

1 Transforming Twists and Spatial Accelerations

$$\begin{aligned}\begin{pmatrix} \omega \\ \mathbf{v} \end{pmatrix}_B &= \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{R}\tilde{\mathbf{r}}^T & \mathbf{R} \end{pmatrix} \begin{pmatrix} \omega \\ \mathbf{v} \end{pmatrix}_A \\ &= \begin{pmatrix} \mathbf{R}\omega_A \\ \mathbf{R}\mathbf{v}_A - \mathbf{R}(\mathbf{r} \times \omega_A) \end{pmatrix}\end{aligned}$$

with

$$\tilde{\mathbf{r}} = \begin{pmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{pmatrix}$$

and

- \mathbf{r} — Translation from frame A to frame B
- \mathbf{R} — Rotation from frame A to frame B
- ω — Angular velocity
- \mathbf{v} — Linear velocity

2 Transforming Wrenches

$$\begin{aligned}\begin{pmatrix} \tau \\ \mathbf{f} \end{pmatrix}_B &= \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{R}\tilde{\mathbf{r}}^T & \mathbf{R} \end{pmatrix}^{-T} \begin{pmatrix} \tau \\ \mathbf{f} \end{pmatrix}_A \\ &= \begin{pmatrix} \mathbf{R} & \mathbf{R}\tilde{\mathbf{r}}^T \\ \mathbf{0} & \mathbf{R} \end{pmatrix} \begin{pmatrix} \tau \\ \mathbf{f} \end{pmatrix}_A \\ &= \begin{pmatrix} \mathbf{R}\tau_A - \mathbf{R}(\mathbf{r} \times \mathbf{f}_A) \\ \mathbf{R}\mathbf{f}_A \end{pmatrix}\end{aligned}$$

with

- τ — Torque
- \mathbf{f} — Force