omega\tau} + E_+ v_- e^{+\imath\Omega\tau} \tag{1.71}$$

We can then calculate the integral in Eq. (1.63)

$$\int_{-\infty}^{0} e^{\imath(kv_\parallel - \omega)\tau} [E_- v_+ e^{-\imath\Omega\tau} + E_+ v_- e^{+\imath\Omega\tau}] \, d\tau = \frac{E_- v_+}{\imath(kv_\parallel - \omega - \Omega)} + \frac{E_+ v_-}{\imath(kv_\parallel - \omega + \Omega)} \tag{1.72}$$

so we can write

$$f_1(k,\mathbf{v},\omega)=\frac{\imath q}{m\omega}\left[k\frac{\partial f_0}{\partial v_\parallel}+\frac{\omega-kv_\parallel}{v_\perp}\frac{\partial f_0}{\partial v_\perp}\right]\left(\frac{E_-v_+}{kv_\parallel-\omega-\Omega}+\frac{E_+v_-}{kv_\parallel-\omega+\Omega}\right)\tag{1.73}$$

The form of the distribution function linearized at first order makes it possible to deduce the form of the conduction current by integration. This is the source term that naturally appears in the Maxwell-Ampère equation. As a reminder,

$$\mathbf{J}_1=q\int_{\mathbb{R}^3}f_1\mathbf{v}\,d\mathbf{v}\tag{1.74}$$

For the sake of simplification, we introduce the new variables

$$\begin{aligned}\mathtt{A}&=\frac{\imath q^2}{m\omega}&(1.75)\\ \mathtt{B}&=k\frac{\partial f_0}{\partial v_\parallel}+\frac{\omega-kv_\parallel}{v_\perp}\frac{\partial f_0}{\partial v_\perp}&(1.76)\\ \mathtt{C}&=kv_\parallel-\omega-\Omega&(1.77)\\ \mathtt{D}&=kv_\parallel-\omega+\Omega&(1.78)\end{aligned}$$

so we can write

$$\mathbf{J}_1=\mathtt{A}\int_{\mathbb{R}}\mathtt{B}\left[\frac{v_+^2E_-}{\mathtt{C}}\hat{\mathbf{e}}_-+\frac{v_+v_-E_+}{\mathtt{D}}\hat{\mathbf{e}}_-+\frac{v_+v_-E_-}{\mathtt{C}}\hat{\mathbf{e}}_++\frac{v_-^2E_+}{\mathtt{D}}\hat{\mathbf{e}}_+\right]\,d\mathbf{v}\tag{1.79}$$

In addition, the combination of the Maxwell-Ampère and Maxwell-Faraday equations, linearized in Fourier space gives

$$\left(1-\frac{k^2c^2}{\omega^2}\right)\mathtt{E}_1=-\frac{\imath}{\omega\varepsilon_0}\mathbf{J}_1\tag{1.80}$$

Using the decomposition $\mathtt{E}_1=E_+\hat{\mathbf{e}}_-+E_-\hat{\mathbf{e}}_+$ and then projecting on the two vectors $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$, we get the system

$$\begin{aligned}\left(1-\frac{k^2c^2}{\omega^2}\right)E_+&=\mathtt{A}\int_{\mathbb{R}}\mathtt{B}\left[\frac{v_+^2}{\mathtt{C}}E_-+\frac{v_+v_-}{\mathtt{D}}E_+\right]\,d\mathbf{v}&(1.81)\\ \left(1-\frac{k^2c^2}{\omega^2}\right)E_-&=\mathtt{A}\int_{\mathbb{R}}\mathtt{B}\left[\frac{v_+v_-}{\mathtt{C}}E_-+\frac{v_-^2}{\mathtt{D}}E_+\right]\,d\mathbf{v}&(1.82)\end{aligned}$$

At this stage, we will consider that the equilibrium distribution function is a Maxwellian. Without this assumption, there is no *a priori* reason to be able to decouple the left and right modes. We therefore set $f_0=n_0F_0$ with

$$F_0(v_\parallel,v_\perp)=\left[\frac{\alpha_\parallel}{\pi}\right]^{1/2}\left[\frac{\alpha_\perp}{\pi}\right]e^{-\alpha_\parallel(v_\parallel-v_0)^2}e^{-\alpha_\perp v_\perp^2}\tag{1.83}$$

and

$$\alpha_{\parallel} = \frac{m}{2k_B T_{\parallel}} \ , \ \alpha_{\perp} = \frac{m}{2k_B T_{\perp}} \tag{1.84}$$

so we have

$$\frac{\partial F_0}{\partial v_{\perp}} = -2\alpha_{\perp} v_{\perp} F_0 \tag{1.85}$$

and the term `B` becomes independent of $v_{\perp}$. In Eq. (1.81), we will show that the integral of the first term of the right hand side is zero. To do this, we decompose $2v_+^2 = v_x^2 - v_y^2 + 2\imath v_x v_y$. As $F_0$ is gyrotropic in the perpendicular plane $(v_x, v_y)$, we have

$$\int_{\mathbb{R}} F_0 v_x^2 dv_x = \int_{\mathbb{R}} F_0 v_y^2 dv_y \tag{1.86}$$

Moreover, the cross term $2\imath v_x v_y$ is an odd power of $v_x$ and $v_y$. $F_0$ being even, its integral on a symmetric support is zero. Thus, all the first term of the right hand side of Eq. (1.81) has a vanishing integral. Symmetrically, it is the same in Eq. (1.82). The two polarizations in $E_+$ and $E_-$ are therefore decoupled.

For the calculation of the remaining integral, with the equality $2v_+v_- = v_{\perp}^2$, we deduce the dispersion relation for the two modes

$$1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{\omega_{Ps}^2}{2\omega^2} \int_{\mathbb{R}} d\mathbf{v}\, v_{\perp} \frac{1}{\omega - kv_{\parallel} \pm \Omega_s} \left[ kv_{\perp} \frac{\partial F_{s0}}{\partial v_{\parallel}} + (\omega - kv_{\parallel}) \frac{\partial F_{s0}}{\partial v_{\perp}} \right] = 0 \tag{1.87}$$

The + sign in the denominator is associated with the $E_+$ mode (right mode) and the - sign with the $E_-$ mode (left mode). The difficulty now lies in calculating the integral $\mathcal{I}$ defined by

$$\mathcal{I} = \int_{\mathbb{R}} d\mathbf{v}\, v_{\perp} \frac{1}{\omega - kv_{\parallel} \pm \Omega} \left[ kv_{\perp} \frac{\partial F_{s0}}{\partial v_{\parallel}} + (\omega - kv_{\parallel}) \frac{\partial F_{s0}}{\partial v_{\perp}} \right] \tag{1.88}$$

With a Maxwell distribution (Eq. 1.83), we have, as for Eq. (1.85),

$$\frac{\partial F_0}{\partial v_{\parallel}} = -2\alpha_{\parallel}(v_{\parallel} - v_0) F_0 \tag{1.89}$$

By rearranging the terms, we thus have

$$kv_{\perp} \frac{\partial F_0}{\partial v_{\parallel}} + (\omega - kv_{\parallel}) \frac{\partial F_0}{\partial v_{\perp}} = 2v_{\perp} F_0 [kv_{\parallel}(\alpha_{\perp} - \alpha_{\parallel}) + k\alpha_{\parallel} v_0 - \omega\alpha_{\perp}] \tag{1.90}$$

Using this form in the expression of $\mathcal{I}$, it appears that this integral depends on $v_{\perp}$ only through $F_0$. By not omitting the term in $v_{\perp}^2$, we calculate the integral

$$\int_{\mathbb{R}} d\mathbf{v}_{\perp} v_{\perp}^2 F_0 = 2\pi \int_0^{\infty} v_{\perp}^3 F_0 \, dv_{\perp} \tag{1.91}$$

This calculation then needs a classical result that we remind here:

$$I_0 = \int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} \quad , \quad I_1 = \int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2\alpha} \quad , \quad I_n = \int_0^\infty x^n e^{-\alpha x^2} dx \tag{1.92}$$

as well as the recurrence relation

$$I_n = \frac{n-1}{2\alpha} I_{n-2} \tag{1.93}$$

These expressions make it possible in particular to calculate the shape of the moments of order $n$ of a Maxwell-Boltzmann distribution. We can deduce

$$\int_{\mathbb{R}^2} d\mathbf{v}_\perp v_\perp^2 F_0 = \frac{1}{\alpha_\perp} \left[\frac{\alpha_\parallel}{\pi}\right]^{1/2} e^{-\alpha_\parallel (v_\parallel - v_0)^2} \tag{1.94}$$

We can then write the form of the integral $\mathcal{I}$,

$$\mathcal{I} = \frac{2}{\alpha_\perp} \left[\frac{\alpha_\parallel}{\pi}\right]^{1/2} \mathcal{J} \tag{1.95}$$

with

$$\mathcal{J} = \int_{\mathbb{R}} dv_\parallel \frac{1}{\omega - k v_\parallel \pm \Omega} [k v_\parallel (\alpha_\perp - \alpha_\parallel) + k \alpha_\parallel v_0 - \omega \alpha_\perp] e^{-\alpha_\parallel (v_\parallel - v_0)^2} \tag{1.96}$$

A change of variable $u^2 = \alpha_\parallel (v_\parallel - v_0)^2$ allows to center this integral and to have only the square of the integrant in the exponential. We introduce

$$\zeta = \frac{\alpha_\parallel^{1/2}}{k} (\omega - k v_0 \pm \Omega) \tag{1.97}$$

so the integral can be written

$$\mathcal{J} = \frac{\alpha_\parallel - \alpha_\perp}{\alpha_\parallel^{1/2}} \int_{\mathbb{R}} \frac{u}{u - \zeta} e^{-u^2}\, du + \frac{\alpha_\perp}{\alpha_\parallel^{1/2}} \int_{\mathbb{R}} \frac{1}{u - \zeta} e^{-u^2}\, du \tag{1.98}$$

As before, this form allows the Fried & Conte function to appear naturally. The integral $\mathcal{J}$ can then be rewritten

$$\mathcal{J} = \left[\frac{\pi}{\alpha_\parallel}\right]^{1/2} \frac{\alpha_\perp - \alpha_\parallel}{2} Z'(\zeta) - \left[\frac{\pi}{\alpha_\parallel}\right]^{1/2} \alpha_\perp \left(v_0 - \frac{\omega}{k}\right) Z(\zeta) \tag{1.99}$$

with the explicit form of $\zeta^\pm$

$$\zeta = \alpha_\parallel^{1/2} \left(\frac{\omega}{k} - v_0 \pm \frac{\Omega}{k}\right) \tag{1.100}$$

that can be written depending on the parallel thermal velocity

$$\zeta_s^\pm = \frac{1}{\sqrt{2} k v_{Ts\parallel}} (\omega - k v_{s0} \pm \Omega_s) \tag{1.101}$$

We can then write the dispersion relation of this mode gien by Eq. (1.87), with Eq. (1.88), (1.95) and (1.99)

$$1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{\omega_{Ps}^2}{\omega^2}\left[\frac{\alpha_\perp - \alpha_\parallel}{2\alpha_\perp} Z'(\zeta_s^\pm) + \alpha_\parallel^{1/2}\left(v_{s0} - \frac{\omega}{k}\right) Z(\zeta_s^\pm)\right] = 0 \tag{1.102}$$

so with the explicit iparallel and perpendicular $\alpha$ values

$$1 - \frac{k^2c^2}{\omega^2} - \sum_s \frac{\omega_{Ps}^2}{\omega^2}\left[\frac{T_\parallel - T_\perp}{2T_\parallel} Z'(\zeta_s^\pm) + \frac{1}{\sqrt{2}v_{Ts\parallel}}\left(v_{s0} - \frac{\omega}{k}\right) Z(\zeta_s^\pm)\right] = 0 \tag{1.103}$$

Eq. (1.103) is the parallel electromagnetic mode dispersion equation for a hot Maxwellian plasma. For an isotropic plasma ($T_\parallel = T_\perp$) it can be rewritten in the more classical form

$$\omega^2 - k^2c^2 + \sum_s \omega_{Ps}^2 \frac{\omega - kv_{s0}}{\sqrt{2}kv_{Ts\parallel}} Z(\zeta_s^\pm) = 0 \tag{1.104}$$

For a cold plasma, *i.e.* $v_T \to 0$, we then have $\zeta \to \infty$. We can then use the asymptotic expansion $Z(\zeta) = -1/\zeta$. The dispersion equation is then

$$\omega^2 - k^2c^2 - \sum_s \omega_{Ps}^2 \frac{\omega - kv_{s0}}{\omega - kv_{s0} \pm \Omega_s} = 0 \tag{1.105}$$

In space plasmas, the instabilities associated with Eq. (1.103) can be encountered in a 3 populations plasma : a dense population of protons with a quasi-zero drift speed called " core ", a more tenuous population of protons with a field-aligned drift speed, so-called " beam ", and a population of electrons which ensures neutrality. For the beam, we can commensurate the thermal speed $v_{Tb}$ to that of drift $v_{0b}$. If $v_{Tb} \ll v_{0b}$, the beam is cold, if $v_{Tb} \sim v_{0b}$, it is warm, and if $v_{Tb} \gg v_{0b}$ it is hot.

The Figure below, is taken from [Gary, 1993], and shows the 3 types of instabilities that can occur. From top to bottom, these are Right Resonant Mode, Non-Resonant Mode, and Left Resonant Mode.

**Right resonnant mode**. This is the mode which has the lowest instability threshold. It appears for an isotropic or weakly anisotropic cold beam. For $v_{0b} = 0$, we find the whistler mode, *i.e.* the high frequency part of the compressional alfvén mode. It is therefore right polarized, hence the first part of its name. This instability develops for $|\zeta_b \lesssim 1|$, which can only occur for the beam. So we have $\omega_r \simeq k_\parallel v_{0b} - \Omega_c$, which justifies the second part of its name. We then have $\omega_r > 0$, $k_\parallel > 0$ and $v_{0b} > 0$, so the mode propagates in the same direction as the beam.

The numerical solutions of this mode show that for $10^{-2} < n_b/n_e < 10^{-1}$, one has $\omega_r \simeq \gamma$ (at the maximum of the growth rate, *i.e.* where $\gamma$ is maximum).

**Exercize 2.** *From Eq. (1.103), show that when $\omega_r \simeq \gamma$, the growth rate of this mode is given by*

$$\gamma = \Omega_c \left(\frac{n_b}{2n_e}\right)^{1/3} \tag{1.106}$$

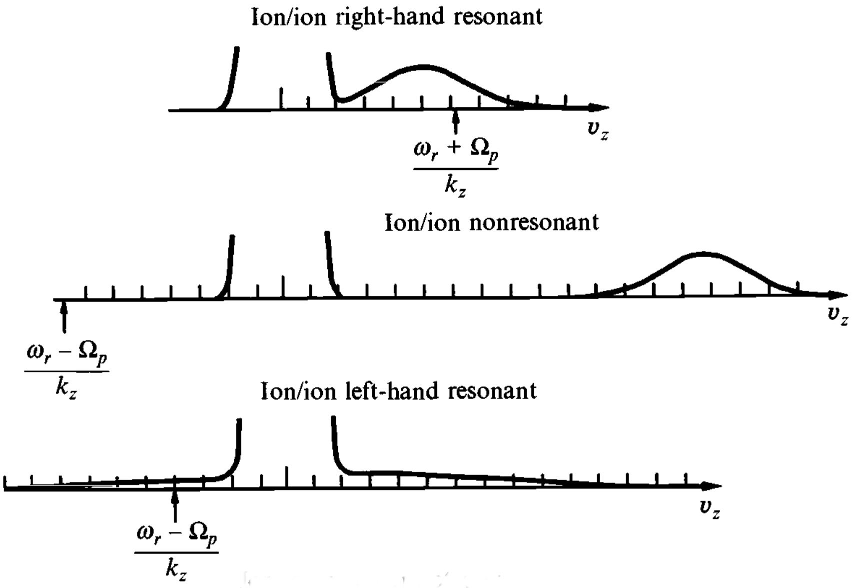


Figure 1.2: Schematic exhibiting the relation between $\omega_r$, $k_z$, and the component of the particle velocity along the DC field (see [Gary, 1993]).

**Left resonant mode**. This mode appears for a sufficiently hot beam, typically $T_b/T_c \sim 10^2$. When $v_{0b}$ approaches zero, this mode approaches AIC mode. It is therefore polarized to the left. As for the previous mode, $|\zeta_c \ll 1|$ and $|\zeta_e \ll 1|$. So only the protons of the beam can resonate ; A very hot beam is needed to have enough of them at $\omega_r \sim k_{\|}v_{0b} + \Omega_c$. So, its growth rate can be as large as the one of the right resonant mode. This mode also propagates in the direction of the beam.

**Non-resonnant mode**. This instability is similar to the firehose mode. For a fast and rather cold beam, we have $|\zeta_s^{-1}| \gg 1$ for the three $s$ species of the plasma. This mode is then non-resonant. But the second order moment associated with the two proton distributions has the same consequences as a high parallel temperature, which favors the firehose mode. Unlike the other 2 modes, this mode propagates in the opposite direction to the beam. In order for its growth rate to become comparable to the right resonant mode, the ratios $n_b/n_e$ and $v_{0b}/v_A$ must be sufficiently large.

## 1.4 The ion fore-schock

Around a magnetized planet plunged into the supersonic and / or superAlfvénic flow of the solar wind, a bow shock is created. A shock is the non-linear limit of a (solitary) wave getting infinitely stiff. One never reaches a profile (of magnetic field for example) whose derivative is infinite, because the dissipative effects often limit this gradient (this is the difference with a soliton for which the non-linearities are counterbalanced by the dispersive effects).

In the case of the Earth, the Mach number (sonic or Alfvenic) is of the order of 8 and the shock is around 15 Earth radii. Downstream of this shock, the plasma still has the characteristics of the solar wind (for its temperature and its density) but a smaller bulk flow velocity. As the plasma downstream of the shock is hotter and denser (this compressive nature shows that it cannot be slow or intermediate shock, so it is a fast shock) and its magnetic field is increased. The physics of collisionless shocks is complex. We can retain that the structure of a shock depends a lot on the angle $\Theta_{Bn}$ between the direction of the magnetic field and that of the normal $\widehat{\mathbf{n}}$ to this shock. One speaks of quasi-parallel shocks for $0 < \Theta_{Bn} < \pi/4$ and of quasi-perpendicular shocks for $\pi/4 < \Theta_{Bn} < \pi/2$. We will only talk about quasi-perpendicular shocks to simplify things.

If we look at the profile of the magnetic field when crossing a quasi-perpendicular shock with $M \gtrsim 3$ (super-critical shocks), we identify 3 characteristic regions : the foot, the ramp and the overshoot. The ramp has a characteristic thickness of around 20 km. But during a gyroperiod, the protons flowing at the speed of the solar wind travel 5000 km while the electrons travel 3 km. Note that being at $\beta \sim 1$, $d_s \sim \rho_s$. The protons therefore behave, when passing through the shock, like demagnetized particles, while the electrons perform several cyclotronic turns. Consequently, the accumulation of protons downstream of the shock (and therefore the “ missing ones ” upstream) creates an electrostatic electric field from downstream to upstream. This field therefore has the consequence of slowing down the protons and accelerating the electrons. In addition, in the frame of reference drifting at the speed of the solar wind, the shock is not stationary. During their reflection, the particles undergo a Fermi acceleration, which increases their perpendicular energy, especially as the solar wind is rapid.

On the other hand, this electrostatic electric field is in the direction normal to the shock. So for $\Theta_{Bn} \neq 0$, this implies a component of the parallel electric field which will be able to efficiently accelerate the reflected protons. With the Fermi process, this is another mechanism to produce a beam. There is a third one ; depending on the value of their gyrophase, the protons which interact with the shock can be trapped. These protons will be efficiently heated. During this trapping, they therefore produce a current, which is on the origin of the deformation of the magnetic field profile. This is the reason why a foot is created in the structure of the shock. Numerous numerical simulations like the one of [Biskamp and Welter, 1972] show that this foot grows and can form a new ramp, while the old ramp becomes evanescent. This double magnetic structure is visible in Fig. 1.3 taken from [Biskamp and Welter, 1972] (1-dimensional PIC simulations). Protons can be trapped there, which is the third mechanism for heating them.

The protons whose gyrophase allows trapping and heating constitute a population that is often called “ gyrating protons ” ; it is a spot of protons which will rotate in the perpendicular plane. When we observe such populations in the foot or the overshoot, we speak of supercritical shock. But the presence of this type of population is also intrinsically linked to the Mach number. Below an Alfvenic Mach number of the order of 2 or 3, the profile of the magnetic field is rather laminar, beyond this it becomes turbulent. This distinction is only valid for quasi-perpendicular shocks. For quasi-parallel shocks, the level of magnetic fluctuations is very high ($\delta B/B \sim 1$). The region of the pre-shock is therefore very extensive.

The reflected protons will therefore create a reflected population upstream of the shock, having a distribution of their perpendicular velocity collected around zero. We can consider that 10 to 20%

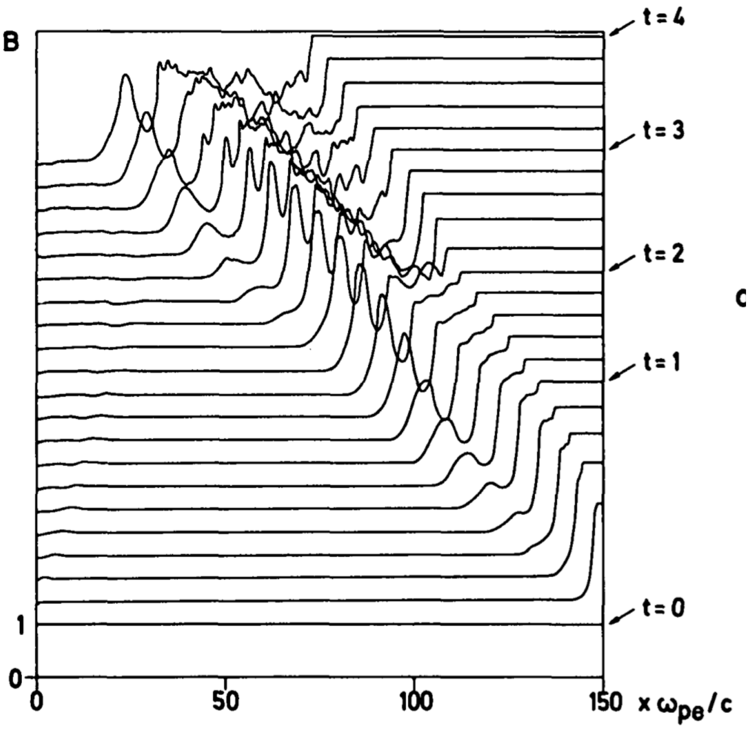


Figure 1.3: Time evolution of the magnetic field then exhibiting the shock reformation from the growth of the foot (see [Biskamp and Welter, 1972]).

of the protons of the solar wind are backscattered by the potential barrier of the shock. On the other hand, to see these protons at a certain distance from the shock, they must be able to get there because of their parallel speed. But the speed (in a perpendicular direction) of the solar wind competes to advect these protons as well as the field line on which they are frozen. This is called the “ time of flight ” effects. Thus, in Fig. **??** taken from [Cairns and Robinson, 1999], at the distance $R$ from the intersection of the field line with the shock, we can only find protons reflected at the depth $D_f$ if their parallel speed is such that

$$v_{\parallel} \geq \frac{R}{D_f} V_{SW} \tag{1.107}$$

where $V_{SW}$ is the solar wind speed.

The observations in the pre-shock show that the parameter $\beta_p$ (of the protons) is high, and that the density of the beam is low compared to that of the core, $n_b \sim 10^{-2} n_e$. As a result, the resonant upright mode is the most likely to grow.

Electromagnetic waves are of very low frequency, in the ULF range, with a period of 20s to 40s. They are often called the “ 30s waves ”. Sometimes observed in the form of “ shocklets ”, they are often almost monochromatic. In the satellite coordinate system, they are often observed as being left modes (see [Fairfield, 1969]). To see it, Fig. 1.5 represents a hodogram of the magnetic field in the plane $xy$, where the direction $z$ is that of the magnetic field. The anti-clockwise direction of rotation is the signature of a right-hand polarized mode. In addition, 90% of observations of this

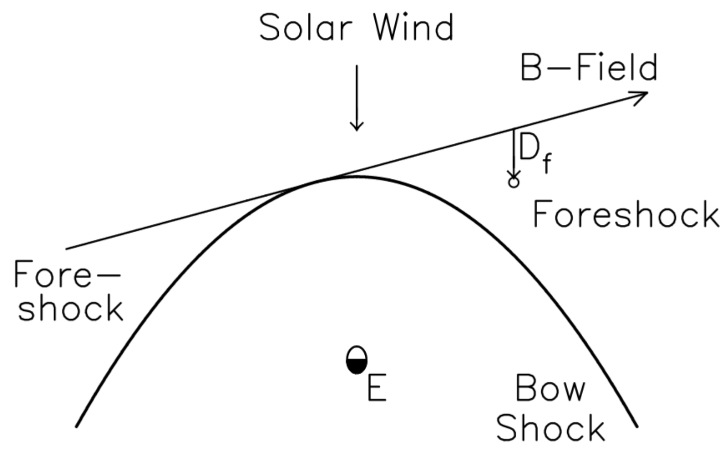


Figure 1.4: Représentation schématique de la position du pré-choc pour illustrer les effets de temps-de-vol. Voir [Cairns and Robinson, 1999].

type of mode are when the magnetic field is locally connected to the pre-shock, which reinforces the hypothesis of a non-local source of these modes.

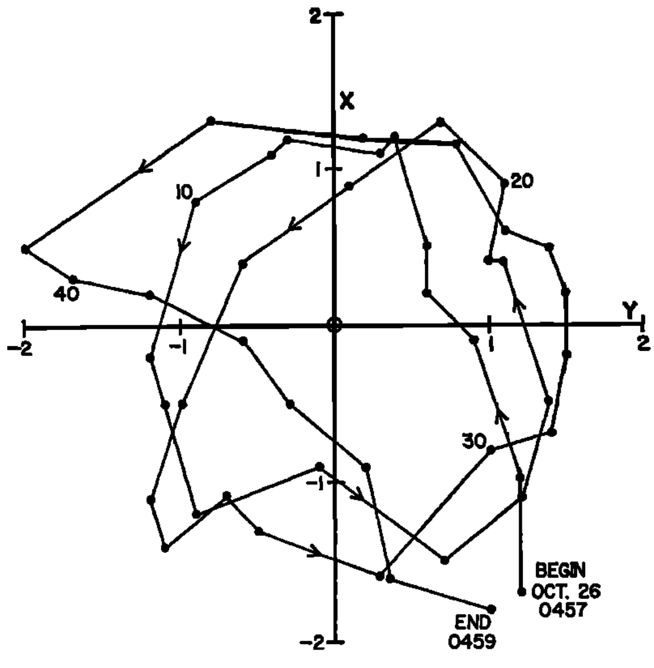


Figure 1.5: Hodograme du champ magnétique dans le plan $xy$. Voir [Fairfield, 1969].

In addition to the speed of the satellite, we must especially consider that of the solar wind. This speed is often significantly greater, as well as that of the satellite than that of the excited mode. As a result, the right mode moving “ slowly ” in the anti-solar direction, will be seen as a left mode moving in the direction of the solar wind, because the solar wind will knock it down quickly. This was shown in the study of [Hoppe et al., 1981]. Considering the Doppler effect, they show that these waves are right-hand polarized and propagate against the solar wind (see [Hoppe et al., 1981]).

# Bibliography


[Biskamp and Welter, 1972] Biskamp, D. and Welter, H. (1972). Structure of the Earth's bow shock. *J. Geophys. Res.*, 77(31):6052.

[Cairns and Robinson, 1999] Cairns, I. H. and Robinson, P. A. (1999). Strong evidence for stochastic growth of langmuir-like waves in earth's foreshock. *Physical Review Letters*, 82:3066–3069.

[Fairfield, 1969] Fairfield, D. H. (1969). Bow shock associated waves observed in the far upstream interplanetary medium. *J. Geophys. Res.*, 74:3541.

[Gary, 1993] Gary, S. P. (1993). *Theory of Space Plasma Microinstabilities.*

[Gary and Feldman, 1978] Gary, S. P. and Feldman, W. C. (1978). A second-order theory for k parallel b sub 0 electromagnetic instabilities. *Phys. Fluids*, 21:72–80.

[Ginzburg and Zhelezniakov, 1958] Ginzburg, V. L. and Zhelezniakov, V. V. (1958). On the possible mechanisms of sporadic solar radio emission (radiation in an isotropic plasma). *Soviet Ast.*, 2:653.

[Hoppe et al., 1981] Hoppe, M. M., Russell, C. T., Frank, L. A., Eastman, T. E., and Greenstadt, E. W. (1981). Upstream hydromagnetic waves and their association with backstreaming ion populations - isee 1 and 2 observations. *J. Geophys. Res.*, 86:4471–4492.

[Reid and Ratcliffe, 2014] Reid, H. A. S. and Ratcliffe, H. (2014). A review of solar type iii radio bursts. *Research in Astronomy and Astrophysics*, 14:773–804.

[Smith and Sime, 1979] Smith, D. F. and Sime, D. (1979). Origin of plasma-wave clumping in type III solar radio burst sources. *Ap. J.*, 233(3):998–1004.