Cap 2 → Sistemas Oscilatórios

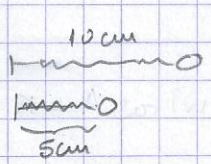
1) $m = 2\,kg$

$\Delta x = 10\,cm = 0,10\,m$

$F = -Kx$

$mg = -Kx \iff K = -196 \ kgN/m$

a) $w = \sqrt{\frac{K}{m}} = 9,9 \ rad/s$

b) 

$w = 2\pi f \iff 9,9 = 2\pi f \iff f = 1,58 \ Hz$

c) $T = \frac{1}{1,58} = 0,63\,s$

d) $A = 10\,cm - 5\,cm = 5\,cm$

e) $x = A\cos(wt + \delta)$ para $t = 0\,s$

$0,05 = 0,05\cos(\delta) \iff \cos(\delta) = 1 \iff \delta = 0 \ rad$

f) $x = 0,05\cos(wt)$

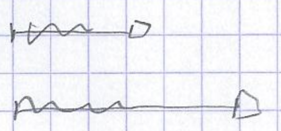
$v = -0,05w\,sen(wt)$ é máxima para

$sen(wt) = -1$

$wt = \frac{3\pi}{2} \iff t = 0,48\,s$

$V = 0,05 \times 9,9 = 0,495 \ m/s$

2) $m = 2\,kg$ $\Delta x = 10\,cm$ $K = 196$



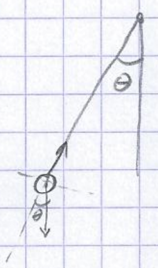
$x = 0{,}05 \cos(wt)$

$x = 0{,}10 \cos(wt)$

$$\left(\cos(wt) = 0 \Leftrightarrow wt = \frac{\pi}{2} \Leftrightarrow t = \frac{\pi}{2w} = \frac{\pi}{2 \cdot \frac{2\pi}{T}} \Leftrightarrow t = \frac{T}{4} \checkmark \right)$$

$x = 0 \Leftrightarrow \cos(wt) = 0$ em ambos $\Rightarrow$ chega ao mesmo tempo.

3)



$$w = \sqrt{\frac{g}{l}} \Leftrightarrow w = \sqrt{\frac{9{,}81}{1}} = 3{,}13\ rad/s$$

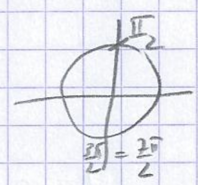
$$w = \frac{2\pi}{T} \Leftrightarrow T = \frac{2\pi}{3{,}13} \Leftrightarrow T = 2{,}007\,s$$

4) $x = 3\cos(5\pi t + \pi)$

a) $w = 5\pi\ rad/s$ $w = 2\pi f \Leftrightarrow 5\pi = 2\pi f \Leftrightarrow f = 2{,}5\ Hz$

$T = \frac{1}{f} = 0{,}4\,s$

b) $\cos(5\pi t + \pi) = 1 \Rightarrow x = 3\,m$



c) $t = 0$; $x = 3\cos\pi = -3\,m$

$t = 0{,}5$; $x = 3\cos(2{,}5\pi + \pi) = 3\cos\left(\frac{7}{2}\pi\right) = 3 \times 0 = 0\,m$

5) $x = 6\,cm$; $t = 0\,s \Rightarrow \delta = 0$; $T = 2\,s$ $w = \frac{2\pi}{t} = \pi\ rad/s$

$x = 0{,}06 \cos(\pi t)$

$v = -0{,}06\pi \operatorname{sen}(\pi t)$ $a = -0{,}06\pi^2 \cos(\pi t)$

(2) b) $x = 4 \operatorname{sen}(2t)$

a) $\operatorname{sen}(2t) = 1 \Rightarrow x = 4\,m$

b) $2t = \frac{\pi}{2} \Leftrightarrow t = \frac{\pi}{4}\,s$

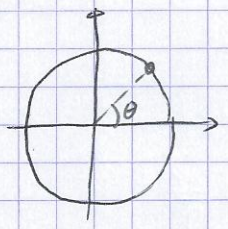
c) $v = +8 \cos(2t)$

d) $t = 0 \;;\; v = 8 \times 1 = 8\,m/s$

e) $a = -16 \operatorname{sen}(2t)$

para $t = 0 \Rightarrow a = -16 \times \operatorname{sen} 0 = 0\,m/s^2$

máxima quando $\operatorname{sen}(2t) = -1 \Rightarrow a = 16\,m/s^2$

7) 

$x = 0{,}4\,m$

$v = 0{,}8\,m/s$

a) $v = \omega R \Leftrightarrow \omega = 2\,rad/s$

b) $\omega = 2\pi f \Leftrightarrow 2\pi f = 2 \Leftrightarrow f = \frac{1}{\pi} \Leftrightarrow f = 0{,}318\,Hz$

$T = \frac{1}{f} = \pi = 3{,}14\,s$

c) $x = 0{,}4 \cos(2t + \delta)$ e $y = 0{,}4 \operatorname{sen}(2t + \delta)$

8) $m = 3\,kg \quad A = 0{,}10\,m \quad f = 2\,Hz \quad v = \omega R$

a) $\omega = \sqrt{\frac{K}{m}} \Leftrightarrow 2\pi f = \sqrt{\frac{K}{3}} \Leftrightarrow K = 3 \times (2\pi \times 2)^2 = 474\,N/m$

b) $E_M = \frac{1}{2} K A^2 = \frac{1}{2} \times 473{,}74 \times 0{,}10^2 = 2{,}37\,J$

$\omega = \sqrt{\frac{K}{m}} \Leftrightarrow K = m\omega^2 \Leftrightarrow K = 3 \times (2\pi \times 2)^2 = 473{,}74\,N/m$

c) $x = 0{,}10 \cos(4\pi t + \delta)$ NÃO!

9) $m = 100g$; $f = 20Hz$ $A = 0,005\ m$

a) $w = \sqrt{\frac{k}{m}} \iff k = mw^2$

$\iff k = 0,1 \times (2\pi \times 20)^2 = 1579\ N/m$

b) $x = 0,005 \cos(40\pi t + \delta)$

$v = -0,2\pi \sin(40\pi t + \delta)$

$a = -8\pi^2 \cos(40\pi t + \delta)$

$a_{max} = 8\pi^2 = 79\ m/s^2$

c) $E_m = \frac{1}{2} k A^2 = \frac{1}{2} \times 1579 \times 0,005^2 = 0,0197\ J$

10) $x = \frac{A}{2}$ $E_M = \frac{1}{2} k A^2$

- $E_M = E_{pe} + E_c$

$\frac{1}{2} k A^2 = \frac{1}{2} k \left(\frac{A}{2}\right)^2 + E_c \iff E_c = \frac{1}{2} k A^2 - \frac{1}{2} k \frac{A^2}{4}$ (4)

$E_c = \frac{4kA^2}{8} - \frac{kA^2}{8} \iff E_c = \frac{3kA^2}{8} \iff E_C = \frac{3}{4} \times \frac{1}{2} k A^2$

$E_C = \frac{3}{4} E_P$

- $E_p = E_c$

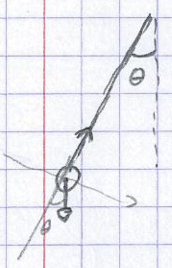
$A^2 = x^2 - A^2$

$x^2 = 2A^2$

$x = \pm \sqrt{2} A$

(3)

11) $T = 1{,}68\ s$ $\quad l = 70\ cm = 0{,}7\ m$



$T = 1{,}68 \Rightarrow w = 2\pi f \Leftrightarrow w = \frac{2\pi}{T} = 3{,}74\ rad/s$

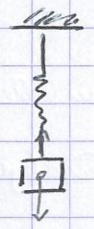
no pêndulo $\quad w = \sqrt{\frac{g}{l}} \Leftrightarrow g = l w^2$

$\Leftrightarrow g = 0{,}7 \times 3{,}74^2$

$\Leftrightarrow g = 9{,}79\ m/s^2$

12) $m = 2\ kg$

$K = 350\ N/m$



a) $y_0 = ?$ $\quad F = -K y_0$

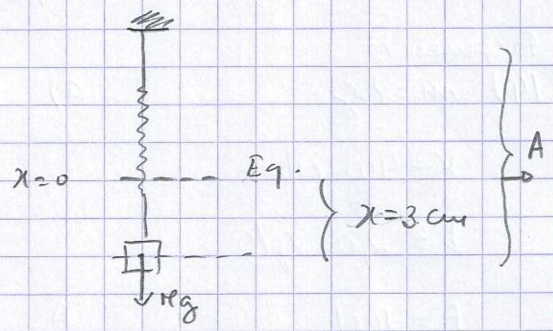
$Mg = -K y_0 \Leftrightarrow 2 \times 9{,}8 = -350\, y_0 \Leftrightarrow y = -0{,}056\ m$

R.: 0,056 m (5,6 cm)

$E_{Pe} = \frac{1}{2} K y^2 = \frac{1}{2} \times 350 \times 0{,}056^2 = 0{,}55\ J$

b) $y' = 3\ cm$ $\quad \Delta E_{Pe}$ $\quad \Delta E_P$

ΔE_g



$\Delta_{Pe} = E_{ef} - E_{ei}$ (0,056 + 0,03)

$= \frac{1}{2} \times 350 \times (0{,}086)^2 - \frac{1}{2} \times 350 \times 0{,}056^2$ (o alongamento)

$= 0{,}7455\ J$

$\Delta_P = E_{ef} - E_{ei}$

$= \frac{1}{2} K A^2 - 0$ (amplitude)

$= \frac{1}{2} \times 350 \times 0{,}03^2$

$= 0{,}1575\ J$

$\Delta_{Pg} = mgh_f - mgh_i$

$= 2 \times 9{,}8 \times (-0{,}03) - 0$

$= -0{,}588\ J$

c) $x = A \cos(wt + \delta)$ $\quad w = \sqrt{\frac{K}{m}} \Leftrightarrow 2\pi f = \sqrt{175} \Leftrightarrow f = 2{,}11\ Hz$;

$T = \frac{2\pi}{\sqrt{175}} = 0{,}475\ s$; $\quad A = 0{,}03\ m$

13) $m = 2\,kg$

$K = 400\ N/m$

$A_i = 0{,}03\,m$

a) $\frac{2\pi}{T} = \sqrt{\frac{K}{m}} \Leftrightarrow T = \frac{2\pi}{\sqrt{\frac{K}{m}}} \Leftrightarrow T = \frac{2\pi}{\sqrt{\frac{400}{2}}} \Leftrightarrow T = \frac{2\pi}{\sqrt{200}} = 0{,}44\,s$

$E_M = E_{Pe} + E_C = \frac{1}{2} K A^2 = \frac{1}{2} \times 400 \times 0{,}03^2 = 0{,}18\,J$

b) $E_{Mf} = 0{,}99 \times 0{,}18 = 0{,}1782\,J$ num período

$\frac{1}{2} K A^2 = 0{,}1782 \Leftrightarrow A = 0{,}02985\,m$ ou J

logo, $A_0 e^{-\frac{b}{2m}T} = A$

$0{,}03 \cdot e^{-\frac{b}{4}T} = 0{,}02985$

$-\frac{b}{4} \times 0{,}44 = \ln\left(\frac{0{,}02985}{0{,}03}\right)$

$b = 0{,}046\ kg/s$

Rascunho

14) $m = 2\,kg$

$K = 400\ N/m$

$b = 2{,}00\ kg/s$

$F = 10\,N$

$w_f = 10\ rad/s$

a) $F_{ext} = F_0 \cos(w_f t)$ — é máx para $\cos(w_f t) = 1$

$10\,N = F_0$

$A = \frac{\frac{10}{2}}{\sqrt{(10^2 - 200)^2 + \left(\frac{2 \times 10}{2}\right)^2}} = \frac{5}{\sqrt{10000 + 100}}$

$A = 0{,}0498\,m \quad (4{,}98\,cm)$

$w_0 = \sqrt{\frac{K}{m}} = \sqrt{\frac{400}{2}} = \sqrt{200} = 14{,}14\ rad/s$

b) $b = 0 \Rightarrow w_f = w_0 = 14{,}14\ rad/s$

c) $A = \frac{5}{\sqrt{0 + \left(\frac{2 \times w_f}{2}\right)^2}} = \frac{5}{w_f} = \frac{5}{14{,}14} = 0{,}354\,m$

$\downarrow$

$35{,}4\,cm$