Rochishnu Banerjee
banerjr2@miamioh.edu

Homework 1
CSE 532A: Machine Learning

09-17-2022
Fall '22

Part A

1. $\overrightarrow{a} = [1, 3, 2, -3] \qquad \overrightarrow{b} = [-2, 1, -1, 2]$
   $\overrightarrow{b} \cdot \overrightarrow{a} = (-2) \cdot 1 + 1 \cdot 3 + (-1) \cdot 2 + 2 \cdot (-3) = -2 + 3 - 2 - 6 = -7$

2. $\overrightarrow{b^T} \cdot \overrightarrow{a} = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 2 & -3 \end{bmatrix} = \begin{bmatrix} (-2) \cdot 1 & (-2) \cdot 3 & (-2) \cdot 2 & (-2) \cdot (-3) \\ 1 \cdot 1 & 1 \cdot 3 & 1 \cdot 2 & 1 \cdot (-3) \\ (-1) \cdot 1 & (-1) \cdot 3 & (-1) \cdot 2 & (-1) \cdot (-3) \\ 2 \cdot 1 & 2 \cdot 3 & 2 \cdot 2 & 2 \cdot (-3) \end{bmatrix}$
   $= \begin{bmatrix} -2 & -6 & -4 & 6 \\ 1 & 3 & 2 & -3 \\ -1 & -3 & -2 & 3 \\ 2 & 6 & 4 & -6 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \qquad \overrightarrow{a} = [1, 3]$
   $\overrightarrow{a} \cdot A = \begin{bmatrix} 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 3 \cdot 0 & 1 \cdot 1 + 3 \cdot 2 & 0 \cdot 1 + 3 \cdot (-1) \end{bmatrix} = \begin{bmatrix} 1 & 7 & -3 \end{bmatrix}$
   $A \cdot \overrightarrow{a} = (2 \times 3) \cdot (1 \times 2)$ cannot be multiplied since number of columns in A do not match number of rows in $\overrightarrow{a}$.

4. $\overrightarrow{a} = [1, 3, 1]$
   $\overrightarrow{a} \cdot A = (1 \times 3) \cdot (2 \times 3)$ cannot be multiplied since number of columns in $\overrightarrow{a}$ do not match number of rows in A.
   $A \cdot \overrightarrow{a^T} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 1 \cdot 3 + 0 \cdot 1 \\ 0 \cdot 1 + 2 \cdot 3 + (-1) \cdot 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 2 & -4 \end{bmatrix}$
   $A \cdot B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 \cdot 3 + 1 \cdot 5 + 0 \cdot 2 & 1 \cdot 2 + 1 \cdot 1 + 0 \cdot (-4) \\ 0 \cdot 3 + 2 \cdot 5 + (-1) \cdot 2 & 0 \cdot 2 + 2 \cdot 1 + (-1) \cdot (-4) \end{bmatrix}$
   $= \begin{bmatrix} 8 & 3 \\ 8 & 6 \end{bmatrix}$

$$B \cdot A = \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 + 2 \cdot 0 & 3 \cdot 1 + 2 \cdot 2 & 3 \cdot 0 + 2 \cdot (-1) \\ 5 \cdot 1 + 1 \cdot 0 & 5 \cdot 1 + 1 \cdot 2 & 5 \cdot 0 + 1 \cdot (-1) \\ 2 \cdot 1 + (-4) \cdot 0 & 2 \cdot 1 + (-4) \cdot 2 & 2 \cdot 0 + (-4) \cdot (-1) \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 7 & -2 \\ 5 & 7 & -1 \\ 2 & -6 & 4 \end{bmatrix}$$

6. $B = \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 2 & -4 \\ 2 & 1 \end{bmatrix}$

   $A \cdot B = (2 \times 3) \cdot (4 \times 2)$ cannot be multiplied since number of columns in A do not match number of rows in B.

   $$B \cdot A = \begin{bmatrix} 3 & 2 \\ 5 & 1 \\ 2 & -4 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 + 2 \cdot 0 & 3 \cdot 1 + 2 \cdot 2 & 3 \cdot 0 + 2 \cdot (-1) \\ 5 \cdot 1 + 1 \cdot 0 & 5 \cdot 1 + 1 \cdot 2 & 5 \cdot 0 + 1 \cdot (-1) \\ 2 \cdot 1 + (-4) \cdot 0 & 2 \cdot 1 + (-4) \cdot 2 & 2 \cdot 0 + (-4) \cdot (-1) \\ 2 \cdot 1 + 1 \cdot 0 & 2 \cdot 1 + 1 \cdot 2 & 2 \cdot 0 + 1 \cdot (-1) \end{bmatrix}$$
   $$= \begin{bmatrix} 3 & 7 & -2 \\ 5 & 7 & -1 \\ 2 & -6 & 4 \\ 2 & 4 & -1 \end{bmatrix}$$

Part B

$$y = \cos\left((3x^2 - 5x)^3\right) \tag{1}$$

To find the derivative of y, we first express Eq. 1 as a system of equations as follows:

$$\begin{aligned} y &= \cos u \\ u &= v^3 \\ v &= 3x^2 - 5x \end{aligned} \tag{2}$$

The derivates (with respect to substituted variable) of the system of equations in Eqs. 2 is as follows:

$$\begin{aligned} \frac{dy}{du} &= -\sin u \\ \frac{du}{dv} &= 3v^2 \\ \frac{dv}{dx} &= 6x - 5 \end{aligned} \tag{3}$$

Now, we can apply the chain rule and simplify using Eqs. 3:

$$\begin{aligned} y' &= \frac{dy}{dx} \\ &= \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx} \\ &= (-\sin u)(3v^2)(6x - 5) \end{aligned}$$

Finally, substituting the functions from Eqs. 2, we get the derivative of y with respect to x as:

$$y' = -3(3x^2 - 5x)^2(6x - 5)\sin\left((3x^2 - 5x)^3\right) \tag{4}$$