Assignment 3
EE321

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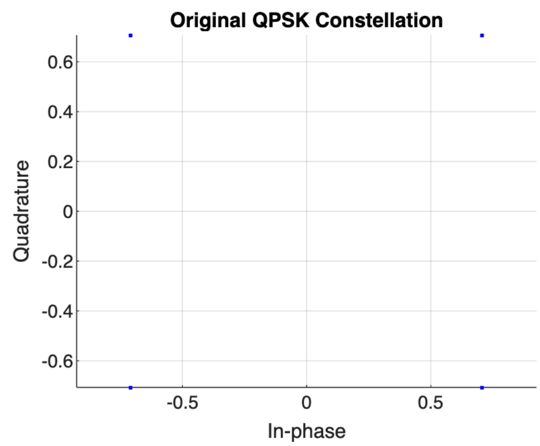
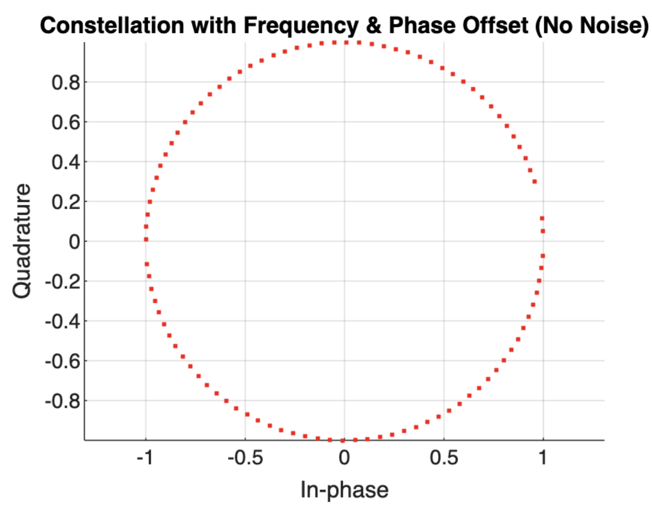
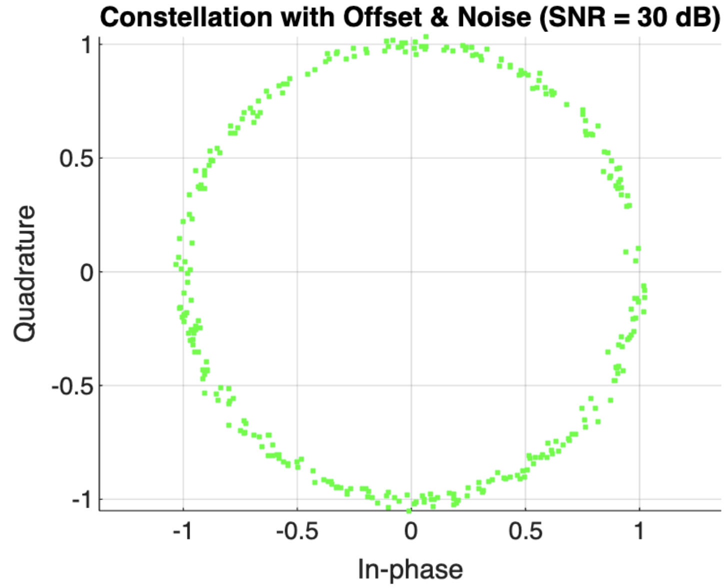
Q2) D part



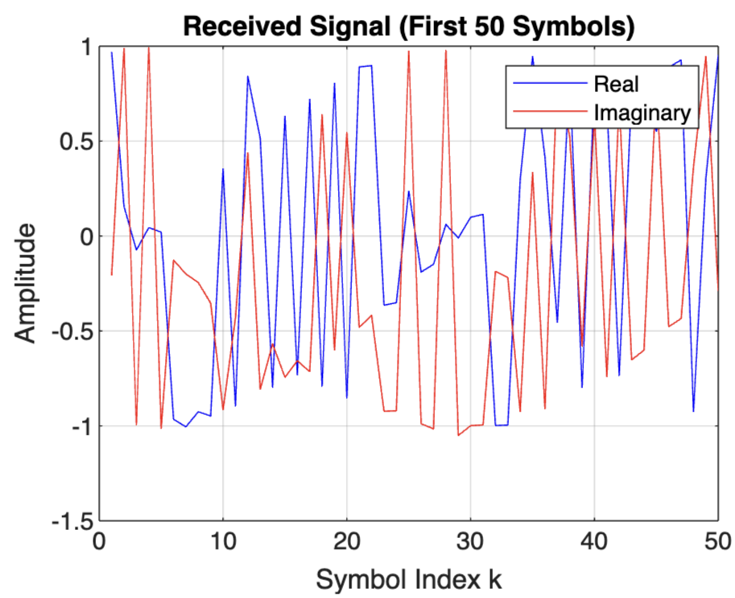
Four clearly defined points form the QPSK pattern
Reference point showing the four distinct QPSK symbols at (±1/√2, ±1/√2)



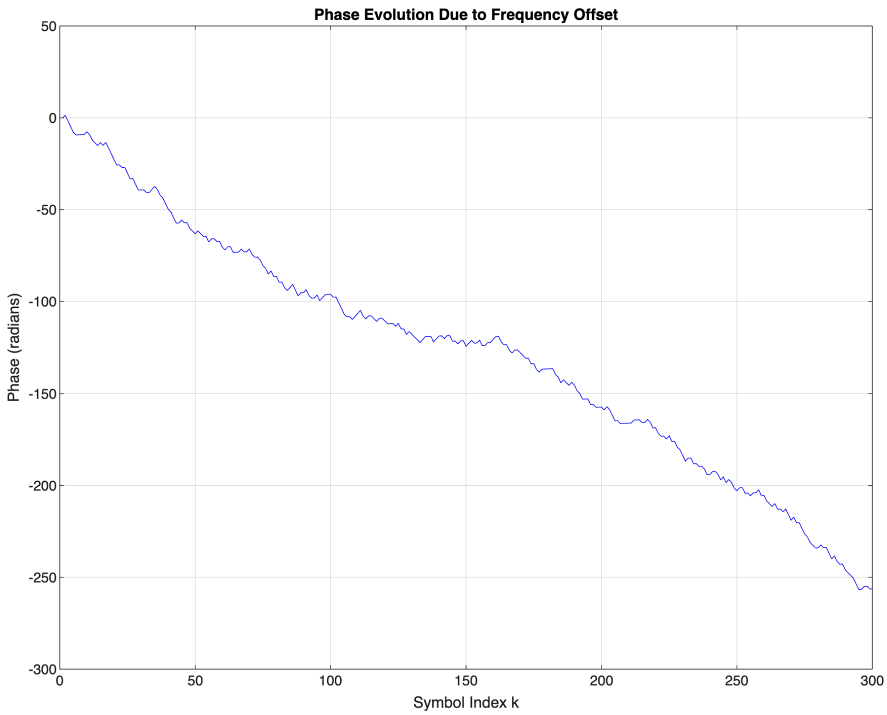
Circular pattern due to the continuous rotation caused by frequency offset
Frequency offset causes constellation points to trace a circle over time, while the phase offset determines the starting angle



Noise affects symbol detection and creates uncertainty in the received signal.



This plots the real and imaginary components of the received signal over time.
Frequency offset appears in the time domain as oscillations



A straight line with a negative slope equal to Γ (= 2πΔfT)

Q3) D part

Here, for estimating the values of Gamma and Theta, I have taken the reference from the book Mathow (screenshots attached below) and have followed the steps below:

DFT-Based Frequency Estimation:

(c) The structure of the cost function is that of maximizing the magnitude of a DFT coefficient. Specifically, if we wish to estimate Γ up to a resolution of $\frac{1}{N}$, we can take an N-point ($N \geq K$) DFT of the sequence $\{y[k]b^*[k]\}$ padded with $N - K$ zeros. The nth DFT coefficient is given by

$$Z[n] = \sum_{k=1}^{K} y[k]b^*[k]e^{-j2\pi nk/N} \ , \quad n = 0, 1, ..., N - 1$$

If the maximum of $\{|Z[n]|\}$ occurs at n_{max}, we obtain the estimate $\hat{\Gamma} = \frac{2\pi n_{max}}{N}$. Note that $\hat{\Gamma} \in [0, 2\pi)$. If the frequency offset $\Gamma = 2\pi\Delta fT$ is large enough to induce more than an offset of 2π per symbol interval, then we can only estimate it modulo 2π. This is a fundamental limitation of our observation model, since we are sampling at the symbol rate.

- We calculate `Z(Γ) = ∑ y[k]b*[k]e^(-jΓk)` for various values of Γ
- This is efficiently done using the FFT algorithm
- The frequency that maximises |Z(Γ)| is our estimate of Γ

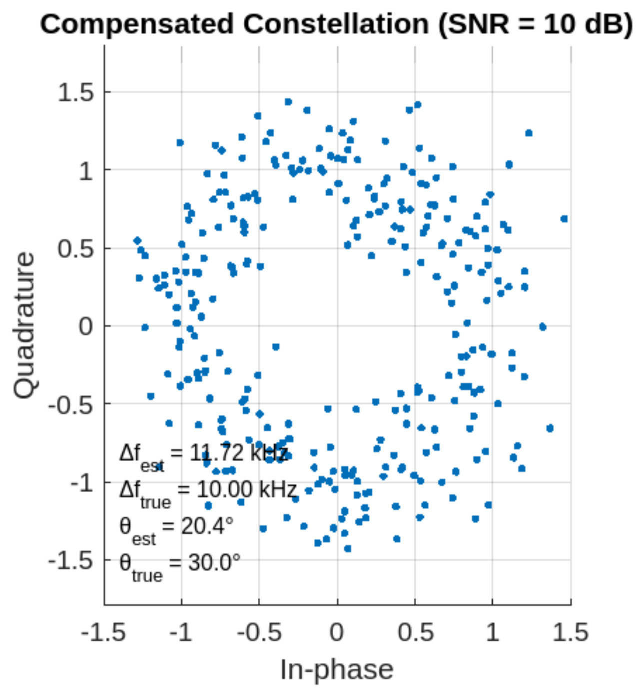
Phase Estimation:

$$\theta = \arg(Z(\Gamma))$$

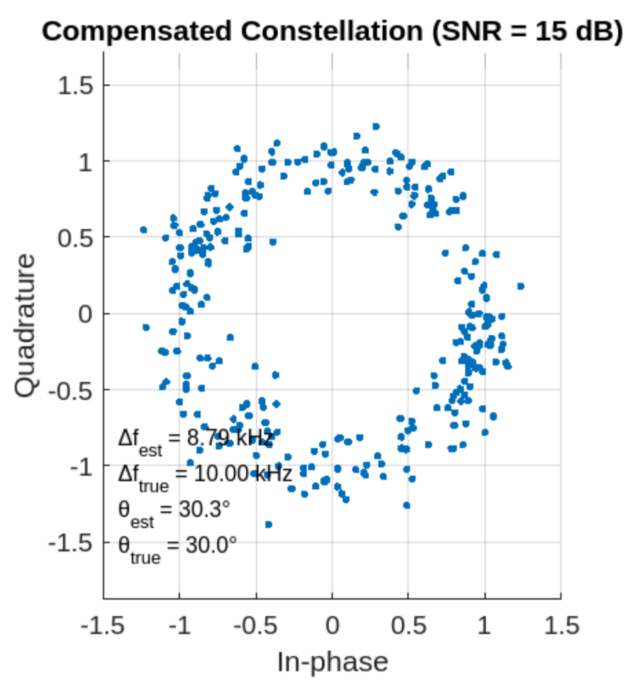
- Once we have Γ, we calculate `Z(Γ)`
- The angle of Z(Γ) gives us our estimate of θ

Compensation:

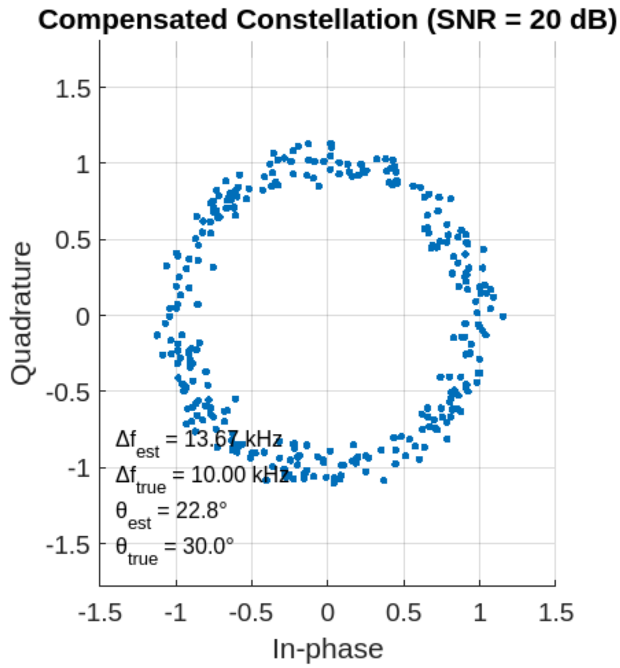
- We remove the estimated frequency and phase offsets by multiplying.
- `y_compensated = y[k]e^(-j(Γk+θ))`



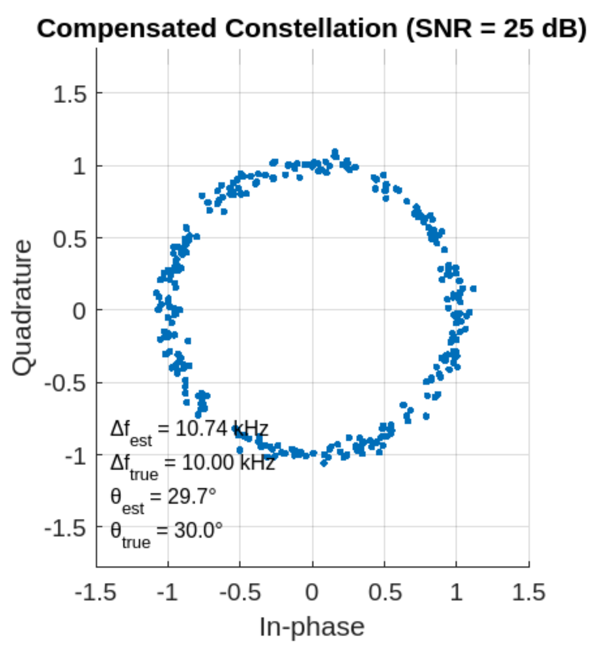
- Heavy noise spreads and overlaps the symbols in the constellation
- Much higher probability of bit errors due to noise and residual offset



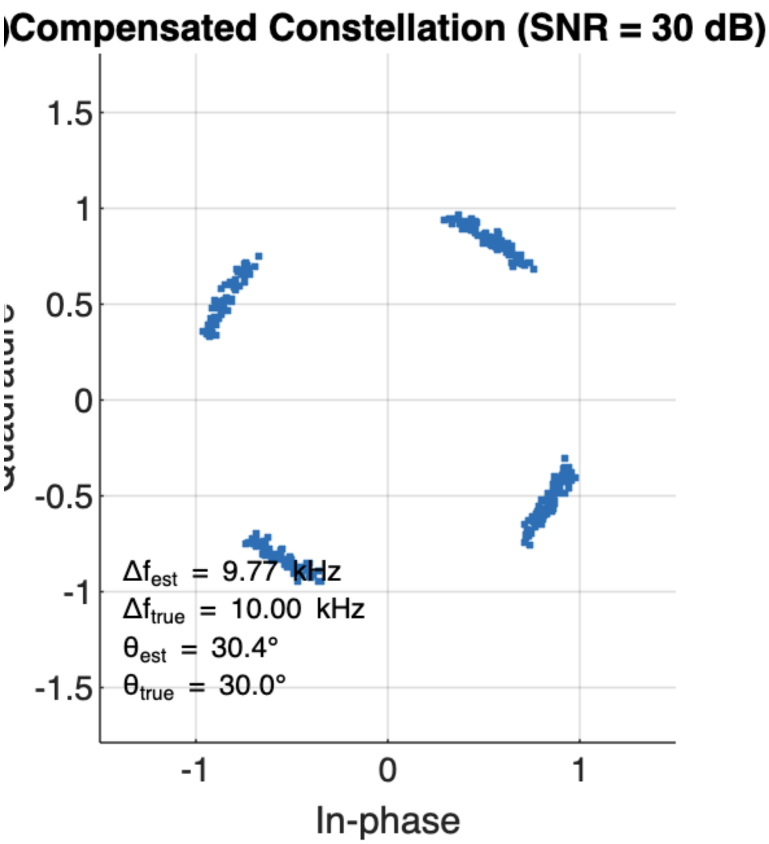
- Moderate estimation errors in both frequency and phase
- Some symbols fall near decision boundaries, increasing the probability of error



- Points in the constellation are noticeably more spread out
- Frequency and phase estimates are still good but show some error



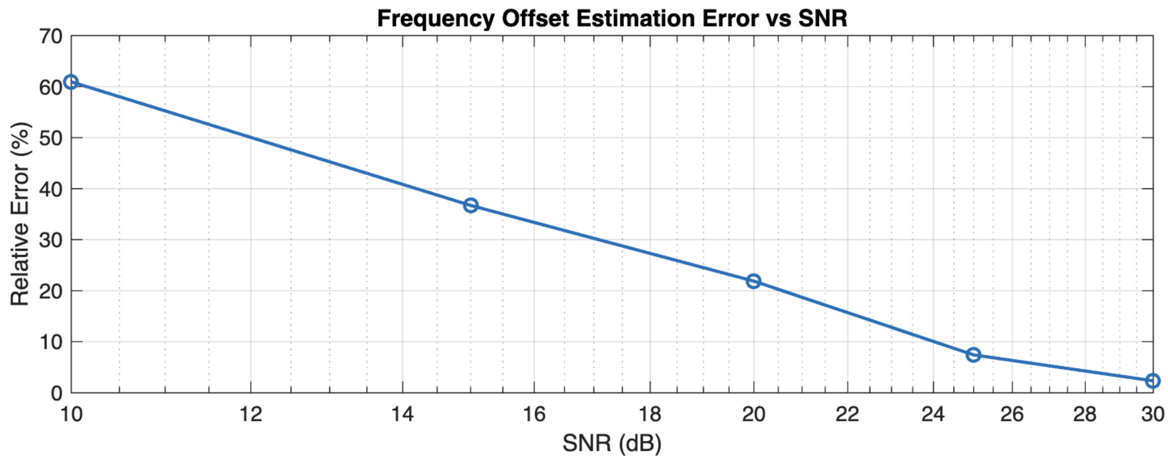
- Slightly more spread in the constellation points
- Frequency and phase estimates are quite accurate

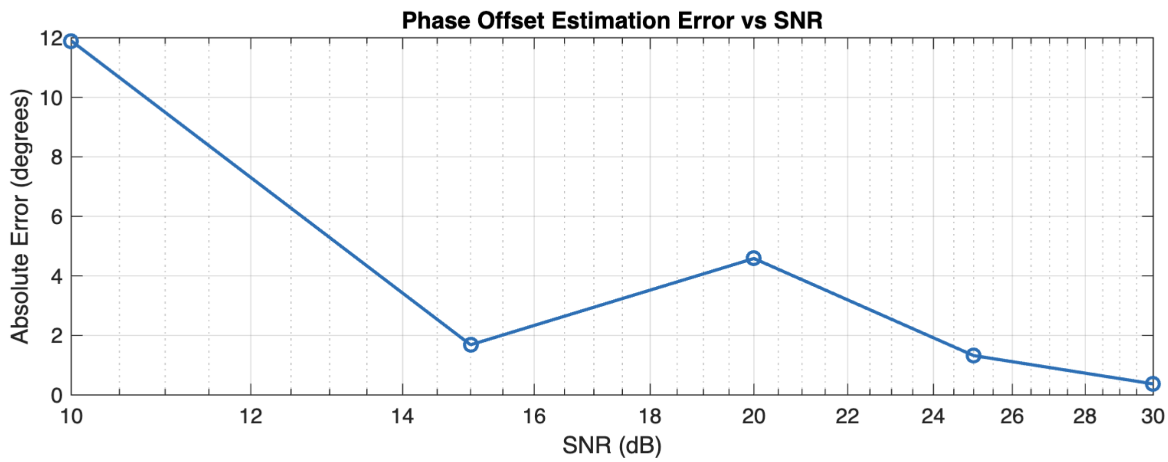


- The constellation shows four distinct clusters perfectly aligned with the original QPSK point.
- Frequency and phase estimates are extremely accurate
- The constellation points have very tight clustering around the ideal values
- Negligible estimation error in both frequency and phase

Conclusion

- Compensation almost perfectly recovers the original constellation at high SNR (>20 dB).
- As SNR decreases, frequency/phase errors combine with noise to degrade performance.
- Using just 8 pilot symbols is sufficient for accurate estimation at high SNR
- For reliable communication with this system, SNR should be at least 15-20 dB
- Below 15 dB, both estimation errors and noise significantly impact performance





The plot above shows how, with the increased SnR value, our absolute error decreases for both Theta and Gamma.

Q3) E part

SNR (dB)	Est. Δf (Hz)	True Δf (Hz)	Est. θ (deg)	True θ (deg)
10	11718.75	10000.00	20.38	30.00
15	8789.06	10000.00	30.31	30.00
20	13671.88	10000.00	22.84	30.00
25	10742.19	10000.00	29.68	30.00
30	9779.49	10000.00	30.4	30.00