(1) 3.3 → lei de Gauss

1) $\vec{E} = a\hat{x} + b\hat{y} + 0\hat{z} \Rightarrow \vec{E} = (a, b, 0)$

a) $\Phi = \int \vec{E} \cdot \vec{n} \, dA$  $\quad x = 0 \Rightarrow \vec{n} = (1, 0, 0)$

$= \vec{E} \cdot \vec{n} \cdot A$

$= (a, b, 0) \cdot (1, 0, 0) \cdot A = aA$

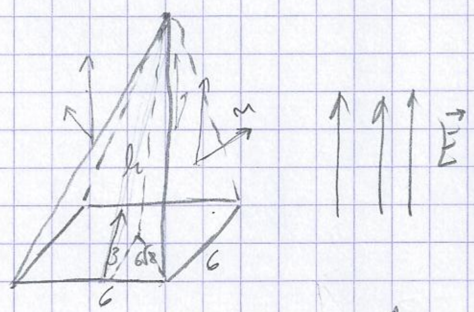
b) $y = 0 \quad \vec{n} = (0, 1, 0) \quad \Phi = bA$

c) $z = 0 \quad \vec{n} = (0, 0, 1) \quad \Phi = 0 \times A = 0$

2) $l = 6{,}00$ cm

$h = 4{,}00$ m

$\vec{E} = 52{,}0\hat{j}$




$A = \dfrac{b \times h'}{2} = \dfrac{6 \times 5}{2} = 15 \text{ cm}^2$

4 faces com o mesmo ângulo

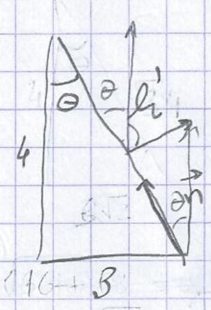
$\Phi = \int \vec{E} \cdot \vec{n} \, dA \quad \times 4$

$= 52 \cos(90° - \theta) \cdot 15 \quad \times 4$

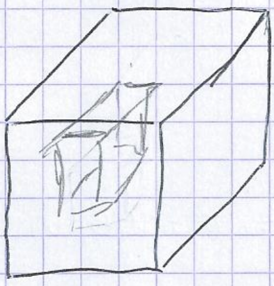
$= 52 \cdot \text{sen}\,\theta \cdot 15 \times 4$

$= 52 \cdot \dfrac{3}{5} \times 15 \times 4$

$= 1872$ N/C




$h' = \sqrt{16 + 9} = 5$

3) 

a) $\Phi = \int \vec{E} \cdot \vec{n} \, dA$

Pela lei de Gauss

$$\oint \vec{E}_n \cdot \vec{n} \, ds = \frac{Q}{\varepsilon_0} \qquad Q = 8q \text{ (carga no interior)}$$

logo $\Phi = \frac{8q}{\varepsilon_0}$

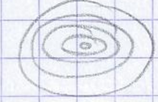
b) Há 6 faces logo $\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = \frac{8q}{\varepsilon_0}$

Como as cargas estão distribuídas uniformemente:

$$\Phi_i = \frac{8q}{6\varepsilon_0} = \frac{4q}{3\varepsilon_0} \quad , i \in \{1,2,3,4,5,6\}$$

c) Não!

4) $R, Q$ $\quad \Phi = \int \vec{E} \cdot \vec{n} \, dA$ $\quad$ A distribuição de carga tem simetria esférica.



a) $r=0$ $\quad E = 0$

b) $r = \frac{R}{2}$ $\quad$ interior $\Rightarrow$ lei de Gauss $\quad \Phi = \frac{q_{int}}{\varepsilon_0}$ $\quad$ $\vec{n}$ tem a direção de $\vec{E}_r$

$$q_{int} = \rho V(r) = \rho \times \frac{4}{3} \pi r^3$$

$$\oint E_r \, dA = E \oint dA = E \times 4\pi \left(\frac{R}{2}\right)^2 \qquad r = \frac{R}{2}$$

$$E \times 4\pi \left(\frac{R}{2}\right)^2 = \frac{\rho \times \frac{4}{3}\pi \times \left(\frac{R}{2}\right)^3}{\varepsilon_0} \Leftrightarrow$$

$$E = \frac{\rho R}{6 \varepsilon_0} = \frac{3QR}{6 \times \varepsilon_0 \times 4\pi R^3} = K_e \frac{Q}{8\pi \varepsilon_0 R^2}$$

(Nota: $K_e = \frac{1}{4\pi\varepsilon_0} \Leftrightarrow \varepsilon_0 = \frac{1}{4\pi K_e}$) $\qquad 4\pi E = \frac{Q}{\varepsilon}$

$$Q = \frac{4}{3}\pi R^3 \rho$$

$$\rho = \frac{3Q}{4\pi R^3}$$

(2) c) $r = R$

$$\oint E_R \, dA = E \oint dA$$
$$= E \times 4\pi R^2$$

$$E \times 4\pi R^2 = \frac{Q}{\varepsilon_0} \Leftrightarrow E = \frac{Q}{4\pi R^2 \varepsilon_0}$$

d) $r = 2R$

$$\oint E_R \, dA = E \oint dA = E \times 4\pi \times (2R)^2$$
$$= 16\pi R^2 E$$

$$16\pi R^2 E = \frac{Q}{\varepsilon_0} \Leftrightarrow E = \frac{Q}{16\pi \varepsilon_0 R^2}$$

(5) $R$, $Q$ $\quad \rho(r) = \alpha . r$

a) Não uniforme!

$V = \frac{4}{3}\pi R^3$

$dv = 4\pi r^2 dr$

$dq = \rho dv$

$q = \int \rho dv$

$$q = \int_0^R \alpha r \times 4\pi r^2 \, dr$$

$$q = \int_0^R \alpha . 4\pi . r^3 \, dr = 4\pi\alpha \left[ \frac{r^4}{4} \right]_0^R$$

$$q = \pi \alpha R^4 \Leftrightarrow \alpha = \frac{Q}{\pi R^4}$$

b) $r = 0 \Rightarrow E = 0$

$r = R/2$

$$\oint E_r \, dA = E_R \oint dA = E \times 4\pi \left(\frac{R}{2}\right)^2 = E \times \pi \times R^2$$

$$E \times \pi R^2 = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow E = \frac{\frac{Q}{16}}{\pi R^2 \varepsilon_0} \Leftrightarrow$$

$q = \pi . \alpha . \frac{R^4}{16}$

$$\Leftrightarrow E = \frac{Q}{16\pi R^2 \varepsilon_0}$$

$q = \frac{Q}{16}$

c) $r = R$ $\qquad q_{int} = \pi \alpha R^4 = Q$

$$\oint E\,dA = E \oint dA = E\, 4\pi R^2$$

$$E \times 4\pi R^2 = \frac{Q}{\varepsilon_0} \iff$$

$$\iff E = \frac{Q}{4\pi \varepsilon_0 R^2}$$

D) $r = 2R$ $\qquad q_{int} = \pi \alpha (2R)^4 = \pi \alpha \times 16 R^4 = 16 Q$

$$\oint E\,dA = E \oint dA = E \,.\, 4\pi (2R)^2$$

$$= E \,.\, 16 \pi R^2$$

$$E \,.\, \cancel{16}\pi R^2 = \frac{\cancel{16} Q}{\varepsilon_0} \iff E = \frac{Q}{\pi R^2 \varepsilon_0}$$

(6) $R$, $\rho$ uniforme $\qquad Q = \rho V = \rho \pi R^2 l$

a) $$\oint E\,dA = \frac{q}{\varepsilon_0}$$


$$E \oint dA = \frac{q}{\varepsilon_0} \qquad V = \pi R^2 l$$

$$E \times 2\pi R \times l = \frac{\rho V}{\varepsilon_0}$$

$$E \times 2\cancel{\pi} r = \frac{\rho \cancel{\pi} R^2}{\varepsilon_0}$$

$$E = \frac{\rho R^2}{2 \varepsilon_0 r} \quad \text{para } r > R$$

Para $r < R$ $\qquad E \times 2 \cancel{\pi} \cancel{r} = \frac{\rho \cancel{\pi} r^2}{\varepsilon_0}$ (carga interior)

$$E = \frac{\rho r}{2 \varepsilon_0} \quad \text{(comprimento no livro)}$$

b) Uniforme $\quad dq = \sigma\, dA$ $\qquad$ Tem que ser $q = Q$

$$q = \sigma \,.\, 2\pi R l \qquad \sigma = \frac{\rho \pi R^{\cancel{2}} \cancel{l}}{2\pi \cancel{R} \cancel{l}}$$

$$\sigma = \frac{q}{2\pi R l} \qquad \sigma = \frac{\rho R}{2} \;//$$

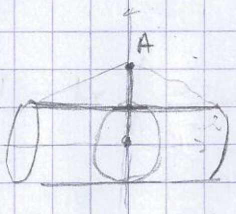
(3) 7) $R$ ; $\rho$ uniforme

a) $$\underbrace{\oint E\,dA}_{\Phi} = \frac{q_0}{\varepsilon_0}$$

$$q = \rho V$$
$$q = \rho\left(\pi R^2 h - \frac{4}{3}\pi R^3\right)$$

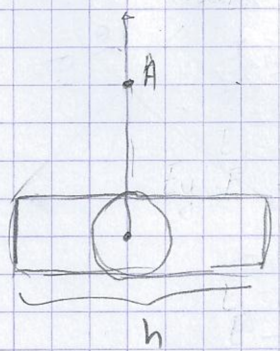
$$\Phi = \frac{\rho\left(\pi R^2 h - \frac{4}{3}\pi R^3\right)}{\varepsilon_0}$$

$$\Phi = \frac{\rho \pi R^2 (h - 4/3 R)}{\varepsilon_0}$$

b) 

Fluxo no cilindro

O campo é $\perp$ à superfície logo $E_x = 0$



$$E_y = K_e \int \frac{1}{r^2}\,dq \quad \text{uniforme}$$

$$= K_e \frac{1}{r^2} Q =$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{4}{9R^2} Q = \frac{Q}{9\pi\varepsilon_0 R^2}$$

$$E_y \cdot A = \Phi \iff \frac{\cancel{Q}}{9\cancel{\pi}\varepsilon_0 R^2} \times 2\cancel{\pi} \times \frac{3}{\cancel{2}} R L = \frac{\cancel{Q}}{\varepsilon_0}$$

$$\iff \frac{3L}{9R} = 1 \iff L = 3R$$

logo

$$Q = \rho\pi R^2\left(3R - \frac{4}{3}R\right) = \rho\pi R^3 \times \frac{5}{3} \quad (C)$$

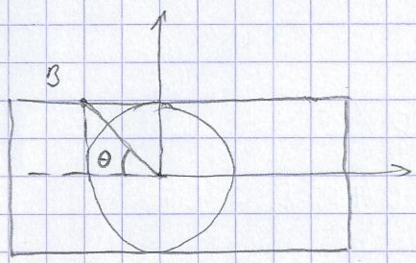
e, consequentemente,

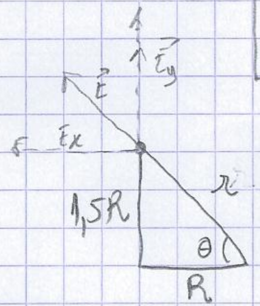
$$E_y = \frac{\rho \cancel{\pi} R^3 \times \frac{5}{3}}{9\cancel{\pi}\varepsilon_0 R^2} = \frac{5\rho R}{27\varepsilon_0}$$ //

Ponto B

$\boxed{L = 3R}$

$$Q = \frac{5}{3}\rho\pi R^3$$




O campo é igual em todos os ponto

logo $E = \sqrt{E_x^2 + E_y^2}$

$$\frac{5\rho R}{27\varepsilon_0} = \sqrt{E_x^2 + E_y^2}$$

$$r^2 = \frac{9}{4}R^2 + R^2$$

$$r^2 = \frac{13}{4}R^2$$

$$r = \frac{\sqrt{13}}{2}R$$

$$\cos\theta = \frac{R}{r} = \frac{2}{\sqrt{13}}$$

$$\operatorname{sen}\theta = \frac{3}{2}\frac{R}{r} = \frac{3}{\sqrt{13}}$$

$$\bar{E}_x = \frac{5\rho R}{27\varepsilon_0}\cos\theta$$

$$\bar{E}_x = \frac{5\rho R}{27\varepsilon_0} \times \frac{2}{\sqrt{13}}$$

$$\frac{5}{27} \times \sqrt{3}\ \frac{8}{39\sqrt{3}}$$

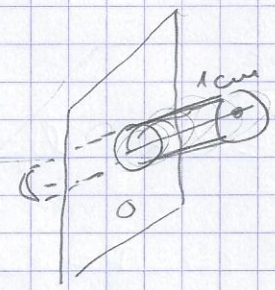
$$x = \frac{216}{195\sqrt{3}}$$

(4)

8) $l = 5{,}0\ cm$

$Q = 50{,}0\ mC = 50 \times 10^{-3} C$

a) $\boxed{\Phi_E = 2E \cdot A}$ duas faces   Uniforme



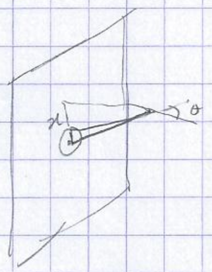

$$\frac{q_{int}}{\varepsilon_0} = 2 \times E \times A$$

uniforme $q = \sigma . A$

$$\frac{\sigma A}{\varepsilon_0} = 2E \times A$$

$$E = \frac{\sigma}{2\varepsilon_0}$$ não depende da distância

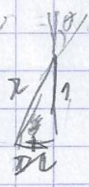
b)



$$E_{disco} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$ Disco

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\sqrt{2}\varepsilon_0}$$



$r^2 = x^2 + 1$

Pelo Princípio da sobreposição:

$E = E_{plano} - E_{disco}$

$$E = \frac{\sigma}{2\sqrt{2}\varepsilon_0}$$

Para $x = 50\ cm$

$$E_{disco} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} \times \frac{50}{\sqrt{50^2 + 1^2}}$$

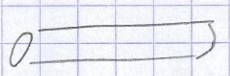
$$E = + \frac{\sigma}{2\varepsilon_0} \times 1 = \frac{\sigma}{2\varepsilon_0}$$

9) $E = 130\ N/C$

a) $EA = \frac{Q}{\varepsilon_0}$

ora $Q = \sigma A$ logo $EA = \frac{\sigma A}{\varepsilon_0}$

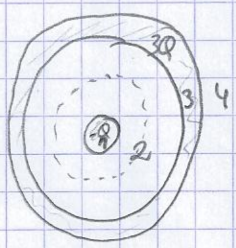
24. 44 $E = \frac{\sigma}{\varepsilon_0} \Leftrightarrow \sigma = \varepsilon_0 E = 8,85 \times 10^{-12} \times 130 = 1,15 \times 10^{-9} \frac{C}{m^2}$

10) 

$EA = \frac{Q}{\varepsilon_0} \Leftrightarrow E \cdot 2\pi R L = \frac{\lambda L}{\varepsilon_0}$

$\Leftrightarrow E = \frac{\lambda}{2\pi R \varepsilon_0}$, para $r > R$

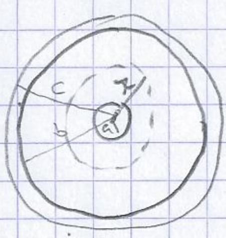
Para $r < R$ $E = 0$

11) 


① Dentro da esfera $E_1 = 0$ A esfera é sólida e está em equilíbrio

② Coroa "oca" $a \leq r \leq b$

$\boxed{q_{in} = -Q}$



$\oint E \cdot dA = \frac{-Q}{\varepsilon_0}$

$E \cdot 4\pi r^2 = -\frac{Q}{\varepsilon_0}$

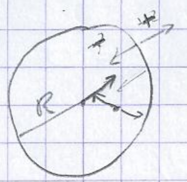
$E = -\frac{Q}{4\pi r^2 \varepsilon_0}$

③ Na coroa condutora $E_3 = 0$

④ Fora: $r > c$ $q = -Q + 3Q = 2Q$

logo, $E \cdot 4\pi r^2 = \frac{2Q}{\varepsilon_0} \Leftrightarrow E = \frac{Q}{2\pi \varepsilon r^2}$

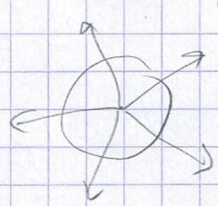
5

12) 

$q = \lambda l$

$Q = 2\lambda l$

a) $q_{int} = \frac{1}{2} Q = \lambda l$ ; $\Rightarrow -\lambda$ fora : $q + Q = 3\lambda l \Rightarrow 3\lambda$



b) $r < R$

$$\oint E\, dA = \frac{\lambda l}{\varepsilon_0}$$

$$E \cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0} \Leftrightarrow E = \frac{\lambda}{2\pi r \varepsilon_0}$$

$r > R$

$$\oint E\, dA = \frac{3\lambda l}{\varepsilon_0} \Leftrightarrow E \cdot 2\pi r l = \frac{3\lambda l}{\varepsilon_0}$$

$$\Leftrightarrow E = \frac{3\lambda}{2\pi \varepsilon_0 r}$$

13) $l = 50{,}0\,cm$

$Q = 4{,}0 \times 10^{-8}\, C$

a) $\sigma = \frac{1}{2}\left(\frac{Q}{A}\right)$ duas faces

$$\sigma = \frac{1}{2}\,\frac{4{,}0\times 10^{-8}}{0{,}5^2} = 8\times 10^{-8}\ C/m^2$$

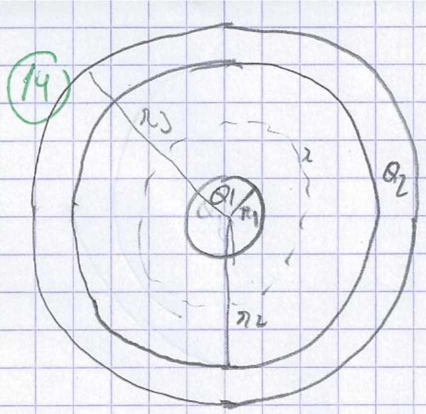
b) Acima da placa:

$$\oint E\, dA = \frac{q}{\varepsilon_0}$$

$$E \cdot A = \frac{\sigma A}{\varepsilon_0} \Leftrightarrow E = \frac{8\times 10^{-8}}{8{,}85\times 10^{-12}} = 9040\ N/C$$

Com direção ↑ $\vec{E} = 9040\,\hat{k}$

Abaixo: $\vec{E} = -9040\,\hat{k}$ ↓

14) 


a) ① Dentro da esfera $r < r_1$ $E_1 = 0$

② Coroa oca: $r_1 < r < r_2$

$$\oint E_2 \, dA = \frac{Q_1}{\varepsilon_0}$$

$$E_2 \cdot 4\pi r^2 = \frac{Q_1}{\varepsilon_0} \Leftrightarrow E_2 = \frac{Q_1}{4\pi \varepsilon_0 r^2}$$

③ Dentro da coroa: $r_2 < r < r_3$ $E_3 = 0$

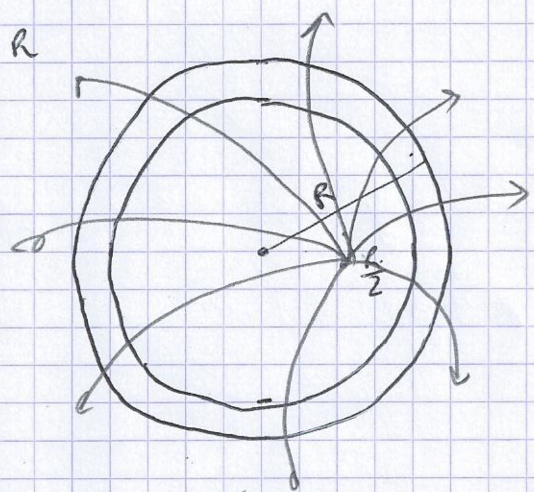
④ Fora: $r > r_3$

$$\oint E_4 \, dA = \frac{Q_1 + Q_2}{\varepsilon_0}$$

$$E_4 = \frac{Q_1 + Q_2}{4\pi \varepsilon_0 r^2}$$

b) Tudo igual menos $E_4 = 0$ (descarrega a carga exterior)

15) $\frac{R}{2}$ , $R$




As linhas do campo elétrico têm que ser perpendiculares ao condutor dentro e fora